CONCEPT DEVELOPMENT

Mathematics Assessment Project

CLASSROOM CHALLENGES

A Formative Assessment Lesson

Representing Data 1: Using Frequency Graphs

Mathematics Assessment Resource Service
University of Nottingham & UC Berkeley
Beta Version

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Representing Data 1: Using Frequency Graphs

MATHEMATICAL GOALS

This lesson unit is intended to help you assess how well students:

- Are able to use frequency graphs to identify a range of measures and make sense of this data in a real-world context.
- Understand that a large number of data points allow a frequency graph to be approximated by a continuous distribution.

COMMON CORE STATE STANDARDS

This lesson relates to the following *Standards for Mathematical Content* in the *Common Core State Standards for Mathematics*:

S-ID Summarize, represent, and interpret data on a single count or measurement variable.

This lesson also relates to the following *Standards for Mathematical Practice* in the CCSS:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.

INTRODUCTION

The unit is structured in the following way:

- Before the lesson, students complete an assessment task individually that is designed to reveal their current understanding.
- A whole-class introduction provides students with guidance on how to work through the task. Students work in pairs or threes on a collaborative discussion task, matching written interpretations and graphs as they begin to link these two representations.
- Towards the end of the lesson there is a whole-class discussion.
- In a follow-up lesson, students work alone on a similar task to the assessment task.

This lesson can be taught in conjunction with the lesson *Representing Data Using Box Plots* or independently.

MATERIALS REQUIRED

Each student will need a copy of the handouts: *Cell Phones 1*, *Cell Phones 1 (revisited)*, and *Drawing Graphs*, a mini-whiteboard, a pen, and an eraser.

Each small group of students will need the following resources:

- The Card Sets: *Frequency Graphs* and *Interpretations*. Cut-up both card sets before the lesson.
- One large sheet of paper for making posters and a glue stick. The paper should be large enough to accommodate three different sets of cards.

There are some projector resources to help with whole-class discussions.

TIME NEEDED

Approximately 20 minutes before the lesson, a 65-minute lesson, and 20 minutes in a follow-up lesson (or for homework). Exact timings will depend on the needs of the class.

BEFORE THE LESSON

Assessment task: *Cell Phones 1* (20 minutes)

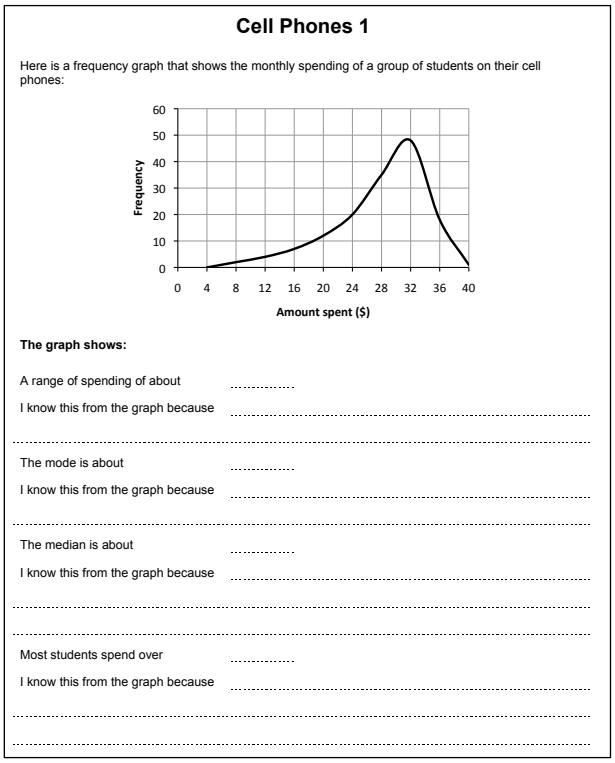
Give this task, in class or for homework, a few days before the formative assessment lesson. This will give you an opportunity to assess the work and to find out the kinds of difficulties students have with it. You will then be able to target your help more effectively in the lesson.

Give each student a copy of the assessment task: *Cell Phones 1*.

> *Read through the questions and try to answer them as carefully as you can.*

It is important that, as far as possible, students are allowed to answer the questions without your assistance.

Students should not worry too much if they cannot understand or do everything because, in the lesson, they will engage in a similar task that should help them. Explain to students that by the end of the next lesson they should expect to answer questions such as these confidently. This is their goal.



Cell Phones 1

Here is a frequency graph that shows the monthly spending of a group of students on their cell phones:

The graph shows:

A range of spending of about .....

I know this from the graph because .....

.....

The mode is about .....

I know this from the graph because .....

.....

The median is about .....

I know this from the graph because .....

.....

.....

Most students spend over .....

I know this from the graph because .....

.....

.....

Assessing students' responses

Collect students' responses to the task and note down what their work reveals about their current levels of understanding, and their different approaches.

We suggest that you do not score students' work. The research shows that this will be counterproductive, as it will encourage students to compare their scores and distract their attention from what they can do to improve their mathematics.

Instead, help students to make further progress by summarizing their difficulties as a series of questions. Some suggestions for these are given on the next page. These have been drawn from common difficulties observed in trials of this unit.

We suggest that you write a list of your own questions, based on your students' work, using the ideas in the *Common issues* table. We recommend that you write questions on each individual student's work. If you do not have time to do this, select a few questions that will be of help to the majority of students. These can be written on the board at the start of the lesson.

Common issues	Suggested questions and prompts
Student provides an incorrect range For example: The student writes 5 - 40. Or: The student assumes the range is the difference between the maximum and zero.	• What is the definition of the term 'range'?
Student uses the frequency readings for the measures of central tendency For example: The student writes that the mode is 48.	• What does the vertical axis represent? • What measure is the question asking for?
Student does not understand the word 'Frequency' For example: The student assumes 'Frequency' represents a percentage.	• What does the term 'frequency' mean? • What does the axis marked 'frequency' represent on the graph?
Student does not understand what median represents or how to use the graph to figure out the median For example: The student assumes the median value is equal to the mode. Or: The student assumes the median value is exactly half way between the maximum and minimum value.	• What does the median represent? • Can the mode and median values be different? • Roughly what proportion of students spends less than the median value? • Sketch on the graph the results of a different survey. In this survey the maximum and minimum values for the amount spent each month remains the same, but the number of students in the survey is half/double. What does this tell you about the area under the graph? • How can you show the median value on your graph? [Draw a vertical line to divide in half the area under the graph. The median amount is the value at the point this line intersects the *x*-axis.]
Student does not contextualize the data For example: The student states the mode is 31, the minimum and maximum values are 5 and 40 respectively, but does not refer to the context.	• What do these figures represent? • Complete this sentence "50% of students spend less than $_ _ _ each month." Explain how you arrived at the figure.

SUGGESTED LESSON OUTLINE

Throughout this lesson, encourage students to use the correct mathematical language to not just **provide** the figure for a measure, but to **place the figure in the context**. For example, rather than "The median is 20", encourage students to say "The median score is 20 out of 100."

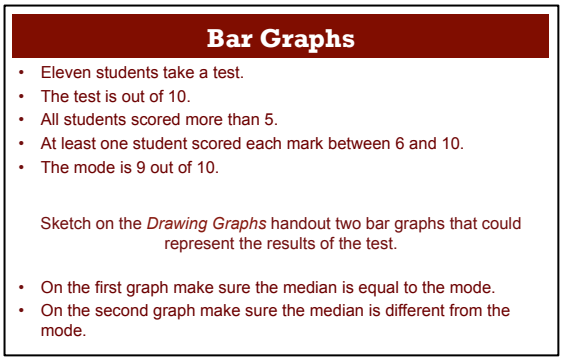
Whole-class interactive introduction: Drawing and Interpreting Graphs (20 minutes)

Give out the sheet *Drawing Graphs*. Maximize participation in the introduction by asking all students to show you their graphs once sketched.

This introduction will provide students with a model of how they should justify their matching of cards in the collaborative activity. It will also help students understand how to identify a range of measures from a graph and in particular the median value.

Often students think that the median should be the middle score of the range of scores. The first task in this introduction helps demonstrate that this is not always the case.

Show Slide P-1 of the projector resource:



Sketch on your sheet two bar graphs that could represent the test results.

On the first make sure the median is equal to the mode.

On the second the median should be different from the mode.

If students struggle, encourage them to discuss the task with a neighbor. It may help if they write down the value of the eleven scores.

After a few minutes ask students to show you their graphs and select two or three students with different graphs to justify them to the class. Ask the rest of the class if they agree with the explanations.

You may want to use Slide P-2 of the projector resource, *Mode and Median*, to support the discussion.

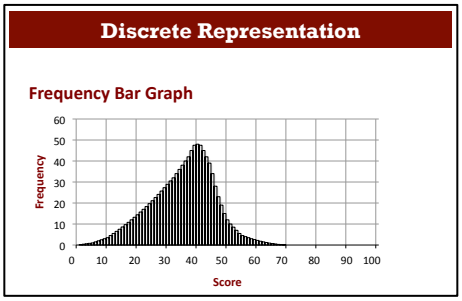
How can you check that eleven students took the test?

How many students achieved the median score or less? How do you know?

What is meant by the statement 'the median is the middle score?' [The middle score of a sorted list of scores.]

Could the median score ever be equal to the minimum or maximum score? [Yes, but the mode would not be 9 out of 10.] How do you know?

Show Slide P-3 of the projector resource:



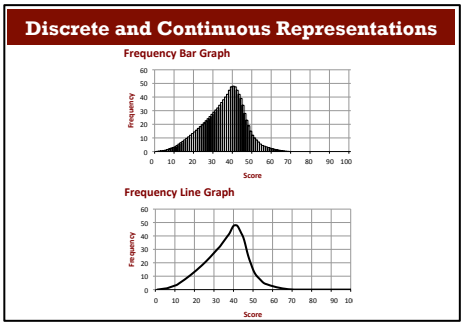
Explain to students that the bar chart represents the scores of students in a test for in which the maximum score was 100. Ask the following questions in turn:

Did anyone achieve the maximum score of 100? How can you tell?

What can you say about the test? Did students find it difficult or easy? How can you tell?

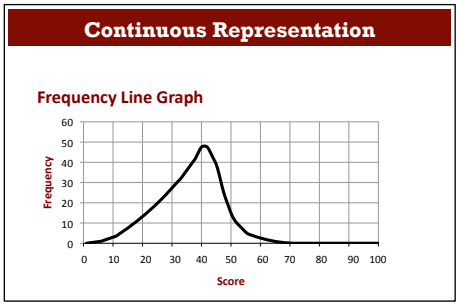
Roughly how many students took the test? [About 1,000.] How can you tell?

Show Slide P-4 of the projector resource:



When there are many bars close together the data can be represented as a continuous line, that is, a frequency graph. This makes it a little easier to read off the values.

Show Slide P-5 of the projector resource:



Students write on their mini-whiteboards all the information they can derive from the graph. After a couple of minutes ask students to show you their answers.

Ask one or two students to justify their answers. Even if their explanations are incorrect or only partially correct, ask students to write them on to the projected graph. Encourage students to challenge these interpretations and then replace them with new ones.

Depending on your class, you may want to ask students a selection of the following questions:

What is the range of scores? [About 64 out of 100.] How do you know?

What is the mode score? [42 out of 100.] How do you know?

What is the range of scores for most students? [Most people scored between 30 and 50 out of 100.]

How do you know? [A large proportion of the area under the graph is between 30 and 50 out of 100.]

What does the area under the graph represent? [The number of people in the survey.]

To confirm that students understand that the area under the graph represents the number of people in the survey you could ask them to sketch a graph on their whiteboard for another set of test results, but this time only half the number of students taking the test. The minimum and maximum scores remain the same

Now return to the original graph:

Roughly what is the median score? [37 out of 100.] How do you know?

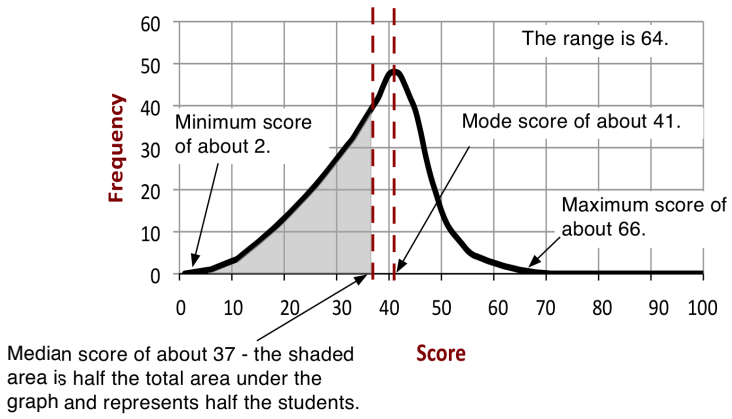
What is the range of scores for the top quarter of students? [About 22 (66 – 44) out of 100.] How do you know?

To answer these final two questions, the area under the graph needs to be divided in half/quarters by vertical lines extending from the *x*-axis. For students to figure out these values it may help if you project the bar graph of the data.

If students find it difficult to figure out an estimate for the median, add a vertical line to the graph that intersects the *x*-axis at, say, a score of 20 out of 100. Then ask student to estimate how many children scored less than 20.

Can this be the median score? How do you know?

The graph may end up looking like this:



Collaborative activity: Matching Card Sets (20 minutes)

Ask students to work in small groups of two or three.

Give each group Card Set: *Frequency Graphs*, Card Set: *Interpretations*, and a large sheet of paper for making a poster.

> *Take turns at matching pairs of cards that you think belong together.*
>
> *Each time you do this, explain your thinking clearly and carefully. Your partner should either explain that reasoning again in his or her own words, or challenge the reasons you gave.*
>
> *Write your reasons for the match on the poster.*
>
> *Place your cards side by side on your large sheet of paper, not on top of one another, so that everyone can see them.*
>
> *You both need to be able to agree on and explain the placement of every card.*

These instructions are summarized on Slide P-6 of the projector resource, *Matching Cards*.

The purpose of this structured work is to encourage each student to engage with their partner's explanations and to take responsibility for their partner's understanding.

While students work in small groups you have two tasks: to note different student approaches to the task and to support student reasoning.

Note different student approaches to the task

In particular, notice any difficulties that students encounter, and the ways they justify and explain to each other. Do students check to see if their match is correct? Do they assume the scores go up the vertical axis? How do they understand how to use the graph to figure out the median and mode? When stating a measure, do students confuse the values on the x-axis and y-axis? Are students using the correct mathematical language? Are students using all the information on the cards or just the first sentence? What do they do if they get stuck?

You can then use this information to focus your questioning in the whole-class discussion towards the end of the lesson.

Support student reasoning

Try not to make suggestions that move students towards particular matches. Instead, ask questions to help students to reason together. You may want to use some of the questions and prompts from the *Common issues* table.

If a student struggles to get started encourage them to ask a specific question about the task. Articulating the problem in this way can sometimes offer a direction to pursue that was previously overlooked. However, if the student needs their question answered, ask another member of the group for a response.

Here are some further questions you may want to use:

> *Rewrite the description in your own words.*
>
> *Write a description of the graph.*
>
> *Tell me how you have used **all** the information [i.e. both sentences] on the Interpretation card to match it with a graph.*
>
> *How can you figure out an approximate value for the median?*

Did many students get a low/high score for this graph? How do you know?

Show me a graph that shows the median score equal to the mode score. How do you know?

Show me a graph that shows the median score different to the mode score. How do you know?

Show me a graph in which a lot of students/few students found it easy. How do you know?

Show me a graph that shows the median score equal to the minimum score. How do you know?

Which graph shows students of a similar ability? How do you know?

Make up five figures where the median is greater than the mode. Now sketch a graph of these figures.

If you find one student has matched two cards, challenge another student in the group to provide an explanation.

Danny matched these cards. Ezra, why does Danny think these two cards go together?

If you find the student is unable to answer this question, ask them to discuss the work further. Explain that you will return in a few minutes to ask a similar question.

If the whole-class is struggling on the same issue you could write a couple of questions on the board and hold a whole-class discussion.

Sharing posters (10 minutes)

As students finish matching the cards, ask one student from each group to visit another group's poster.

If you are staying at your desk, be ready to explain the reasons for your group's decisions.

If you are visiting another group, copy your matches onto a piece of paper. Go to the other group's desk and check to see which matches are different from your own. If there are differences, ask for an explanation. If you still don't agree, explain your own thinking. When you return to your own desk, you need to consider as a group whether to make any changes to your poster.

Slide P-7 of the projector resource, *Sharing Posters*, summarizes these instructions.

When students are satisfied with all their matches give them a glue stick and ask them to glue the cards onto the poster.

Whole-class discussion (15 minutes)

You may want to use transparencies of the cards or Slide 8 of the projector resource to support the discussion.

The intention is that this discussion focuses on the justification of a few examples, rather than checking students all have the correct solutions. You may want to first select a pair of cards that most groups matched correctly as this may encourage good explanations. Then select one or two matches that most groups found difficult. In trials students have had difficulty matching graphs B, C, and H.

How did you decide to match this card?

Can someone else put that into his or her own words?

Could this card be matched with another one?

After discussing two or three matches ask:

Which graph card do you think is unrealistic? Why?

Now ask students to sketch on their whiteboard a graph that shows the test results of a different class. The first piece of information about the test is that there is a in a huge range of scores.

Once students have shown you their whiteboards ask them to swap whiteboards with a neighbor and write a second piece of information about the test on their neighbors' whiteboard. This piece of information, combined with the first, should make their existing graph incorrect.

Once whiteboards are returned students will need to re-draw their graph so that it represents both pieces of information.

Ask students to show you their whiteboards. Ask a few students with differing graphs to explain why they were forced to re-draw it.

Follow-up lesson: Reviewing assessment (20 minutes)

Return the original assessment *Cell Phones 1* to the students together with a copy of *Cell Phones 1 (revisited)*. If you have not added questions to individual pieces of work, write your list of questions on the board. Students should select from this list only those questions they think are appropriate to their own work. Some students may struggle to identify which questions they should consider from this list. If this is the case it may be helpful to give students a printed version of the list of questions so that you can highlight the ones that you want them to focus on.

Look at your original responses and the questions (on the board/written on your script.) Think about what you have learned.

Now look at the new task sheet, Cell Phones 1 (revisited). Use what you have learned to answer these questions.

When you revise your work, write as if you are explaining the solutions to someone unfamiliar with this type of math.

SOLUTIONS

Assessment task: *Cell phones 1*

All answers are approximate.

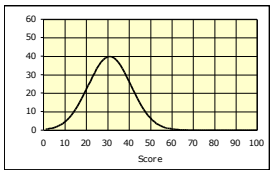
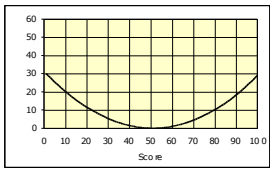
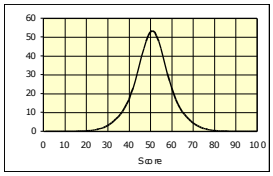
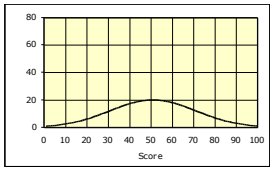
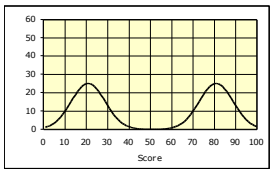
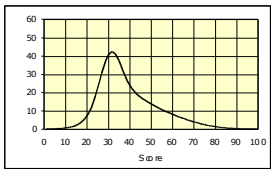
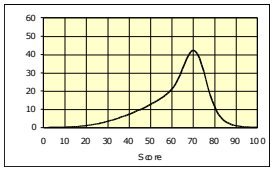
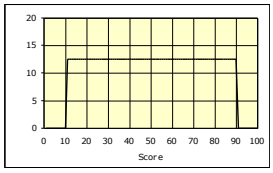
The range is about $35 a month. I know this from the graph because the range represents the difference between the maximum ($40 a month) and minimum ($5 a month) amount spent each month.

The mode is about $31 a month. I know this from the graph because the maximum frequency is about $31 a month.

The median is about $29 a month. I know this from the graph because if I drew a vertical line from this point on the *x*-axis it would divide in half the area under the graph. The area under the graph represents the number of students in the survey. This means about half the students spent less than $29 a month and half spent more than $29 a month.

Most students spend over $24 a month. I know this from the graph because the area under the graph up to $24 is a lot less than the area under the graph between $24 and $40.

Collaborative activity:

A 	8. This test was much too difficult for most people.	B 	1. This was the sort of test where you could either do everything or you couldn't get started.
C 	2. This test did not sort out the stronger students from the weaker ones. They all got similar scores.	D 	7. In this test, the median and the mode scores were the same. There was a very big range of scores.
E 	3. Two groups of students took the test. One group had studied the work for two years. The other group had only just begun.	F 	5. In this test, the median score was greater than the mode score.
G 	6. In this test, the median score was smaller than the mode score.	H 	4. This test resulted in a huge range of scores. Everyone could do something but nobody could do it all.

Assessment task: *Cell phones 1 (revisited)*

All answers are approximate.

The range is about $38 a month. I know this from the graph because the range represents the difference between the maximum ($44 a month) and minimum ($6 a month) amount spent.

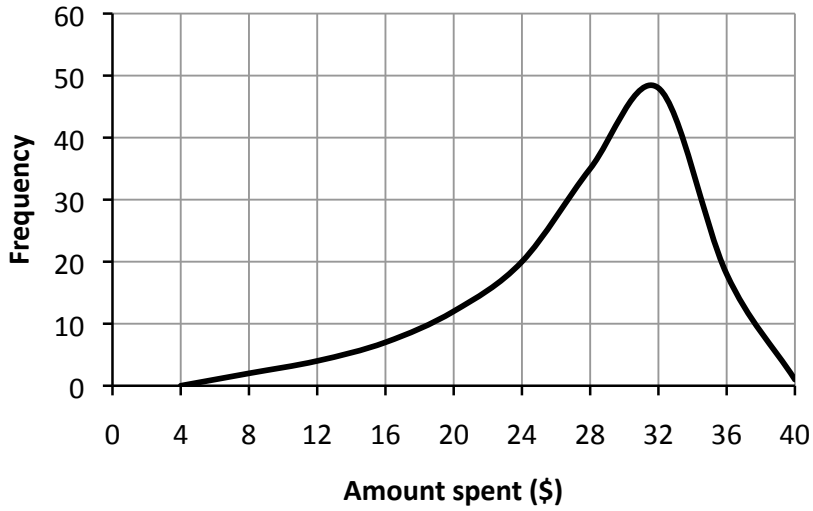
The mode is about $23 a month. I know this from the graph because the maximum frequency is about $23 a month.

The median is about $18 a month. I know this from the graph because if I drew a vertical line from this point on the *x*-axis it would divide in half the area under the graph. The area under the graph represents the number of students in the survey. This means about half the students spent less than $18 a month and half spent more than $18.

Very few students spend over $30 a month. I know this from the graph because the area under the graph from $30 to $44 is a lot less than the area under the graph between $6 and $30.

Cell Phones 1

Here is a frequency graph that shows the monthly spending of a group of students on their cell phones:



The graph shows:

A range of spending of about _____

I know this from the graph because _____

_____

The mode is about _____

I know this from the graph because _____

_____

The median is about _____

I know this from the graph because _____

_____

_____

Most students spend over _____

I know this from the graph because _____

_____

_____

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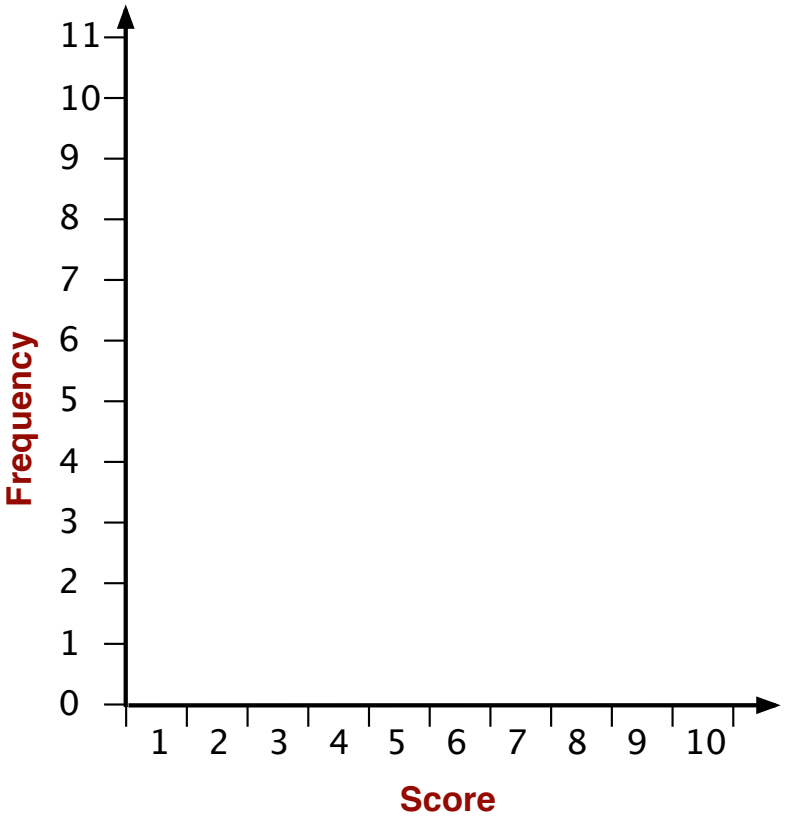
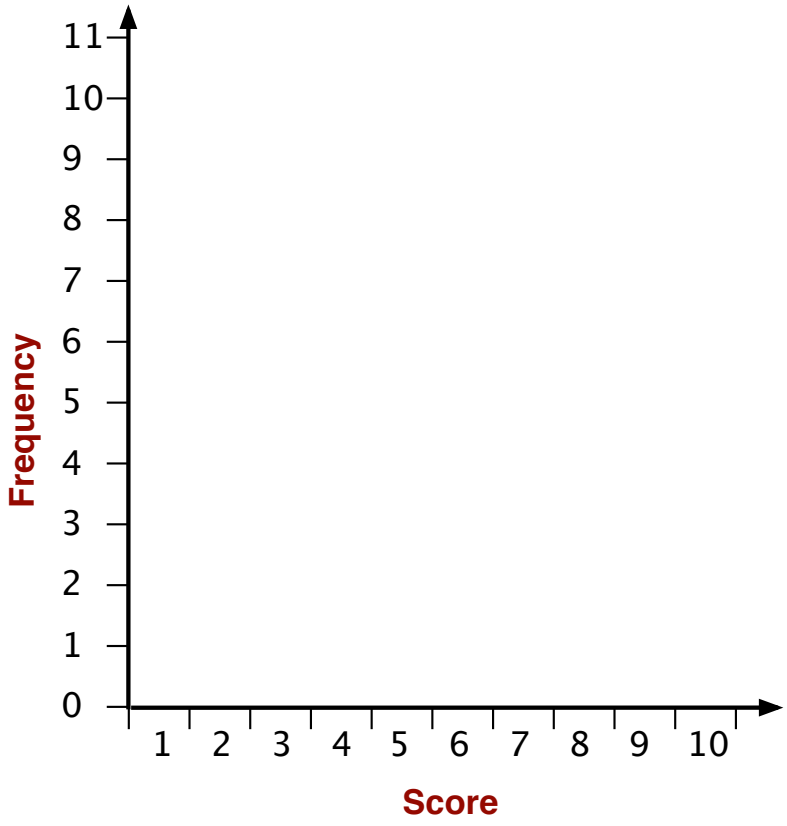
Drawing Graphs





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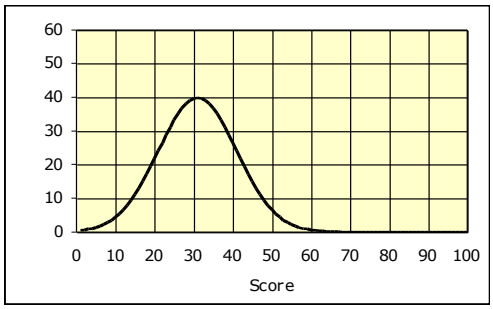
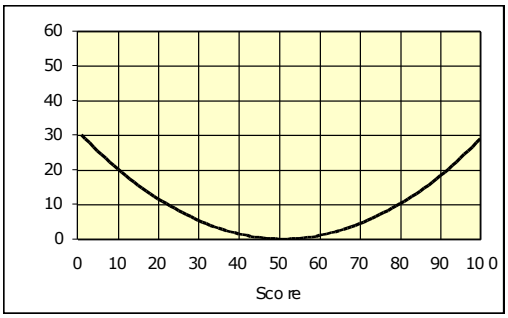
Card Set: Frequency Graphs

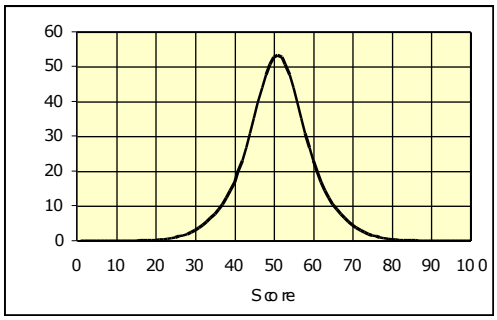
Frequency Graph A



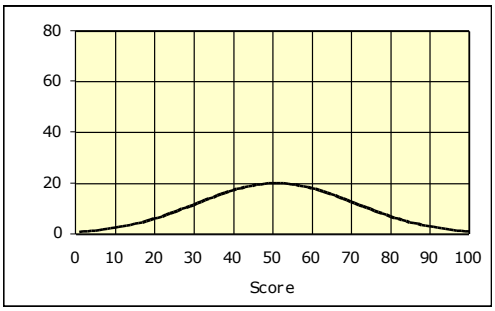
Frequency Graph B



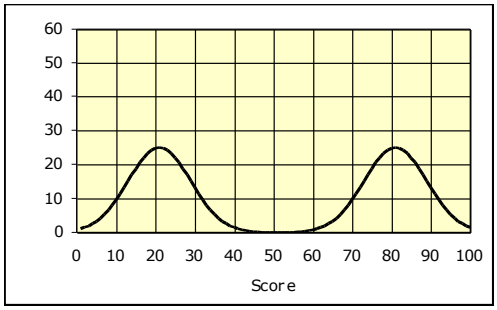
Frequency Graph C



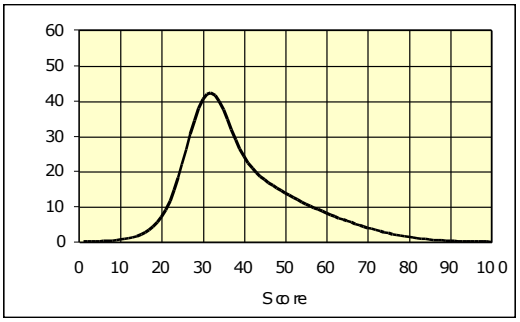
Frequency Graph D



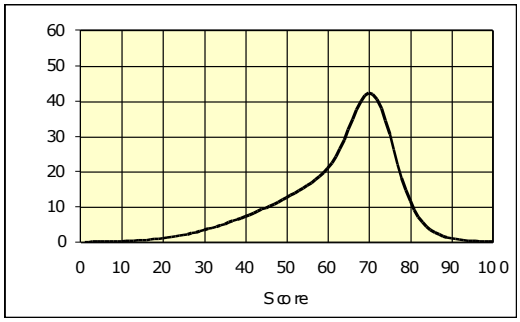
Frequency Graph E



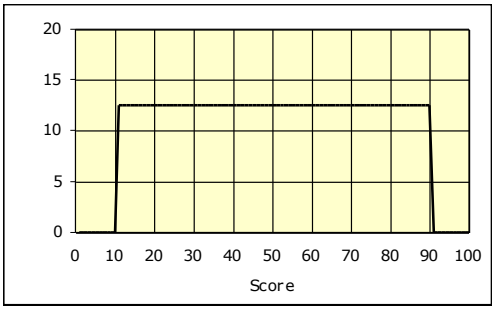
Frequency Graph F



Frequency Graph G



Frequency Graph H



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Card Set: Interpretations

1. This was the sort of test where you could either do everything or you couldn't get started.	2. This test did not sort out the stronger students from the weaker ones. They all got similar scores.
3. Two groups of students took the test. One group had studied the work for two years. The other group had only just begun.	4. This test resulted in a huge range of scores. Everyone could do something but nobody could do it all.
5. In this test, the median score was greater than the mode score.	6. In this test, the median score was smaller than the mode score.
7. In this test, the median and the mode scores were the same. There was a very big range of scores.	8. This test was much too difficult for most people.

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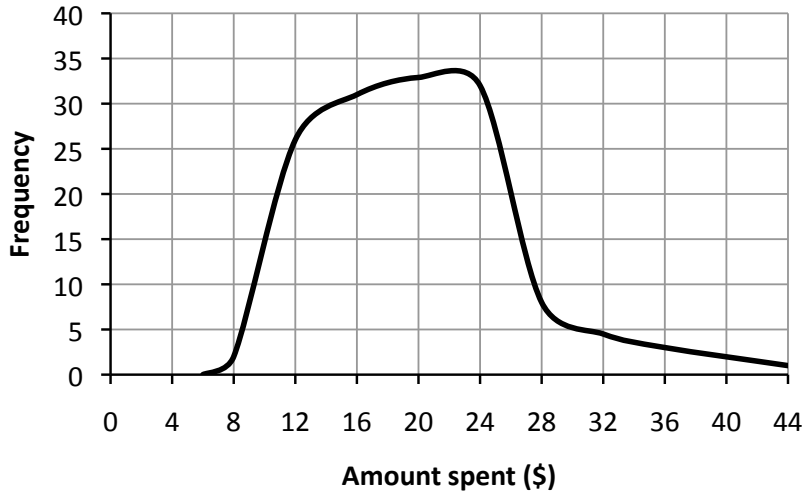
Cell Phones 1 (revisited)

Here is a frequency graph that shows the monthly spending of a group of students on their cell phones:



The graph shows:

A range of spending of .....

I know this because .....

.....

The mode is .....

I know this because .....

.....

The median is .....

I know this because .....

.....

.....

Very few students spend over .....

I know this because .....

.....

.....

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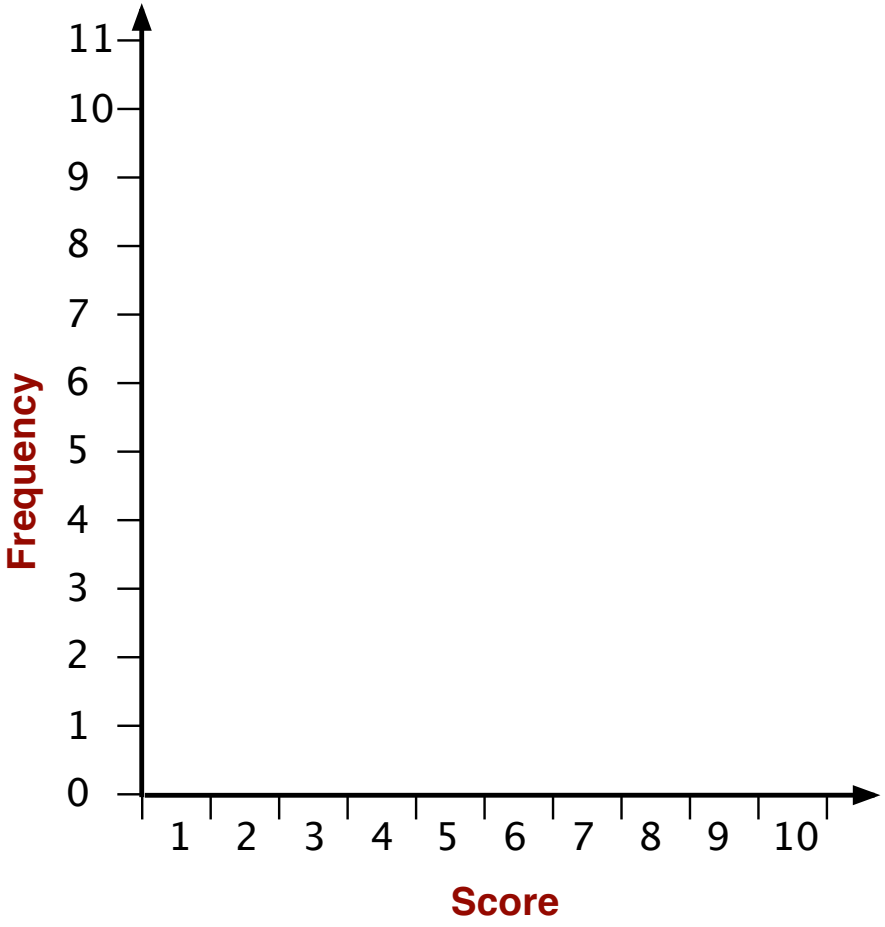
Bar Graphs

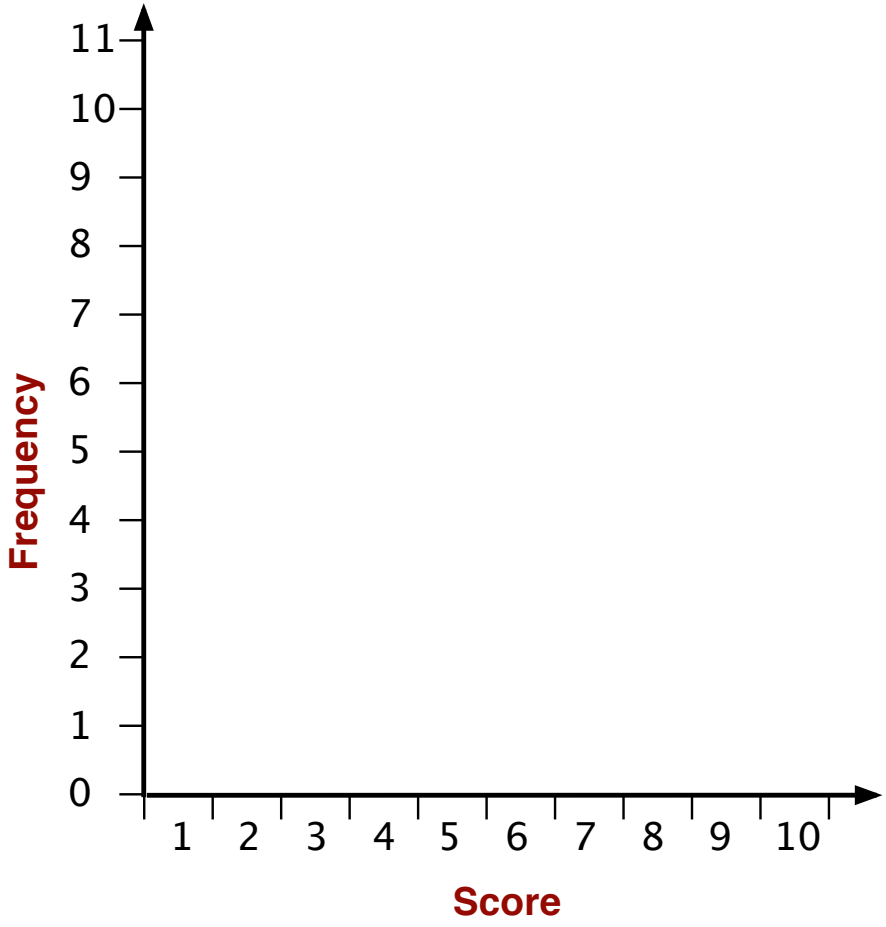
- Eleven students take a test.
- The test is out of 10.
- All students scored more than 5.
- At least one student scored each mark between 6 and 10.
- The mode is 9 out of 10.

Sketch on the *Drawing Graphs* handout two bar graphs that could represent the results of the test.

- On the first graph make sure the median is equal to the mode.
- On the second graph make sure the median is different from the mode.

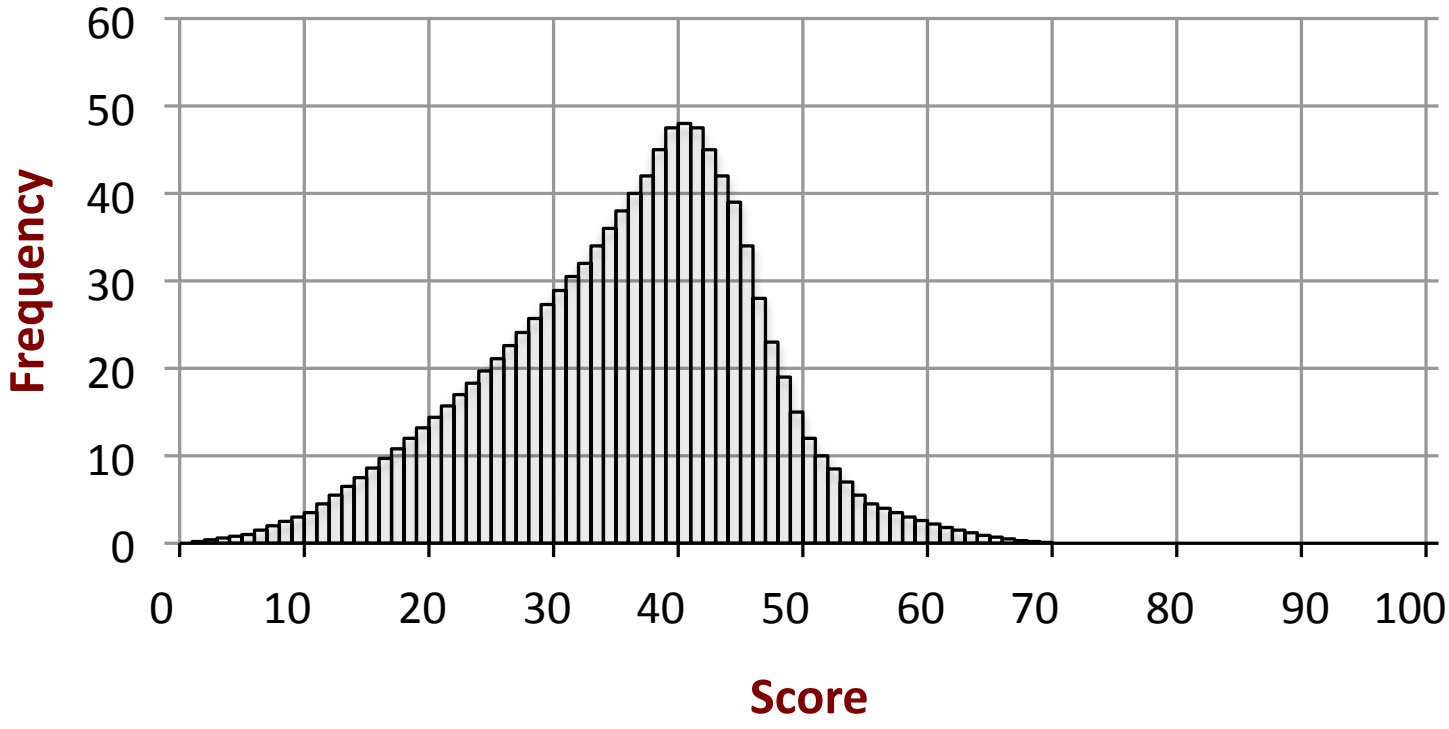
Mode and Median





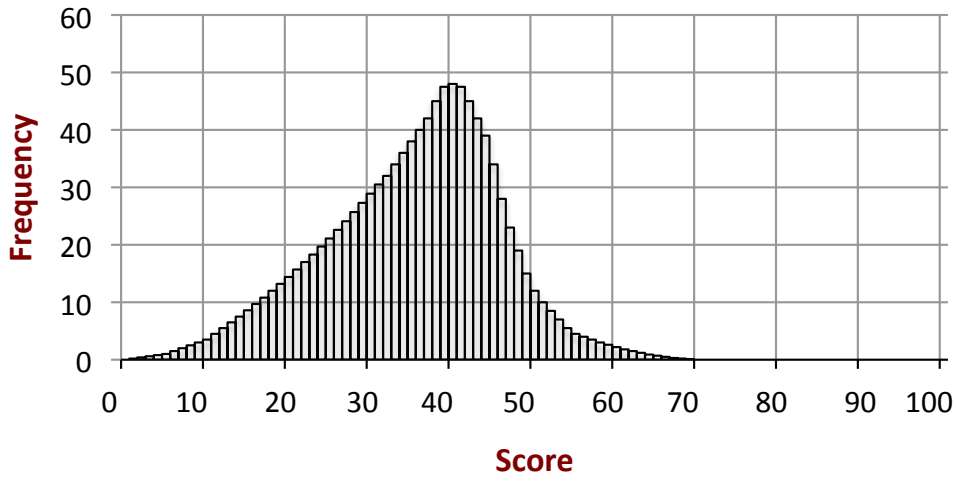
Discrete Representation

Frequency Bar Graph

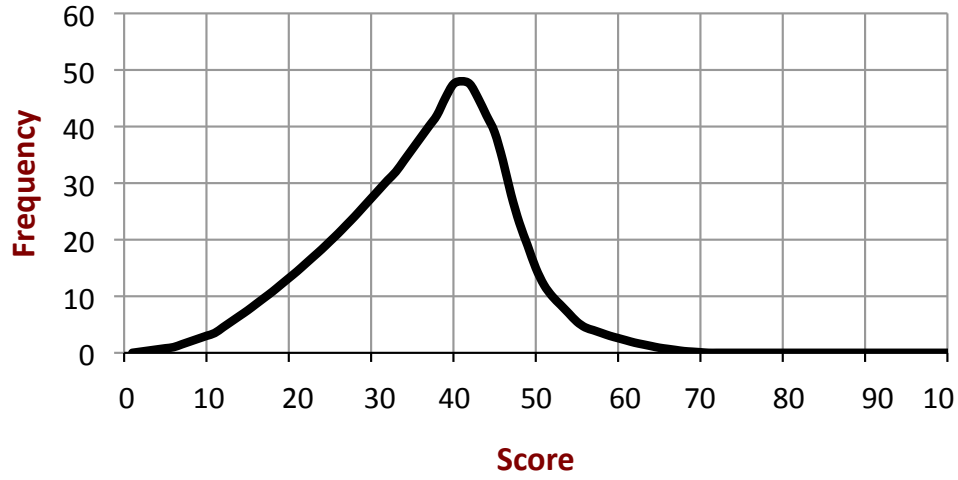


Discrete and Continuous Representations

Frequency Bar Graph

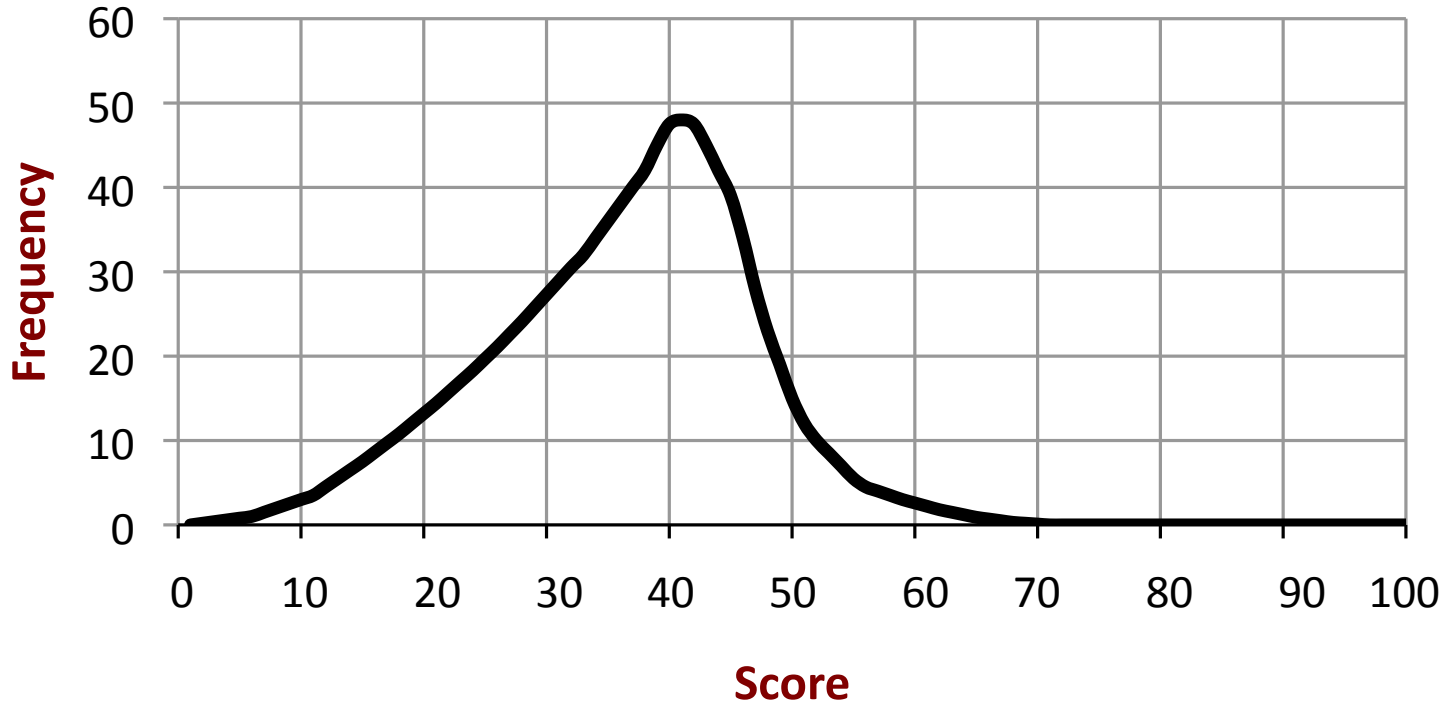


Frequency Line Graph



Continuous Representation

Frequency Line Graph



Matching Cards

1. Take turns at matching pairs of cards that you think belong together.

2. Each time you do this, explain your thinking clearly and carefully.

3. Your partner should either explain that reasoning again in his or her own words, or challenge the reasons you gave.

4. Write your reasons for the match on the poster.

5. You both need to be able to agree on and explain the placement of every card.

You may find some of 'Word' cards match two graphs.

This problem will be resolved as you match more cards.

Be prepared to change your mind about the matches.

Sharing Posters

1. If you are staying at your desk, be ready to explain the reasons for your group's matches.

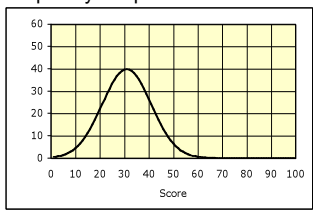
2. If you are visiting another group:
   - Copy your matches onto a piece of paper.
   - Go to another group's desk and check to see which matches are different from your own.

3. If there are differences, ask for an explanation. If you still don't agree, explain your own thinking.

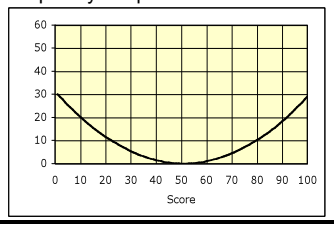
4. When you return to your own desk, you need to consider as a group whether to make any changes to your poster

Card Set: Frequency Graphs

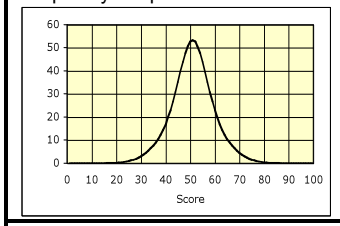
Frequency Graph A



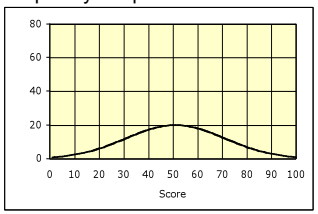
Frequency Graph B



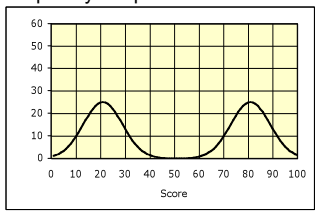
Frequency Graph C



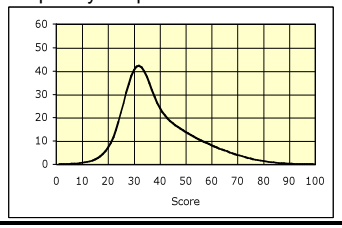
Frequency Graph D



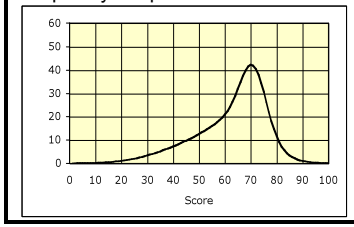
Frequency Graph E



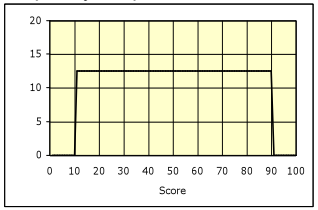
Frequency Graph F



Frequency Graph G



Frequency Graph H



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Mathematics Assessment Project

CLASSROOM CHALLENGES

This lesson was designed and developed by the
Shell Center Team
at the
University of Nottingham
Malcolm Swan, Nichola Clarke, Clare Dawson, Sheila Evans
with
Hugh Burkhardt, Rita Crust, Andy Noyes, and Daniel Pead

It was refined on the basis of reports from teams of observers led by
David Foster, Mary Bouck, and Diane Schaefer
based on their observation of trials in US classrooms
along with comments from teachers and other users.

This project was conceived and directed for
MARS: Mathematics Assessment Resource Service
by
Alan Schoenfeld, Hugh Burkhardt, Daniel Pead, and Malcolm Swan
and based at the University of California, Berkeley

We are grateful to the many teachers, in the UK and the US, who trialed earlier versions
of these materials in their classrooms, to their students, and to
Judith Mills, Carol Hill, and Alvaro Villanueva who contributed to the design.

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