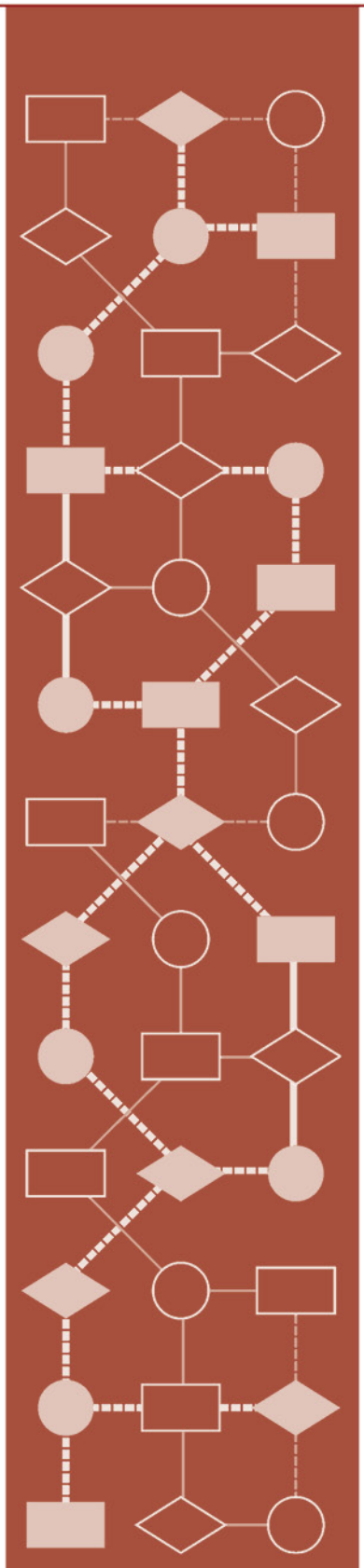
PROBLEM SOLVING



Mathematics Assessment Project

CLASSROOM CHALLENGES

A Formative Assessment Lesson

Optimizing: Security Cameras

Mathematics Assessment Resource Service
University of Nottingham & UC Berkeley
Beta Version

Optimizing: *Security Cameras*

MATHEMATICAL GOALS

This lesson unit is intended to help you assess how well students are able to:

- Analyze a realistic situation mathematically.
- Construct sight lines to decide which areas of a room are visible or hidden from a camera.
- Find and compare areas of triangles and quadrilaterals.
- Calculate and compare percentages and/or fractions of areas.

COMMON CORE STATE STANDARDS

This lesson relates to the following *Mathematical Practices* in the *Common Core State Standards for Mathematics*:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.

This lesson gives students the opportunity to apply their knowledge of the following *Standards for Mathematical Content* in the *Common Core State Standards for Mathematics*:

6-RP: Understand ratio concepts and use ratio reasoning to solve problems.

6-G: Solve real-world and mathematical problems involving area, surface area, and volume.

INTRODUCTION

The lesson unit is structured in the following way:

- Before the lesson, students attempt the *Security Cameras* task individually. You review their responses, and formulate questions for students to consider in order to improve their work.
- At the start of the lesson, students think individually about their responses to the questions set.
- Next, students work in small groups to combine their thinking and work together to produce a collaborative solution to the *Security Cameras* task, in the form of a poster.
- In the same small groups, students evaluate and comment on sample responses, identifying the strengths and weaknesses in these responses and comparing them with their own work.
- In a whole-class discussion students compare and evaluate the methods they have seen and used.
- In a follow-up lesson, students spend ten minutes reflecting on their work and what they have learned.

MATERIALS REQUIRED

- Each individual student will need a copy of the assessment task: *Security Cameras,* a mini-whiteboard, a pen, and an eraser. Provide rules if requested.
- Each small group of students will need at least two copies of the *Plan View,* a sheet of poster paper, felt tipped pens, a glue stick, and copies of *Sample Responses to Discuss*.

There is a projector resource to help to introduce activities and support whole-class discussions.

TIME NEEDED

20 minutes before the lesson, a 70-minute lesson and 10 minutes in a follow-up lesson (or for homework). Timings given are only approximate and will depend on the needs of the class.

BEFORE THE LESSON

Assessment task: *Security Cameras* (20 minutes)

Have the students complete this task, in class or for homework, a few days before the formative assessment lesson. This will give you an opportunity to assess the work, and identify the kinds of difficulties students have with it. You should then be able to target your help more effectively in the follow-up lesson.

Give each student a copy of the assessment task *Security Cameras*. Provide rules if requested.

Introduce the task briefly, helping the class to understand the problem and its context.

Have you ever seen a security camera in a shop? What did it look like?

Some may not look like cameras at all, but rather like small hemispheres. They may be fixed but swivel around. The camera in this problem can turn right round through 360°.

The diagram shows a plan view of the shop. This means that we are looking down on the shop from above.

The little circles represent people standing in the shop.

Now explain what you are asking students to do.

Spend twenty minutes on your own answering the questions.

Don't worry if you can't do everything. There will be a lesson on this material [tomorrow] that will help you to improve your work.

Your goal is to be able to answer these questions with confidence by the end of that lesson.

Security Cameras

A shop owner wants to prevent shoplifting.

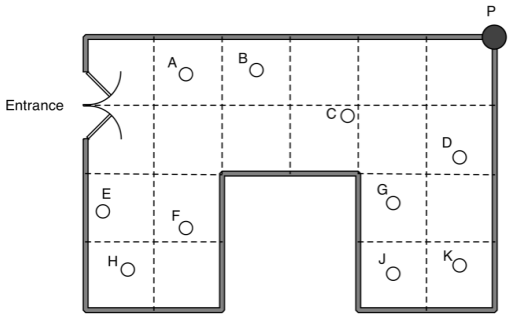
He decides to install a security camera on the ceiling of his shop.

The camera can turn right round through 360° in all directions.

The shop owner places the camera at point P, in the corner of the shop.

The plan view below shows where ten people are standing in the shop.

Plan view of the shop:



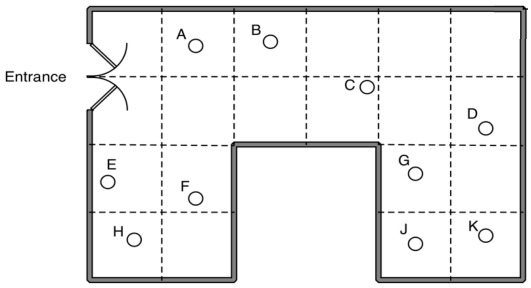
1. Which people in the shop cannot be seen by the camera at P?
   Explain your answer, showing clearly on the diagram how you know.

2. The shop owner says, "15% of the shop is hidden from the camera."
   Show clearly that he is correct.

Security Cameras (continued)

3. The shop owner decides to move the camera so that it can see as much of the shop as possible.

   a. On the plan view below show the best place for the camera so that it can see as much of the shop as possible.



   b. Explain how you know that this is the best place for the camera. What percentage of the shop cannot be seen with the camera in this new position?

It is important that, as far as possible, students are allowed to answer the questions without your assistance.

Students who sit together often produce similar answers, and then when they come to compare their work, they have little to discuss. For this reason, we suggest that when students do the task individually, you ask them to move to different seats. At the beginning of the formative assessment

lesson, allow them to return to their usual seats. Experience has shown that this produces more profitable discussions.

Assessing students' responses

Collect students' responses to the task. Make some notes on what their work reveals about their current levels of understanding and their different problem-solving approaches.

We suggest that you do not score students' work. The research shows that this will be counterproductive, as it will encourage students to compare their scores and distract their attention from what they can do to improve their mathematics.

Instead, help students to make further progress by summarizing their difficulties as a series of questions. Some suggestions for these are given in the *Common issues* table below. These have been drawn from common difficulties observed in trials of this unit.

We suggest you make a list of your own questions, based on your students' work. We recommend you either:

- Write one or two questions on each student's work, or
- Give each student a printed version of your list of questions, and highlight the questions for each individual student.

If you do not have time to do this, you could select a few questions that will be of help to the majority of students, and write these on the board when you return the work to the students.

Common issues	Suggested questions and prompts
Incorrectly identifies people which cannot be seen by the camera at P For example: The student thinks that person E cannot be seen.	• How do you know which parts of the shop cannot be seen by the camera? Can you shade this area?
Considers full squares only For example: The student thinks that the full square containing person F/H cannot be seen.	• Which 'full' squares cannot be seen by the camera? Which 'part' squares cannot be seen? Show on the diagram exactly where the camera can/not see.
Considers people rather than area (Q.2 & Q.3) E.g. Draws sight lines to the people rather than considering places in the shop that cannot be seen	• What does '15% of the shop is hidden' mean? • You have identified which people can / cannot be seen by the camera. Can you show which parts of the shop cannot be seen?
Does not draw sight lines or show evidence of any other suitable method	• How could you show on the diagram which parts of the shop cannot be seen?
Considers the problem in 2D rather than 3D For example: The student thinks people may be hidden from the camera by others.	• Will the height the camera affect what it can see? Why? • If a person is immediately behind person C will that person be hidden from the camera? Why?

Common issues	Suggested questions and prompts
Incomplete justification For example: The student states that three squares cannot be seen when the camera is at P, but does not justify this as area rather than whole squares. Or: The student provides an incomplete explanation of why the shop owner is correct. Or: The student identifies a new position for the camera without evidence of investigating other positions or calculating the percentage/fraction of the shop that cannot be seen.	• Can you show which three squares are hidden from the camera at P? • Explain your thinking about the 15%? What mathematical calculations are involved in showing this? • How can you be sure you have found the best place for the camera? Are there any other places that are equally as good? • What percentage of the shop cannot be seen? • What would happen if you moved the camera along the back wall? How would the percentage of the shop unseen vary?
Identifies a new position for the camera, resulting in less coverage of the shop than at P For example: The student states that the best place for the camera is in the center of the shop.	• Can you shade in the parts of the shop that cannot be seen from the center of the room? What percentage is this?
Makes incorrect assumptions For example: The student thinks that the camera cannot see underneath itself and includes the square directly under the camera as hidden.	• The camera can turn right round through 360° in all directions. What does this tell you about the parts of the shop that the camera can see?
Completes the task	• There are several places that the camera might be placed that are as good as the one you have found. Try to find all the solutions. Can you convince me that these are all possible solutions? Can you explain why they all give the same coverage of the shop?

SUGGESTED LESSON OUTLINE

Reviewing individual solutions to the problem (10 minutes)

Give each student a mini-whiteboard, a pen, and an eraser and return their work on the *Security Cameras* task.

If you have not added questions to individual pieces of student work, either give each student a printed version of your list of questions with the questions that relate to their work highlighted or write your list of questions on the board so that students can select questions that are appropriate to their own work.

Recall what we were working on previously. What was the task about?
In the diagram, what do the circles represent? What do the squares represent?

I have had a look at your work and have some questions I would like you to think about.

On your own, carefully read through the questions I have written.
I would like you to use the questions to help you to think about ways of improving your work.

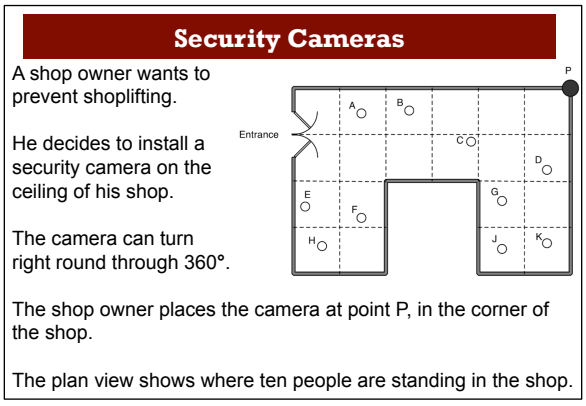
Use your mini-whiteboards to make a note of anything you think will help to improve your work.

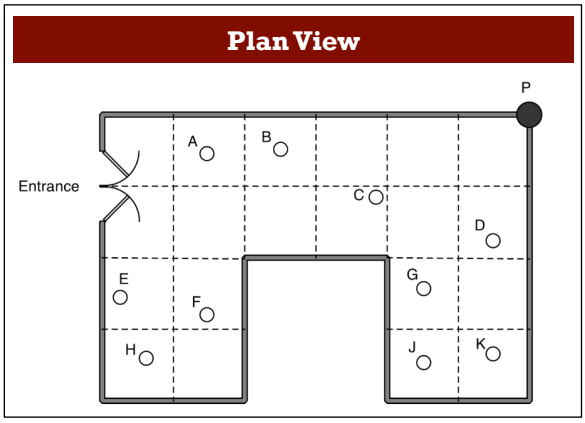
If mini-whiteboards are not available, students may want to use the back of their assessment task to jot down their ideas about ways to improve their work.

This is an opportunity for students to review their own work before working collaboratively on producing a group solution.

Whilst students are reviewing their work, it may be appropriate to ask individual students questions that help them to clarify their thinking. However, at this stage there is no need to address all student misunderstandings. They will benefit from the opportunity to overcome these misunderstandings for themselves during the collaborative activity.

You may want to show the class Slide P-1 or Slide P-2 of the projector resource.





Collaborative small-group work: making posters (20 minutes)

Organize the class into groups of two or three students. Give each group a large sheet of paper, and some felt-tipped pens.

Today you are going to work together on the Security Cameras task to produce a joint solution that is better than your individual work.

Deciding on a Strategy

Ask students to share their ideas about the task, and plan a joint solution.

You each have your own, individual solution and have been thinking about how you might improve this. I want you to share your work with your group and your ideas for improving it.

Listen carefully to each other and ask questions if you don't understand or agree.

Once students have evaluated the relative merits of each approach, ask them to agree on a strategy and write an outline of what they plan to do on their large sheet of paper.

Slide P-3 of the projector resource, *Planning a Joint Method*, summarizes these instructions.

Implementing the Strategy

Once students have had chance to discuss their work, hand out two copies of the *Plan View* and a glue stick to each group of students. Provide rules if students request them.

Display Slide P-4 of the projector resource:

Security Cameras

1. Which people cannot be seen by the camera at P? Explain your answer, showing clearly on the diagram how you know.
2. The shop owner says, "15% of the shop is hidden from the camera." Show clearly that he is correct.
3. The shop owner decides to move the camera so that it can see as much of the shop as possible.
   a. Show on the plan view the best place for the camera, so that it can see as much of the shop as possible.
   b. Explain how you know that this is the best place for the camera.

Having discussed the work you have done individually, in your group, agree together on the best method for completing the problem and produce a poster that shows a joint solution to the Security Cameras task that is better than your individual work.

You have two plan view diagrams. Use one to show the coverage of the shop when the camera is at P and the other one for the new position of the camera.

Stick the Plan View diagrams on your poster and use felt tipped pens to write your explanations and reasons for your choice of camera position.

Make sure that you answer all the task questions clearly and include on your poster a list of any assumptions you have made.

Again check that students understand what they are being asked to do by asking someone to explain the task to the rest of the class. In particular, students may not understand the term 'assumption'.

Have extra copies of the *Plan View* available for students should they request them.

While students are working in small groups you have two tasks: to note different student approaches to the task, and to support student problem solving.

Note different student approaches

In particular, note whether students' original methods are the same or different. If they are different, how do they decide which method to use for their joint solution? What are their reasons for the choice of method? Some students may not take part squares into consideration. If they do, how do they find the total number of squares? Do students use math to justify their answers? Are there any common mistakes being made? Are students aware of any assumptions they have made? How do they show that they have found the optimal place for the camera? Do they consider more than one possible position?

Support student problem solving

If students are struggling to produce a joint solution to the task, encourage them to identify the strengths and weaknesses of the methods employed in their individual responses. Can any of these methods be improved to produce a group solution that is better than the original individual response? Can they think of any other approaches to try?

Try not to make suggestions that move students towards a particular approach to the task. Instead, ask questions that help students to clarify their thinking. For example, you may ask them to consider these questions:

What have you done that you both [all] agree on?

What else do you need to find?

Have you used all the information given in the task?

What do you now know that you didn't know before?

Why do you think this is the best position for the camera? Write an explanation on the diagram.

Do your calculations make sense?

What assumptions have you made? Do you think they are reasonable?

How do you know for sure you have found the best place for the camera?
Are there any other positions as good as the one you already have?

You may also want to use some of the questions in the *Common issues* table to support your questioning. The purpose of these questions is to help students to track and review their problem solving strategies. They should be encouraged to give reasons for the choices they have made.

Sharing different approaches (10 minutes)

Once groups have completed their posters, display them at the front of the room. Hold a whole-class discussion on the methods used to produce a group solution. Ask two groups of students to describe the method used and the ways in which this method differs to their initial individual responses. Did the students check their work? If they did, what checking method did they use?

Collaborative analysis of *Sample Responses to Discuss* (20 minutes)

Once students have had sufficient time to discuss some different approaches, distribute copies of the *Sample Responses to Discuss* to each group. It may not be appropriate, or there may not be enough time, for all students to analyze all three sample responses. Each response highlights different misconceptions and so depending on the progress already made on the task, it may be appropriate to issue different sample responses to different groups. For example, groups that have failed to justify the shop coverage when the camera is at P could be given Simon's work, while groups that have failed to check the coverage of the shop with the camera in the new position could be given Ellie's work.

The whole-class discussion held after the collaborative work should help to inform your decision on whether or not to be selective about which sample responses students are given.

> *In your groups you are now going to look at some student work on the task.*
> *Notice in which ways this work is similar to yours and in which ways it is different.*
>
> *There are some questions for you to answer as you look at the work. You may want to add annotations to the work to make it easier to follow.*

This task gives students an opportunity to evaluate a variety of possible approaches to the task, without providing a complete solution strategy. Students should thoughtfully answer the questions below each piece of sample student work and be encouraged to think carefully about ways in which the work could be improved rather than focus on whether the student has neat handwriting etc.

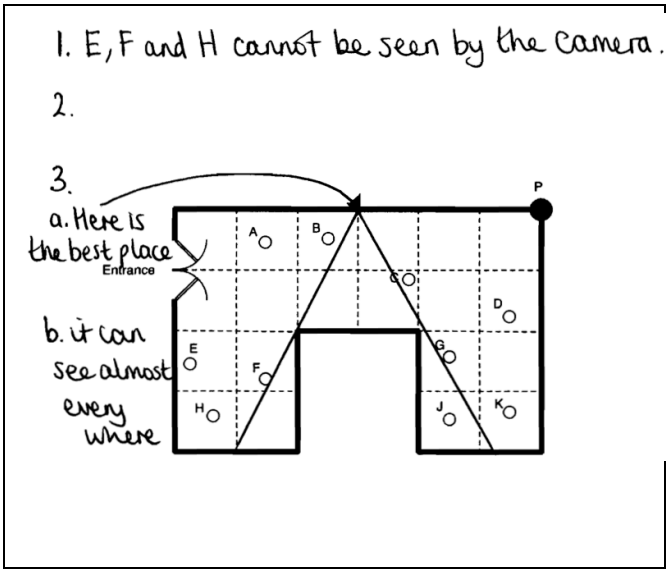
Slide P-5 of the projector resource provides an overview of how students should work together.

Sample Response to Discuss

1. Each choose a sample response to work through and write your answers on your mini-whiteboards.
2. Explain your answers to the rest of the group.
3. Listen carefully to explanations. Ask questions if you don't understand.
4. Once everyone is satisfied with the explanations, write the answers to the questions below the sample response.
5. Try to ensure that the student who writes the answers is not the student who explained them.

Simon realizes that F and H cannot be seen, but incorrectly thinks that E also cannot be seen. He does not show any work to justify his thinking. Simon fails to explain whether or not he agrees with the shop owner about the percentage of the shop hidden from the camera. He identifies the center of the back wall as the best place for the camera but does not investigate further. No calculations are shown.

Simon could improve his work by finding the percentage of the shop not seen by the camera at P. He also needs to compare areas from various viewpoints to find all optimum positions for the camera.



Ellie correctly states that F and H cannot be seen and that three squares cannot be seen. However, it is possible that she thinks that three whole squares are hidden from the camera. Ellie does not show any sightlines to justify her answer to Q1 and her justification for Q2 is incomplete and poorly explained. Ellie suggests a new position for the camera and suggests that the camera cannot see two squares when it is placed here. She again omits to use sightlines to justify this new position.

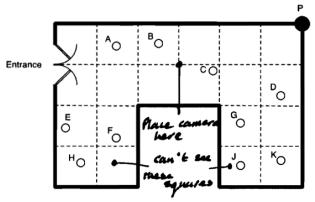
Ellie could improve her work by showing the three squares that are hidden when the camera is at P. She also needs to calculate the percentage of the shop that cannot be seen when the camera is placed in the center of the room. This will show her that she has not found the optimal position.

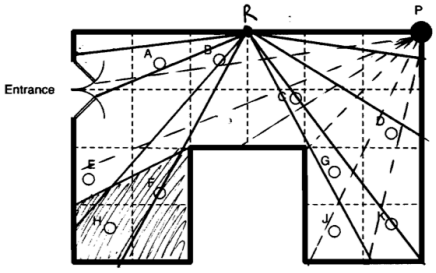
Rhianna correctly shows that F and H cannot be seen and that three squares (= 15% of the area) cannot be seen. Her diagram shows that the three squares are made up of two whole squares and two part squares, but this is not explained. Rhianna investigates the best place for the camera and shows that the center of the back wall is good. She has included sightlines on her diagram and calculations to try and justify her findings.

Rhianna could improve her work by comparing areas of the shop that cannot be seen from different camera positions to find the optimum points.

1. F+H
2. This is true because if there are 20 squared areas to make up the shop and 3 cannot be seen by the camera then that means the 3 squared areas would have to equal 15%.

3a+b I think the best place for the camera is in the center of the room because it only can't see two squares.





1. He cannot see F+H.
2. There are 20 squares. 3 squares are hidden from the camera.
   Each square represents 5%
   3 × 5% = 15%
   This proves 15% of the shop is hidden.
3. a) R = My camera — 5% is hidden on one half. 5% is hidden on the other half. This way only 10% is hidden + that space could be used for a till/trolley.
   b) I know this is the best place because it has full view of all around the shop it can go

Whole-class discussion (10 minutes)

You may want to compare students' own work with the different approaches used in the sample student responses as part of the whole-class discussion. Ask the students to compare the different solution methods and comment on their strengths and weaknesses.

What did Simon/Ellie/Rhianna do?

How is it similar/different to what you did?

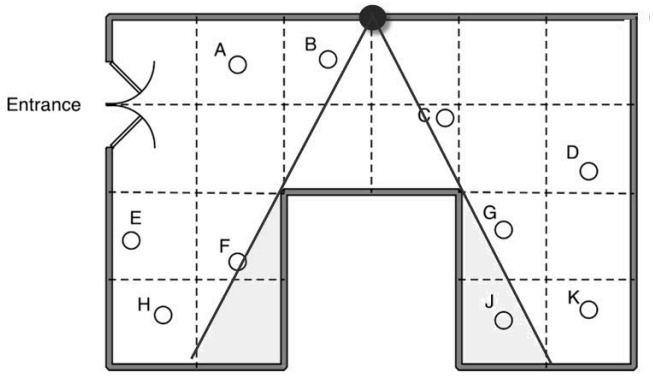
Did analyzing the responses enable anyone to see errors in their own work?

What is unclear about Simon's/Ellie's/Rhianna's work?

Of the three sample pieces of work, which do you think has the most complete and accurate solution? Why? In what ways could it be improved further?

You may want to use Slides P-6, P-7 and P-8 of the projector resource and the questions in the *Common issues* table to support this discussion.

Finally, if students conclude that only the following position as the best place for the camera, encourage them to consider moving the camera up to one square to the left or right. This does not decrease the amount of the shop that may be covered. Set students the challenge of proving this, if they can.



Follow-up lesson (or possible homework): individual reflection (10 minutes)

Once students have had a chance to discuss the sample responses as a whole class, distribute the questionnaire *How Did You Work?* Ask students to spend a few minutes, individually, answering the questions.

Think carefully about your work this lesson and the different methods you have seen and used.

On your own, answer the review questions as carefully as you can.

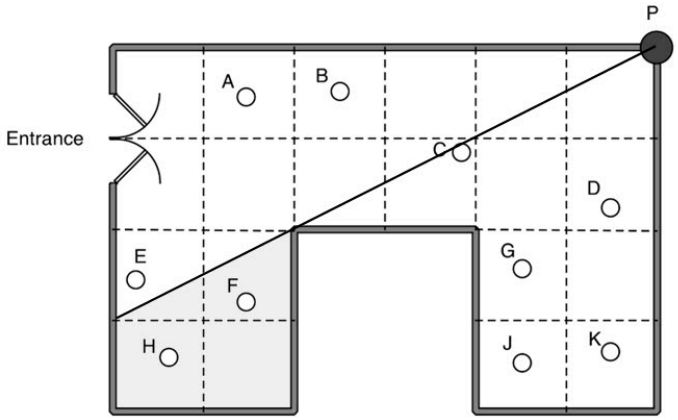
Some teachers set this as a homework task.

SOLUTIONS

Assessment task: *Security Cameras*

1) F and H cannot be seen by camera at P.

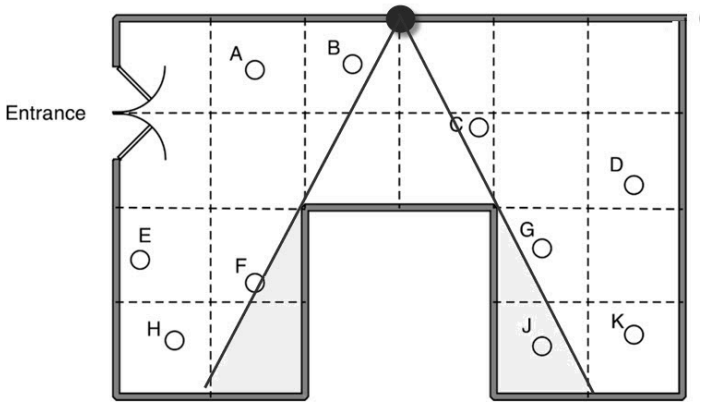
2) Drawing a sight line from the camera at P shows that there are two whole squares and two part squares (which make up a whole square) that cannot be seen; a total of three squares altogether.



As the total area of the shop is 20 squares, the percentage that cannot be seen is three squares out of a possible 20 squares, which is 15%, so the shopkeeper is correct.

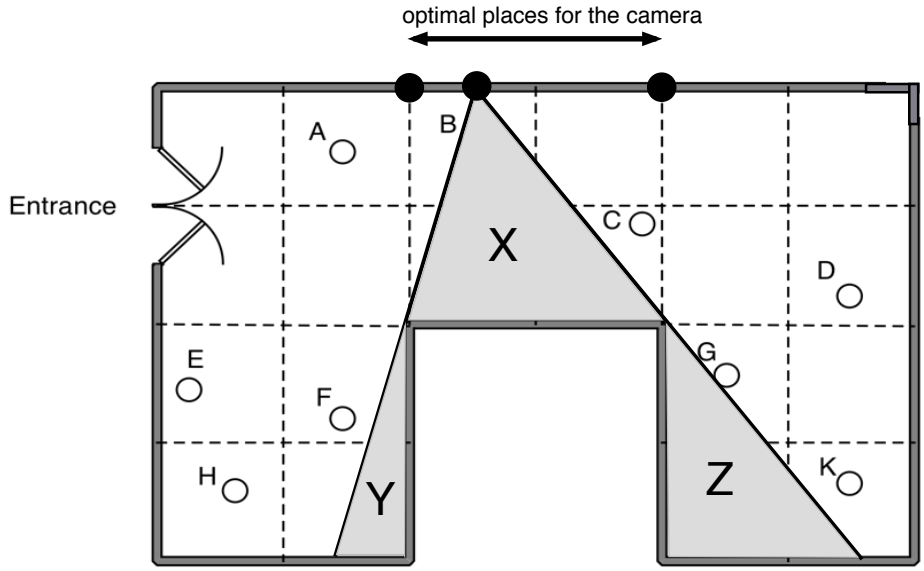
3) The maximum possible coverage of the shop with one security camera is where 10% (two out of a possible 20 squares) of the shop cannot be seen.

An example of this is positioning the camera in the middle of the back wall:



Students need to explain how they know they have found the best place for the camera. They should compare the coverage from various viewpoints and recognize that there are multiple places that the camera can be positioned to give the optimum coverage of the shop.

If the camera is positioned along the back wall of the shop, anywhere either on or between the two points shown below 10% of the shop is hidden.



To see this, students need to recognize that the two areas of the shop that cannot be seen by the camera (Y and Z) have a combined area equal to the area labeled X in the diagram above, and this area is invariant as the camera is moved between the two points shown. (This is seen by dropping a perpendicular from the apex of triangle X to its base. The two triangles so formed are congruent to Y and Z respectively).

The area of X is invariant and is equal to ½ × 2 squares x 2 squares = 2 squares, which is 10% of the total 20 squares.

When the camera is positioned on the extreme end of the interval, either X = Z and Y = 0 (left hand end), or Y = X and Z = 0 (right hand end). So 10% of the shop is still hidden.

Security Cameras

A shop owner wants to prevent shoplifting.

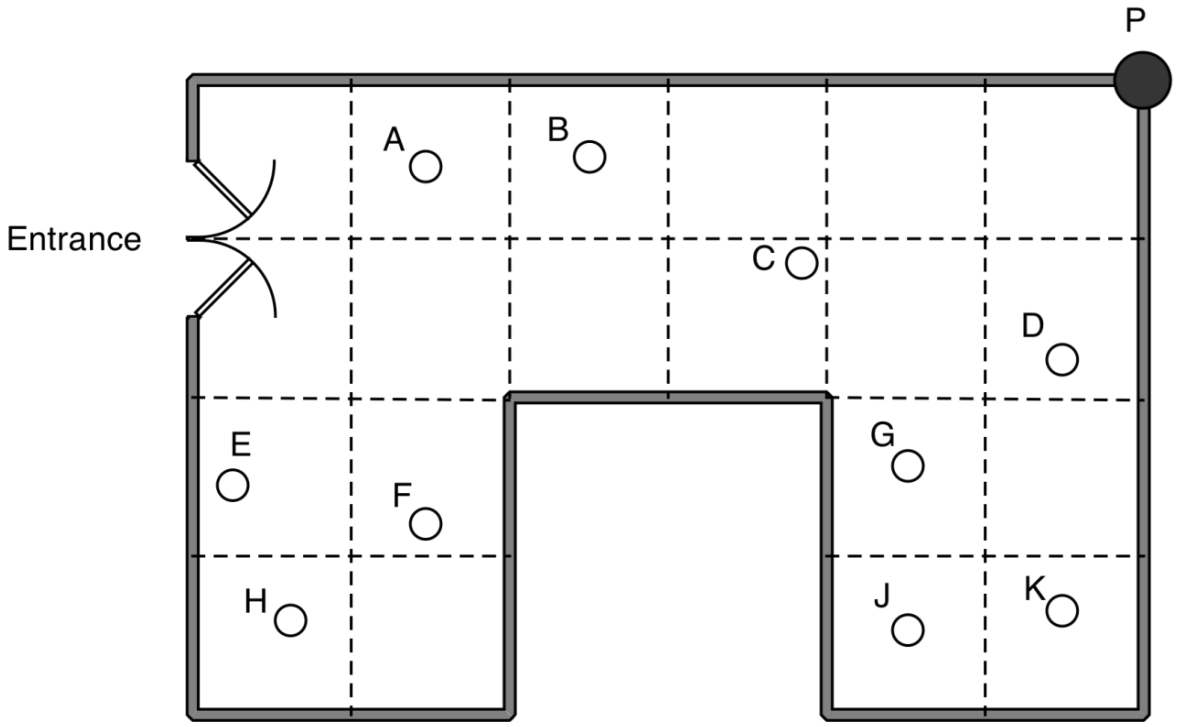
He decides to install a security camera on the ceiling of his shop.

The camera can turn right round through 360° in all directions.

The shop owner places the camera at point P, in the corner of the shop.

The plan view below shows where ten people are standing in the shop.

Plan view of the shop:



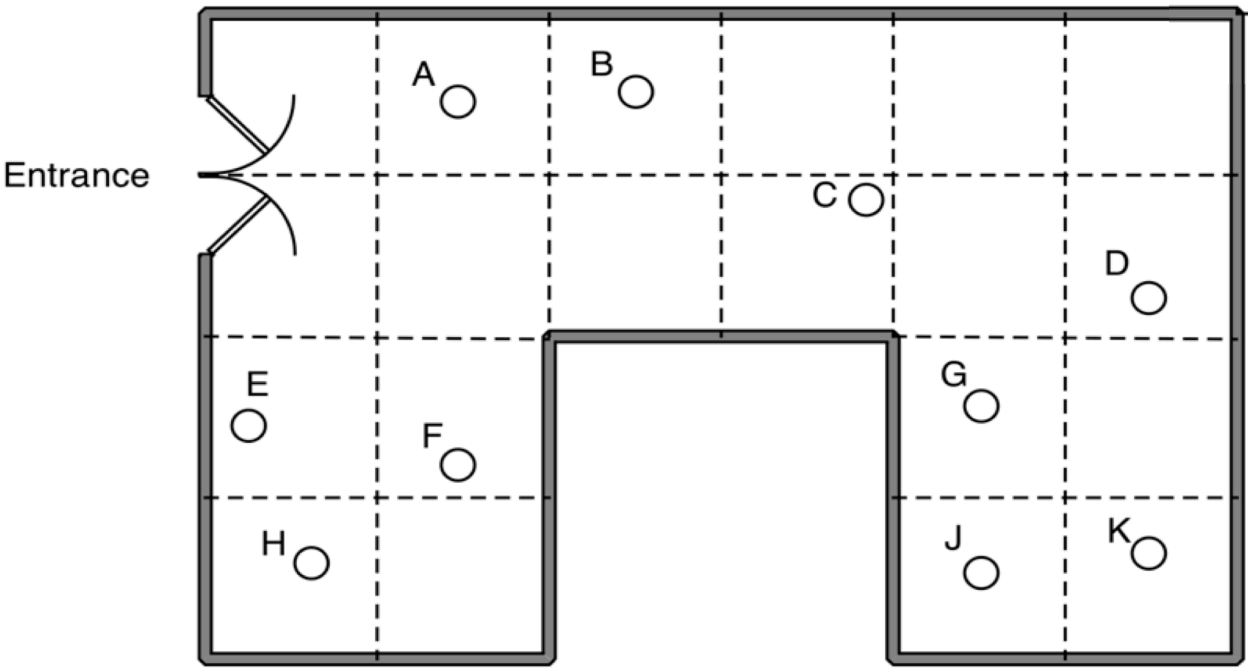
1. Which people in the shop cannot be seen by the camera at P?
   Explain your answer, showing clearly on the diagram how you know.

2. The shop owner says, “15% of the shop is hidden from the camera.”
   Show clearly that he is correct.

Security Cameras (continued)

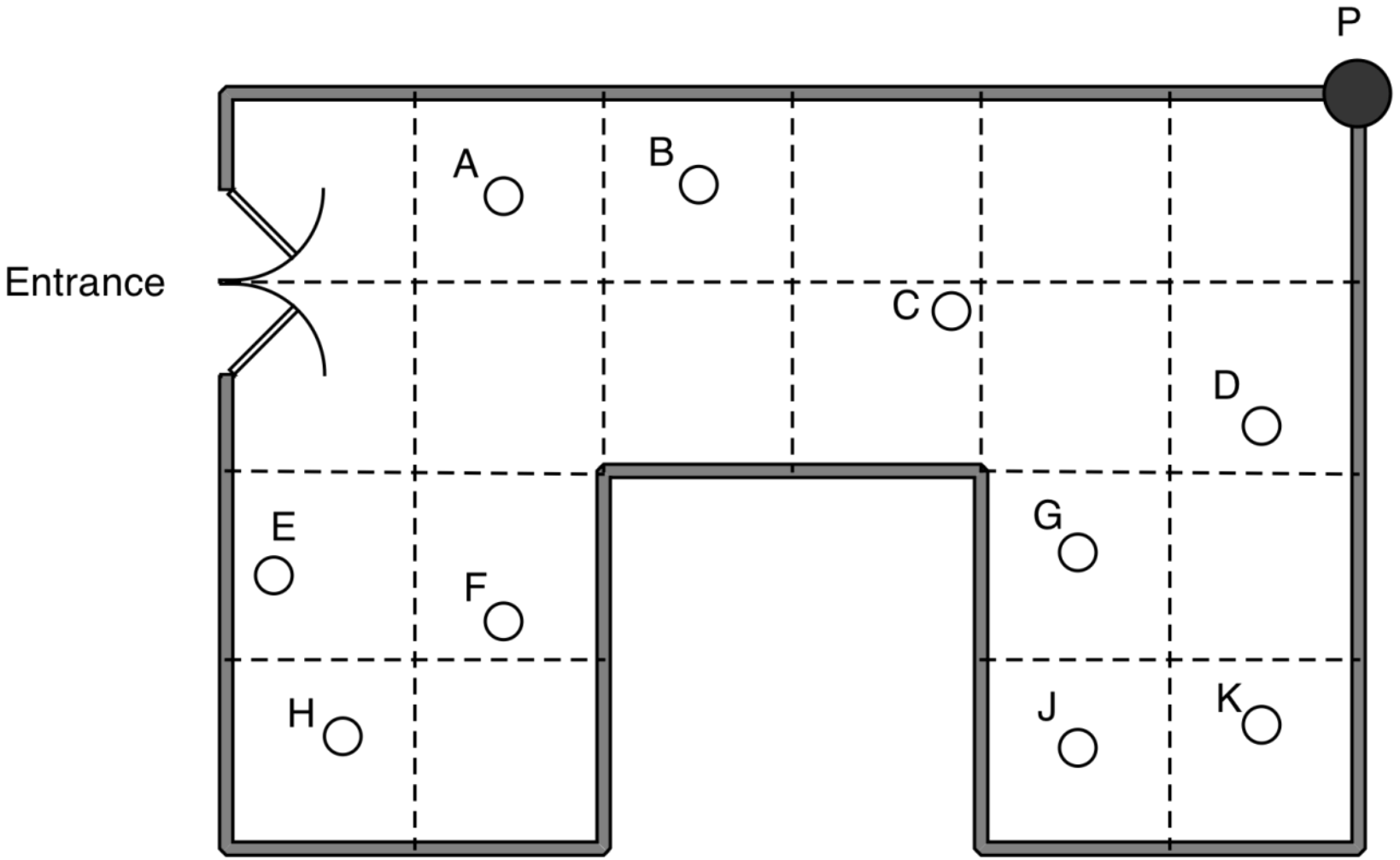
3. The shop owner decides to move the camera so that it can see as much of the shop as possible.

   a. On the plan view below show the best place for the camera so that it can see as much of the shop as possible.

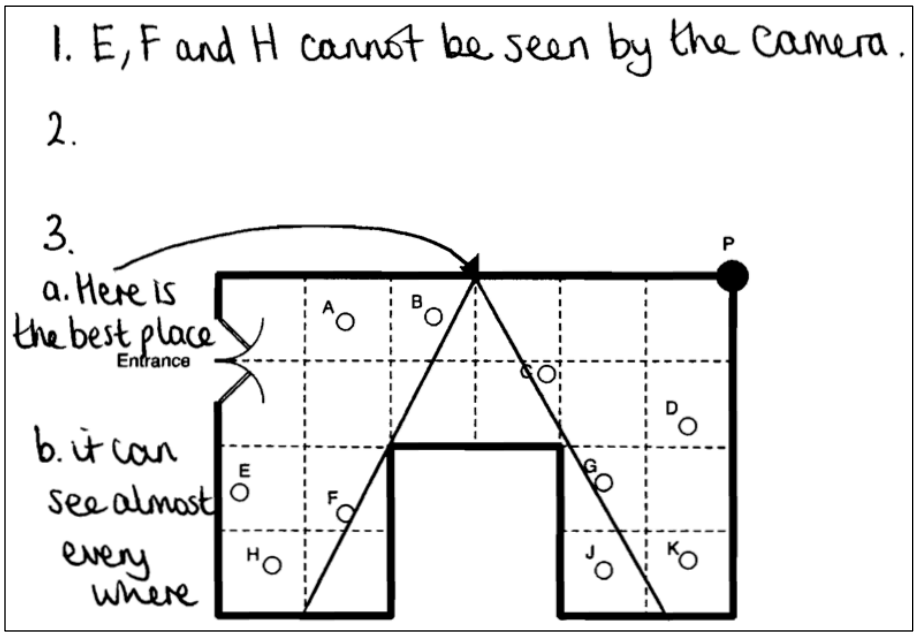


   b. Explain how you know that this is the best place for the camera. What percentage of the shop cannot be seen with the camera in this new position?

Plan View



Sample Responses to Discuss: Simon



What mistake has Simon made?

What do the lines on Simon's diagram show?

In what ways could Simon's work be improved?

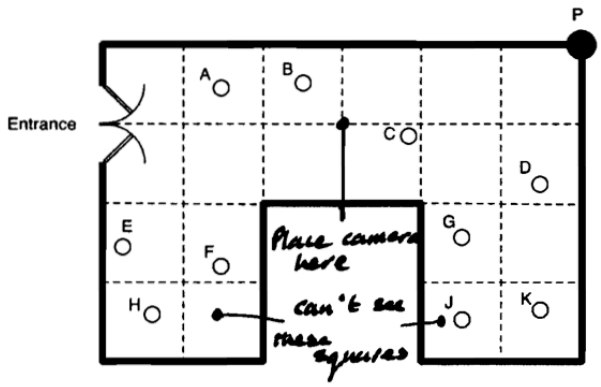
To help you to understand Simon's work, what question(s) could you ask him?

Sample Responses to Discuss: Ellie

1. F + H

2. This is true because if there are 20 squared areas to make up the shop and 3 cannot be seen by the camera then that means the 3 squared areas would have to equal 15%.

3a+b I think the best place for the camera is in the center of the room because it only can't see two squares.

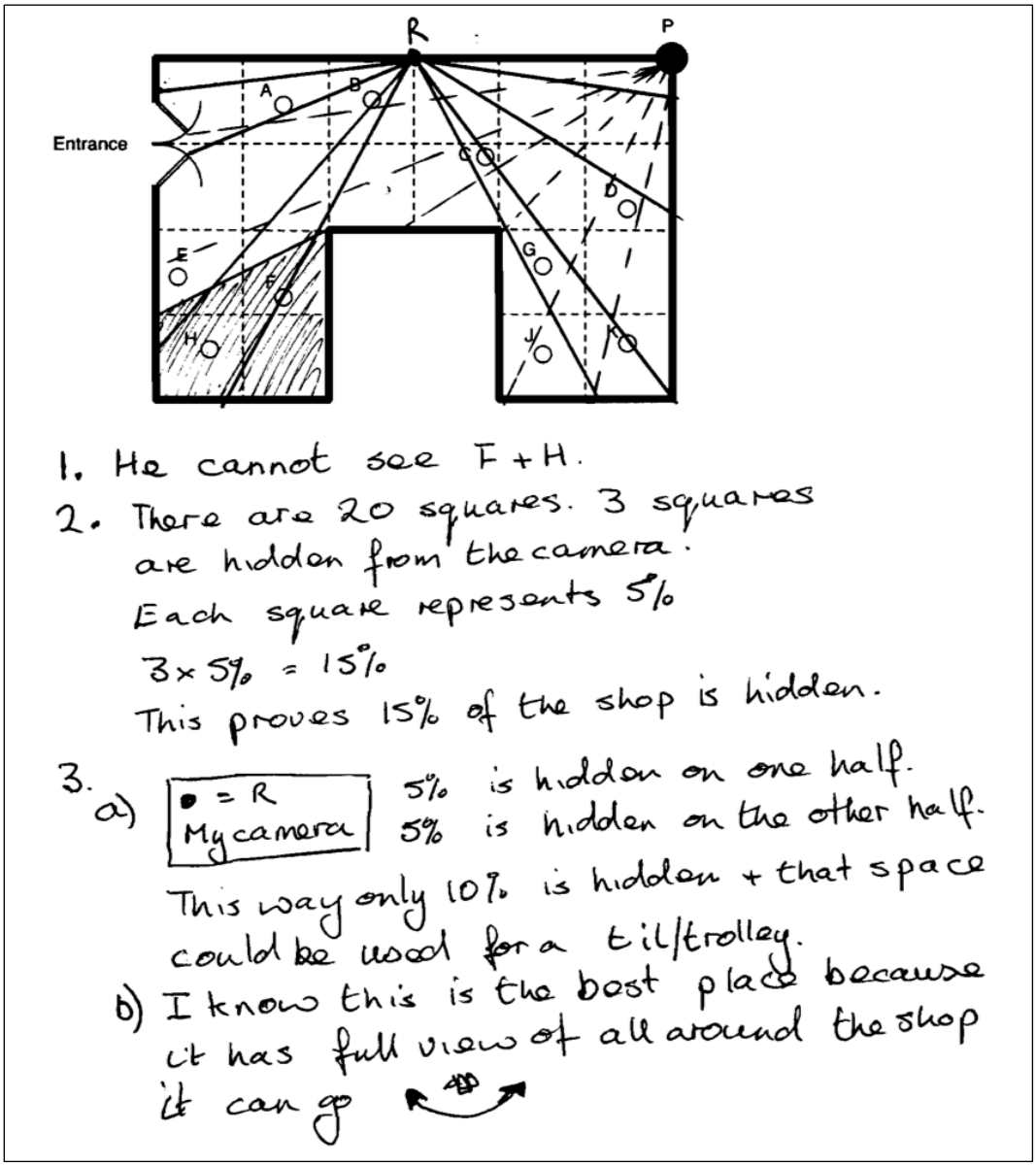


Why is Ellie's response to question 2 unclear?

In what ways could Ellie's work be improved?

To help you to understand Ellie's work, what question(s) could you ask her?

Sample Responses to Discuss: Rhianna



1. He cannot see F + H.
2. There are 20 squares. 3 squares are hidden from the camera.
   Each square represents 5%
   3 × 5% = 15%
   This proves 15% of the shop is hidden.
3. a) ● = R
   My camera
   5% is hidden on one half.
   5% is hidden on the other half.
   This way only 10% is hidden + that space could be used for a till/trolley.
   b) I know this is the best place because it has full view of all around the shop it can go

In what ways could Rhianna's work be improved?

To help you to understand Rhianna's work, what question(s) could you ask her?

How Did You Work?

Mark the boxes, circle an option and complete the sentences that apply to your work.

1. Our group work was better than my own work **Yes/ No**

   This is because .....

   .....

2. In our solution **we checked / could have checked** (*circle*) our work by .....

   .....

3. Our solution is similar to one of the sample responses ☐ **OR** Our solution is different from **all** of the sample responses ☐

   Our solution is similar to ..... *(add name of the student)* Our solution is different from all of the sample responses

   I prefer **our solution / the student's solution** (*circle)* because .....

   This is because .....

   .....

   .....

4. I made some assumptions ☐

   My assumptions were: .....

   .....

5. What advice would you give a student new to this task to help them with difficulties?

   .....

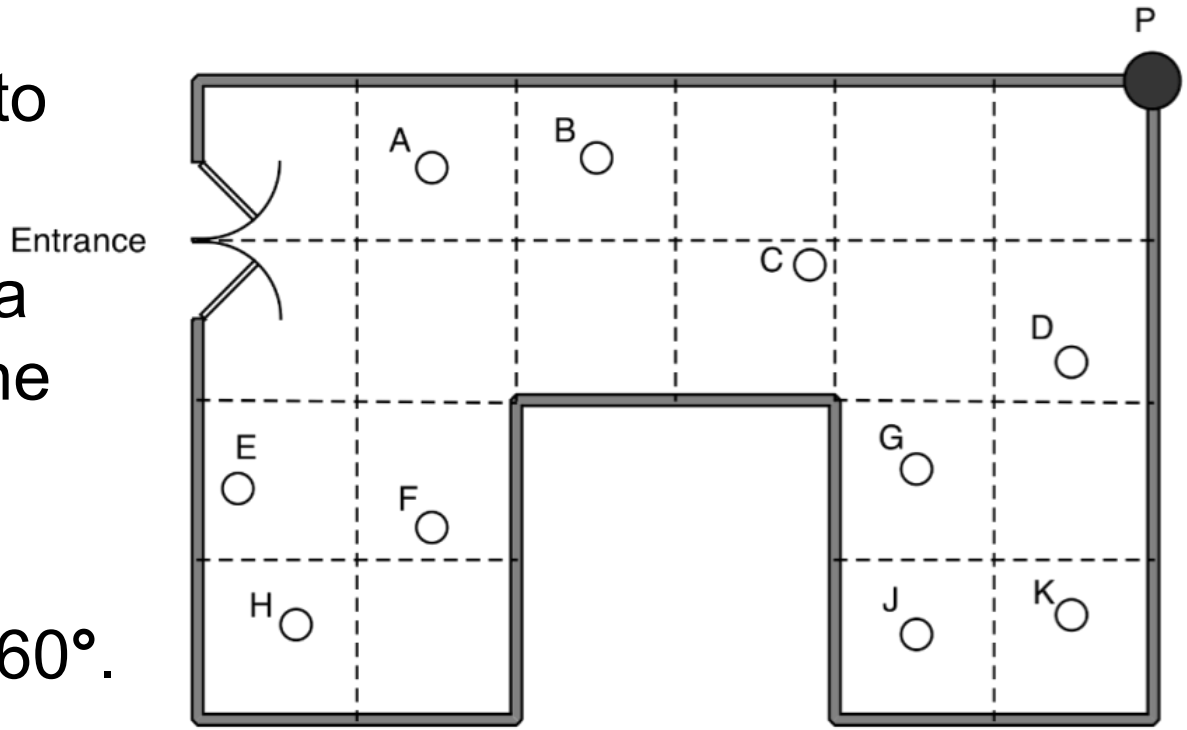
   .....

Security Cameras

A shop owner wants to prevent shoplifting.

He decides to install a security camera on the ceiling of his shop.

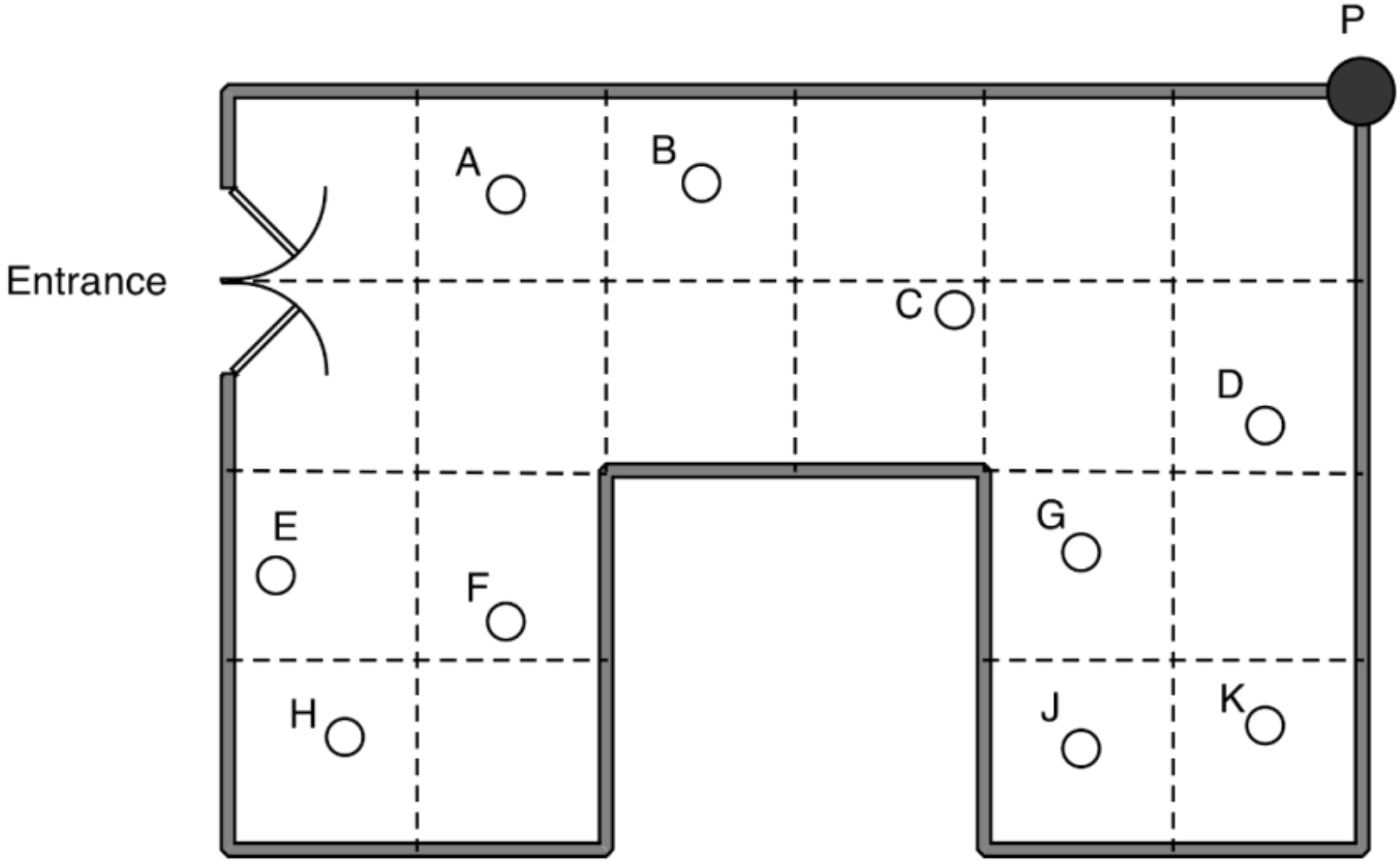
The camera can turn right round through 360°.



The shop owner places the camera at point P, in the corner of the shop.

The plan view shows where ten people are standing in the shop.

Plan View



Planning a Joint Solution

1. Take turns to explain your work and how you think it could be improved.
2. Listen carefully to each other and ask questions if you don't understand or agree.
3. Once everyone in the group has explained their method, plan a joint method that is better than each of your separate ideas.
4. Make sure that everyone in the group can explain the reasons for your chosen method.
5. Write a brief outline of your planned method on your large sheet of paper.

Security Cameras

1. Which people cannot be seen by the camera at P? Explain your answer, showing clearly on the diagram how you know.
2. The shop owner says, “15% of the shop is hidden from the camera.” Show clearly that he is correct.
3. The shop owner decides to move the camera so that it can see as much of the shop as possible.
   a. Show on the plan view the best place for the camera, so that it can see as much of the shop as possible.
   b. Explain how you know that this is the best place for the camera.

Sample Response to Discuss

1. Each choose a sample response to work through and write your answers on your mini-whiteboards.
2. Explain your answers to the rest of the group.
3. Listen carefully to explanations. Ask questions if you don't understand.
4. Once everyone is satisfied with the explanations, write the answers to the questions below the sample response.
5. Try to ensure that the student who writes the answers is not the student who explained them.

Sample Responses to Discuss: Simon

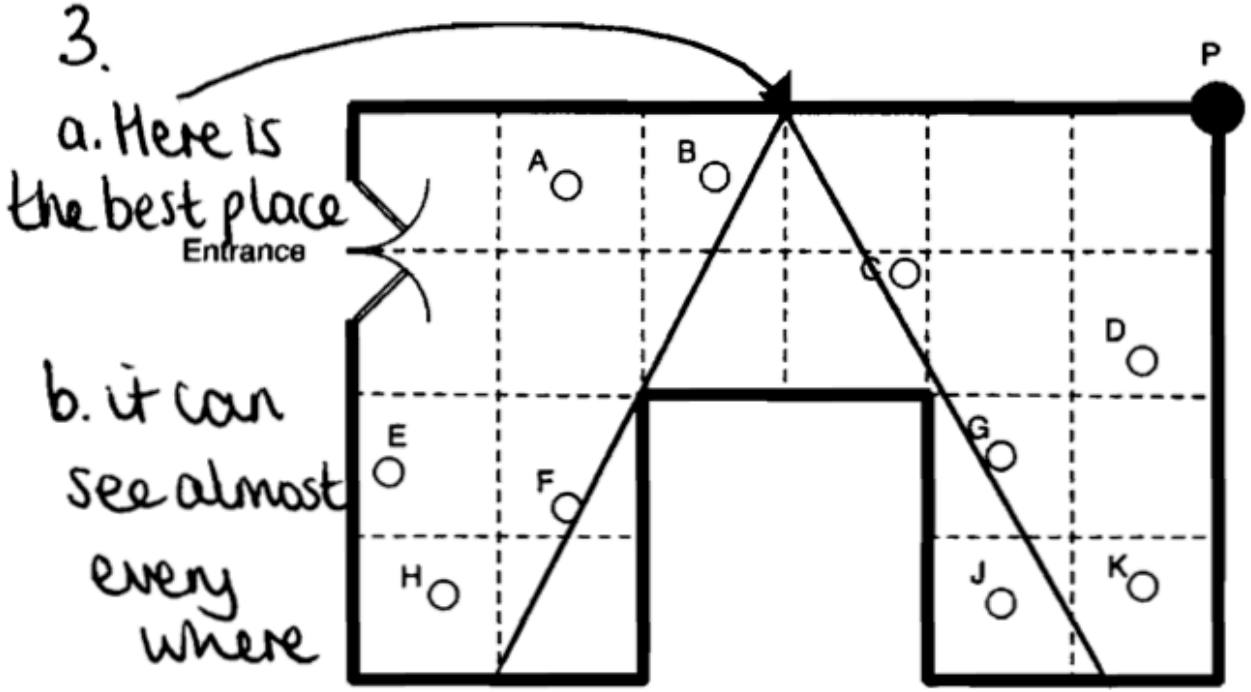
1. E, F and H cannot be seen by the camera.

2.

3.

a. Here is the best place

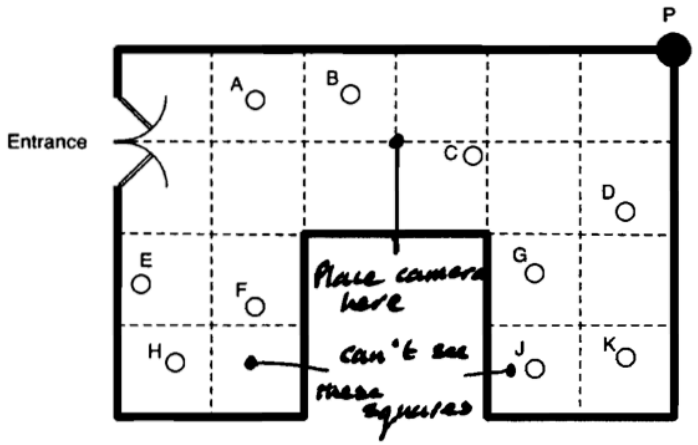
b. it can see almost every where



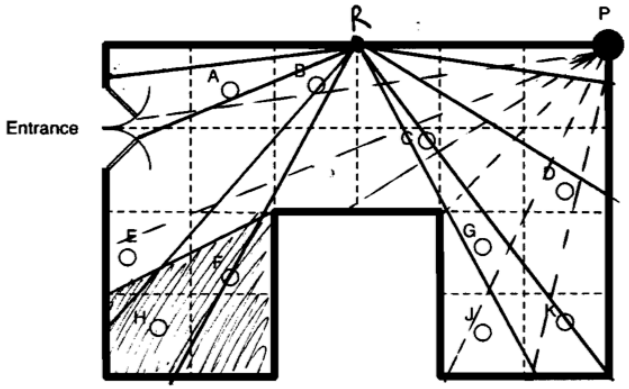
Sample Responses to Discuss: Ellie

1. F + H
2. This is true because if there are 20 squared areas to make up the shop and 3 cannot be seen by the camera then that means the 3 squared areas would have to equal 15%.

3a+b I think the best place for the camera is in the center of the room because it only can't see two squares.



Sample Responses to Discuss: Rhianna



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   b) I know this is the best place because it has full view of all around the shop it can go

Mathematics Assessment Project

CLASSROOM CHALLENGES

This lesson was designed and developed by the
Shell Center Team
at the
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Malcolm Swan, Nichola Clarke, Clare Dawson, Sheila Evans,
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It was refined on the basis of reports from teams of observers led by
David Foster, Mary Bouck, and Diane Schaefer
based on their observation of trials in US classrooms
along with comments from teachers and other users.

This project was conceived and directed for
MARS: Mathematics Assessment Resource Service
by
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and based at the University of California, Berkeley

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