CONCEPT DEVELOPMENT

Mathematics Assessment Project

CLASSROOM CHALLENGES

A Formative Assessment Lesson

Fractions, Decimals and Percents

Mathematics Assessment Resource Service
University of Nottingham & UC Berkeley
Beta Version

Fractions, Decimals and Percents

MATHEMATICAL GOALS

This lesson unit is intended to help students to:

- Compare, convert between and order fractions, decimals and percents.
- Use area and linear models of fractions, decimals and percents to understand equivalence.

COMMON CORE STATE STANDARDS

This lesson relates to the following *Standards for Mathematical Content* in the *Common Core State Standards for Mathematics*:

6.NS Apply and extend previous understandings of numbers to the system of rational numbers.

This lesson also relates to the following *Standards for Mathematical Practice* in the *Common Core State Standards for Mathematics*:

2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
7. Look for and make use of structure.

INTRODUCTION

This lesson unit is structured in the following way:

- Before the lesson, students work individually on an assessment task designed to reveal their current understanding. You then review their responses and create questions for students to consider when improving their work.
- Students work in small groups on a collaborative discussion task, placing decimal/percent, and fraction cards in order, along with area and linear diagrams that assist them in justifying and explaining their thinking.
- In a whole-class discussion, students discuss what they have learned.
- Finally, students revisit their initial work on the assessment task and work alone on a similar task to the introductory task.

MATERIALS REQUIRED

- Each student will need some blank paper and copies of the assessment tasks *Fractions, Decimals and Percents* and *Fractions, Decimals and Percents (Revisited)*.
- Each small group of students will need cut-up copies of all of the *Card Sets*, some poster paper and a glue stick.

TIME NEEDED

15 minutes before the lesson for the assessment task, an 80-minute lesson and 20 minutes in a follow-up lesson (or for homework). Exact timings will depend on the needs of the class.

BEFORE THE LESSON

Assessment task: *Fractions, Decimals and Percents* (15 minutes)

Have the students complete this task, in class or for homework, a few days before the formative assessment lesson. This will give you an opportunity to assess the work and to find out the kinds of difficulties students have with it. You should then be able to target your help more effectively in the follow-up lesson.

Give each student a copy of the assessment task *Fractions, Decimals and Percents* and briefly introduce the task:

> *You are asked to put numbers in order, or to check which number is greater, and then explain why.*
>
> *You may draw diagrams to help you explain, if you wish.*
>
> *Make sure that you explain your method clearly. I want to understand how you are working them out.*

It is important that, as far as possible, students are allowed to answer the questions without assistance. They should not have access to calculators.

Students should not worry too much if they cannot understand or do everything, because in the next lesson they will work on a similar task that should help them. Explain to students that by the end of the next lesson they should be able to answer questions such as these confidently. This is their goal.

Fractions, Decimals and Percents

1. Put the following decimals in order of size, starting with the one with the least value:

0.125 0.4 0.62 1.05 0.05

....., ..., ..., ..., ...,

Least **Greatest**

Explain your method for doing this.

2. Put the following fractions in order of size, starting with the one with the least value:

$\frac{3}{4}$ $\frac{9}{16}$ $\frac{3}{16}$ $\frac{7}{8}$ $\frac{3}{8}$

....., ..., ..., ..., ...,

Least **Greatest**

Explain your method for doing this.

3. Put a check mark in each case to say which number is larger.

Explain your answer each time on the dotted lines underneath.

(a) 40% ☐ or $\frac{1}{4}$ ☐

Explain how you know.

(b) 0.7 ☐ or $\frac{3}{5}$ ☐

Explain how you know.

(c) 33% ☐ or 0.4 ☐

Explain how you know.

Assessing students' responses

Collect students' responses to the task. Make some notes on what their work reveals about their current levels of understanding and their different problem-solving approaches.

We suggest that you do not score students' work. Research shows that this will be counterproductive, as it will encourage students to compare their scores and distract their attention from what they can do to improve their mathematics.

Instead, help students to make further progress by summarizing their difficulties as a series of questions. Some suggestions for these are given on the next page. These have been drawn from common difficulties observed in trials of this unit. We suggest you make a list of your own questions, based on your students' work.

We recommend you either:

- write one or two questions on each student's work, or
- give each student a printed version of your list of questions, and highlight appropriate questions for each student.

If you do not have time to do this, you could select a few questions that will be of help to the majority of students, and write these on the board when you return the work to the student at the end of the lesson.

Common issues	Suggested questions and prompts
Assume the 'length' of a decimal determines its relative size (Q1) For example: Students believe that 0.4 < 0.125 because 4 < 125 ('longer' decimals are greater). Or: Students believe that 0.4 > 0.62 because 0.4 is in tenths and 0.62 is in hundredths and tenths are greater than hundredths ('longer decimals are smaller'). Or: Students believe that 0.4 > 0.62 because 1/4 > 1/62 ('longer decimals are smaller').	• Can you show me on this number line where you would place 0.4, 0.125 and 0.62? • Which is greater in value: 0.5 or 0.50? Why? • Is it true that you can tell which of two decimals is bigger by counting the digits?
Compare numerators and denominators independently when comparing fractions For example: Students may reason that 3/4 < 9/16 because 3 < 9 and 4 < 16.	• Which is greater 2/3 or 4/6? Why? • If you double the numerator and denominator does this change the size of the fraction? Why? • Can you tell how big a fraction is by looking at the size of the numerator and denominator separately?
Ignore the numerator or denominator when comparing fractions For example: Students may reason that 9/16 < 3/8 because sixteenths are smaller than eighths (ignores numerator).	• Which is greater 3/8 or 6/16? Why? • Now which is greater 9/16 or 3/8?
Assumes that *n*% is the same as 1/*n* For example: Student appears to believe that 40% and ¼ are equivalent.	• Can you draw a diagram to show the meaning of ¼? • Can you draw a diagram to show the meaning of 40%?
Assumes that fractions are always smaller/greater than percents For example: Student says that ¼ is smaller than 40% because ¼ is a fraction.	• What do you understand by ¼? • What do you understand by 40%?

Focuses on the size of the digits rather than the entire number For example: Student says that 33% is greater than 0.4 because 33 is a greater number than 4 (or 0.4). Or: Student says that 0.7 is greater than 3/5 (which it is) because 7 is greater than 3 or 5.	• What is the difference between 33 and 33%? • What is the difference between 4 and 0.4? • Can you draw a diagram to show the meaning of 0.7? • Can you draw a diagram to show the meaning of 3/5?
Student answers all questions correctly with valid explanations	• For each of these pairs of numbers, can you find a number in between them in size? • Can you write each of your numbers as a fraction, decimal and percentage?

SUGGESTED LESSON OUTLINE

Introduction (5 minutes)

Remind students of the task they completed last lesson.

Do you remember the work you did on fractions, decimals and percents?

Today you are going to develop your understanding of fractions, decimals and percents further.

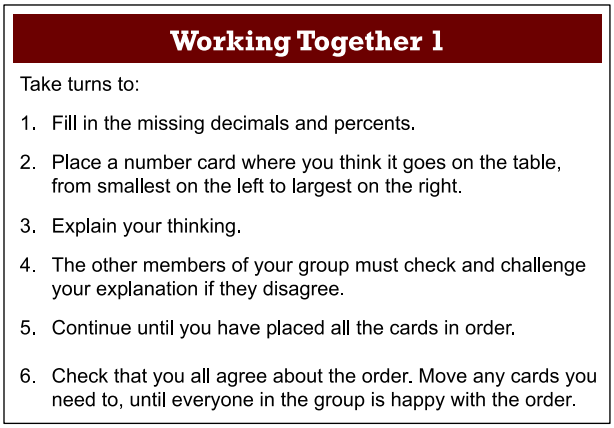
Collaborative small-group work (40 minutes)

Ask students to work in groups of two or three. Give each group cut-up copies of *Card Set A: Decimals and Percents* and some blank paper. Ask them to fill in the blanks on the cards.

Figure out the missing decimals or percents and fill them in on the cards.
Leave the card with neither a decimal nor percent for now.

When you have done that, place the cards in order from the smallest on the left to the largest on the right.

Explain how students are to work together, using Slide P-1:



Students may use their blank paper for rough calculations and to explain their thinking to each other. They should not use calculators.

The purpose of this task is to see what misconceptions students may have, so do not correct them if they place the cards in the wrong order. If students cannot agree on an order, you do not need to help them to resolve this at this stage, as the subsequent work in the lesson will help with that.

When most groups have reached a consensus about the cards, give out *Card Set B: Areas*.

Most groups have placed some of the cards correctly and some incorrectly. That's okay for the moment. Please leave the cards on the table.

I am going to give you some more cards, and I want you to match these to the decimals/percents cards.

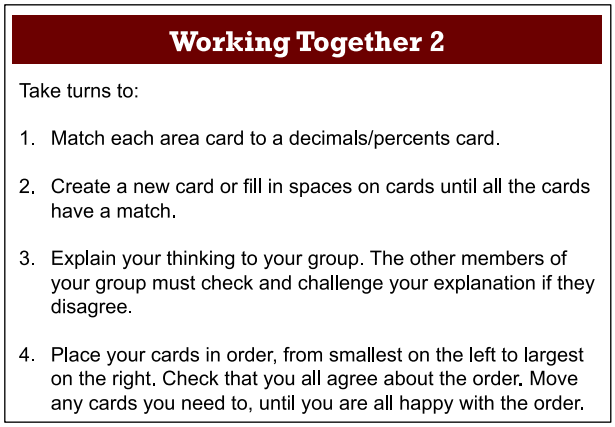
Cards that have the same value should go at the same position, underneath each other. Look up and down to make sure.

Fill in any gaps on the cards so that every card has a match. You will need to complete the blank card from Card Set A now too.

Check that the cards are in the right order, from smallest to greatest.

If you change your mind, then make a note of what you did wrong the first time.

Slide P-2 summarizes the instructions:



As groups continue to work, give out *Card Set C: Fractions* and *Card Set D: Scales*. The instructions are the same and should not need repeating. However, do make sure that students are completing the blank cards so that each number is represented in each of the four ways (decimal/percent; area; fraction; scale.)

The reason we are suggesting that you give the cards out in this order is that students usually associate decimals with number lines and fractions with areas. The process we describe here should encourage them to make connections that they do not normally make.

While students are working you have two tasks: to note different student approaches to the task and to support student problem solving.

Note different student approaches

Listen to and watch students carefully. Notice how students make a start on the task, where they get stuck, and how they overcome any difficulties. Which card sets do they find easiest/hardest to order? Which matches do they find easiest/hardest to make? What calculations do they perform? What sketches do they find helpful/unhelpful? What misconceptions are manifest? What disagreements are common?

In particular, notice whether students are addressing the difficulties they experienced in the assessment task. Note also any common mistakes. You may want to use the questions in the *Common issues* table to help address any misconceptions that arise.

Support student problem solving

Help students to work constructively together. Remind them to look at Slide P-2 for instructions on how to work. Check that students listen to each other and encourage them to do any necessary calculations or drawings on their blank paper.

Try not to solve students' problems or do the reasoning for them. Instead, you might ask strategic questions to suggest ways of moving forward:

> *If you're stuck with that card, you could put it to one side and place the others first.*
>
> *Can you find a fraction equivalent to this one?*
>
> *Which fraction card goes with this diagram?*
>
> *How might you express that card in words? Could you express it any other way? Could you make a drawing to represent it?*

Some groups may not manage to place all of the cards and it is not essential that they do so. It is more important that every student learns something from the cards that they try to place.

If a group of students finish placing all the cards and complete all the blank ones, ask them to create additional matching cards, perhaps with some constraints to make the challenge harder:

Can you make me a set of cards (decimal/percent; fraction; area; scale) that would lie between 3/4 and 6/10?

Can you make me a set of cards that would lie exactly half way between 1/20 and 3/4?

Making posters (15 minutes)

Once students have had a chance to match/order all 4 sets of cards, give them a piece of poster paper and a glue stick and ask them to glue their cards down in the agreed order. On their poster they need to justify their matches. If they changed their mind about the placement of a card(s) during the activity they should be encouraged to include details of this on their poster as well.

Whole-class discussion (20 minutes)

Conduct a whole-class discussion about what has been learned and explore the different orders in which the cards have been placed. What methods have students used? Have you noticed some interesting misconceptions? If so, you may want to focus the discussion on these.

Slides P-3 to P-6, which contain the different card sets, may be useful here.

Can someone tell us a card that they were very sure where to place? Why were you so sure?

Who agrees/disagrees? Why?

Does anyone have a different way of explaining it?

Does anyone have a card that they couldn't place or were very unsure about? Which one? Why? What do other people think?

Can someone say which card or cards they have at the far left, lowest, end?
What about at the highest end?

Which kinds of cards did you find the easiest/hardest to place? Why do you think that was?

If the class is confident with this work, you could ask more demanding questions:

Someone suggest a fraction, decimal, or percent that isn't on any of the cards.
Which cards will it lie between?
Which other representations should go with it? Why?

Draw out any issues you noticed as students worked on the activity, making specific reference to any misconceptions you noticed. You may want to use the questions in the *Common issues* table to support your discussion.

Follow-up lesson: reviewing the assessment task (20 minutes)

Give each student a copy of the review task, *Fractions, Decimals and Percents (Revisited)*, and their original scripts from the assessment task, *Fractions, Decimals and Percents*. If you have not added questions to individual pieces of work then write your list of questions on the board. Students should select from this list only those questions they think are appropriate to their own work.

Look at your original responses and the questions [on the board/written on your script]. Think about what you have learned.

Now look at the new task sheet, Fractions, Decimals and Percents (Revisited). Can you use what you have learned to answer these questions?

Some teachers give this as homework.

SOLUTIONS

Assessment task: *Fractions, Decimals and Percents*

1. The correct order is:

   0.05 (least); 0.125; 0.4; 0.62; 1.05 (greatest)

Watch for students who use an algorithm without understanding. For example, they fill in zeros so that each decimal has an equal length, then compare as if these are whole numbers. This method may give correct answers but does not usually contribute to the understanding of place value.

2. The correct order is:

   $\frac{3}{16}$ (least); $\frac{3}{8}$; $\frac{9}{16}$; $\frac{3}{4}$; $\frac{7}{8}$ (greatest)

One method is to convert them all to sixteenths: $\frac{3}{16}$; $\frac{6}{16}$; $\frac{9}{16}$; $\frac{12}{16}$; $\frac{14}{16}$.

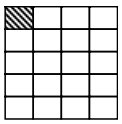
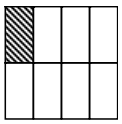
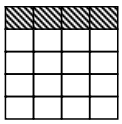
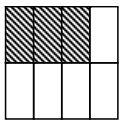
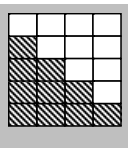
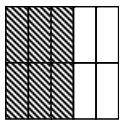
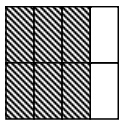
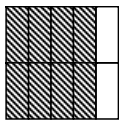
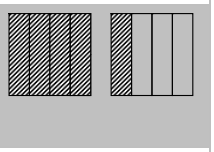
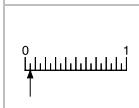
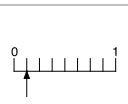
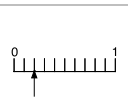
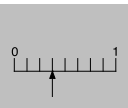
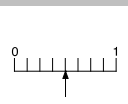
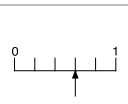
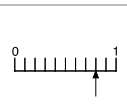
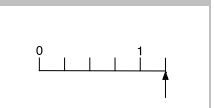
3(a) 40% is greater than $\frac{1}{4}$. Students might explain this by converting 40% to $\frac{2}{5}$ or $\frac{1}{4}$ to 25% or both to decimals. Alternatively they may draw pictures to illustrate.

(b) 0.7 is greater than $\frac{3}{5}$. Students might use similar methods.

(c) 0.4 is greater than 33%. Some students might think that 33% is $\frac{1}{3}$, which is approximately but not exactly true.

Collaborative task

The answers are given below, from smallest number on the left to largest on the right.

0.05 5%	0.125 12.5%	0.2 20%	0.375 37.5%	0.5 50%	0.6 60%	0.75 75%	0.8 80%	1.25 125%
$\frac{1}{20}$	$\frac{1}{8}$	$\frac{1}{5}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{6}{10}$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{5}{4}$
								
								

Shaded answers indicate missing ones that students needed to create for themselves. In the case of $\frac{1}{5}$

this could be $\frac{2}{10}$ etc. and for the area representing $\frac{1}{2}$ and the number line representing $\frac{3}{8}$ there are many possibilities. Equally there are various possible area cards for 1.25.

Assessment task: *Fractions, Decimals and Percents (Revisited)*

1. The correct order is:

   0.04 (least); 0.258; 0.4; 0.52; 1.25 (greatest)

2. The correct order is:

   $\frac{1}{4}$ (least); $\frac{5}{16}$; $\frac{1}{2}$; $\frac{5}{8}$; $\frac{3}{4}$ (greatest)

   One method is to convert them all to sixteenths: $\frac{4}{16}$; $\frac{5}{16}$; $\frac{8}{16}$; $\frac{10}{16}$; $\frac{12}{16}$.

3(a) 80% is greater than $\frac{1}{8}$. Students might explain this by converting 80% to $\frac{4}{5}$ or $\frac{1}{8}$ to 12.5% or both to decimals. Alternatively they may draw pictures to illustrate.

(b) $\frac{3}{4}$ is greater than 0.6. Students might use similar methods.

(c) 0.7 is greater than 7%. Students might use similar methods.

Fractions, Decimals and Percents

1. Put the following decimals in order of size, starting with the one with the least value:

0.125 0.4 0.62 1.05 0.05

.....,	.....,	.....,	.....,	.....,
Least				**Greatest**

Explain your method for doing this.

2. Put the following fractions in order of size, starting with the one with the least value:

$\frac{3}{4}$ $\frac{9}{16}$ $\frac{3}{16}$ $\frac{7}{8}$ $\frac{3}{8}$

.....,	.....,	.....,	.....,	.....,
Least				**Greatest**

Explain your method for doing this.

3. Put a check mark in each case to say which number is larger.

   Explain your answer each time on the dotted lines underneath.

   (a) 40% ☐ or $\frac{1}{4}$ ☐

   Explain how you know.

   (b) 0.7 ☐ or $\frac{3}{5}$ ☐

   Explain how you know.

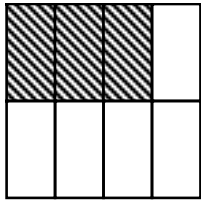
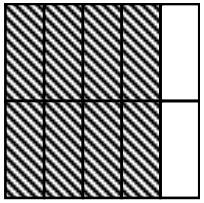
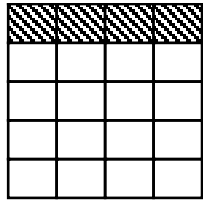
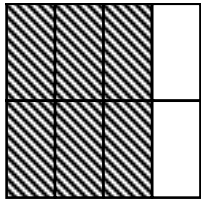
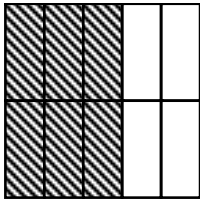
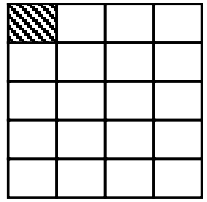
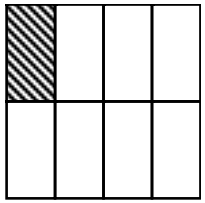
   (c) 33% ☐ or 0.4 ☐

   Explain how you know.

Card Set A: Decimals and Percents

0.2 ____%	0.05 ____%	__.____ 80%
0.375 ____%	__.____ 12.5%	0.75 ____%
1.25 ____%	__.____ 50%	__.____ ____%

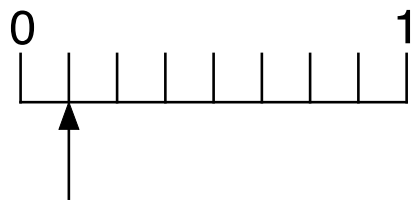
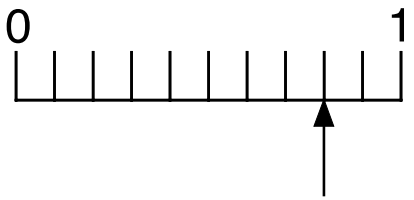
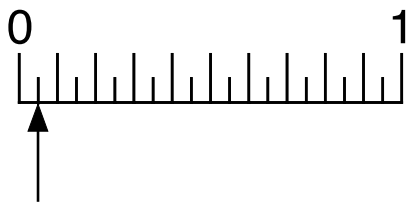
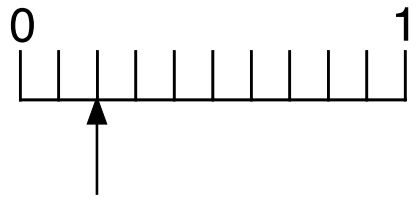
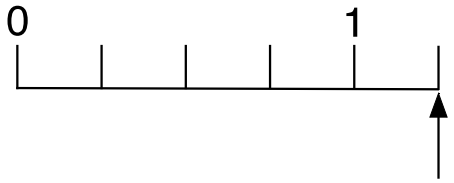
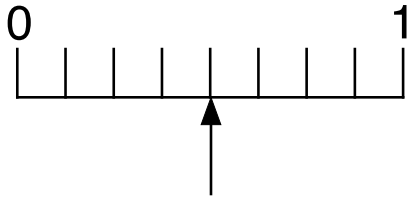
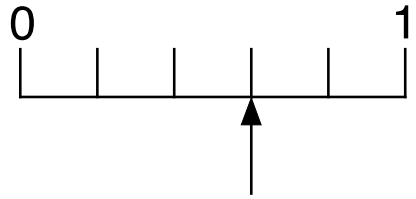
Card Set B: Areas

Area A 	Area B 	Area C 
Area D 	Area E 	Area F 
Area G 	Area H	Area I

Card Set C: Fractions

$\frac{3}{8}$	$\frac{4}{5}$	$\frac{1}{2}$
$\frac{3}{4}$	$\frac{6}{10}$	$\frac{5}{4}$
$\frac{1}{8}$		

Card Set D: Scales

Scale A 	Scale B 	Scale C 
Scale D 	Scale E 	Scale F 
Scale G 	Scale H	Scale I

Fractions, Decimals and Percents (Revisited)

1. Put the following decimals in order of size, starting with the one with the least value:

0.258 0.4 0.52 1.25 0.04

.....,	.....,	.....,	.....,	.....,
Least				**Greatest**

Explain your method for doing this.

2. Put the following fractions in order of size, starting with the one with the least value:

$\frac{5}{16}$ $\frac{1}{2}$ $\frac{3}{4}$ $\frac{5}{8}$ $\frac{1}{4}$

.....,	.....,	.....,	.....,	.....,
Least				**Greatest**

Explain your method for doing this.

3. Put a check mark in each case to say which number is larger.

   Explain your answer each time on the dotted lines underneath.

   (a) 80% ☐ or $\frac{1}{8}$ ☐

   Explain how you know.

   (b) 0.6 ☐ or $\frac{3}{4}$ ☐

   Explain how you know.

   (c) 7% ☐ or 0.7 ☐

   Explain how you know.

Working Together 1

Take turns to:

1. Fill in the missing decimals and percents.
2. Place a number card where you think it goes on the table, from smallest on the left to largest on the right.
3. Explain your thinking.
4. The other members of your group must check and challenge your explanation if they disagree.
5. Continue until you have placed all the cards in order.
6. Check that you all agree about the order. Move any cards you need to, until everyone in the group is happy with the order.

Working Together 2

Take turns to:

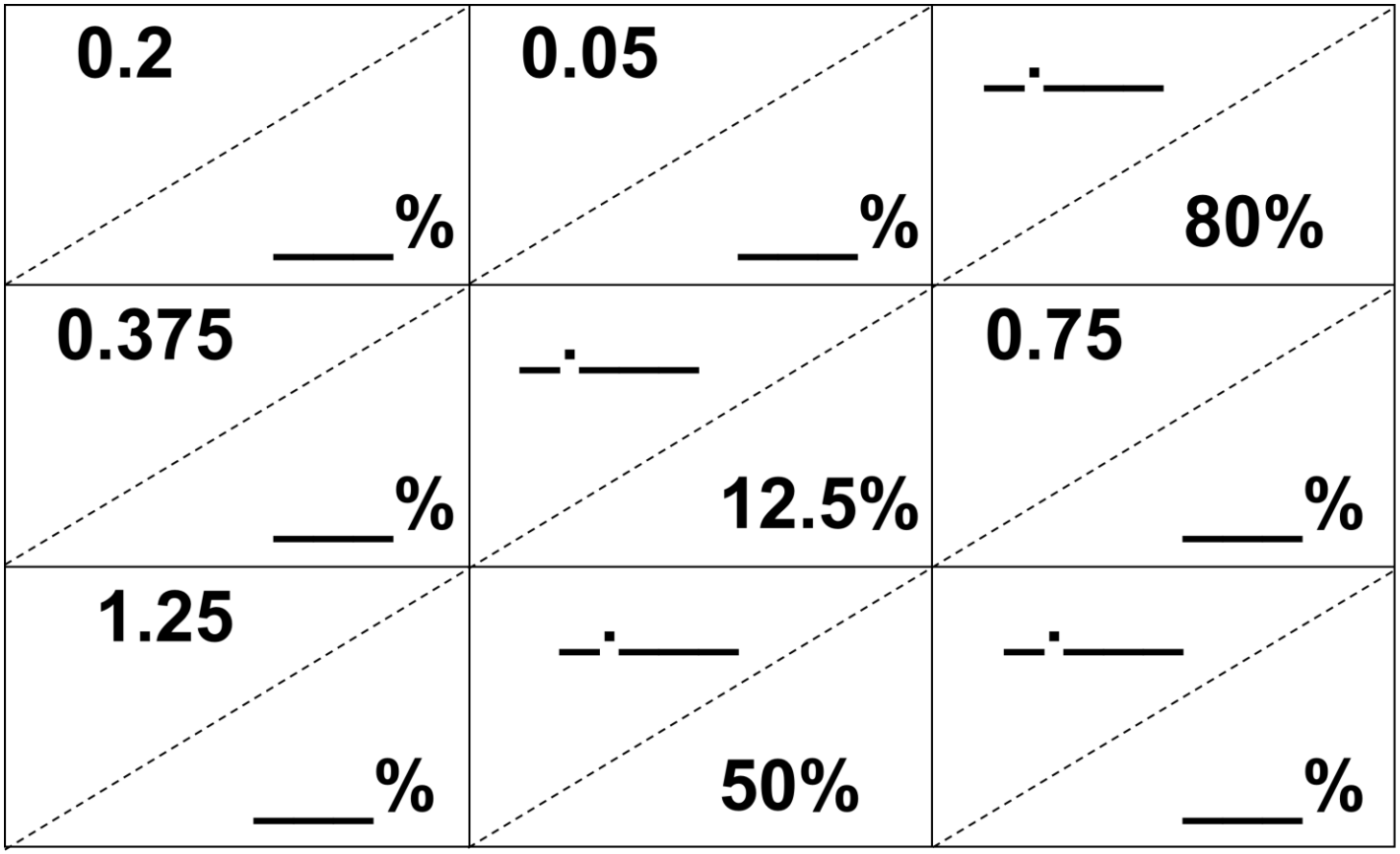
1. Match each area card to a decimals/percents card.

2. Create a new card or fill in spaces on cards until all the cards have a match.

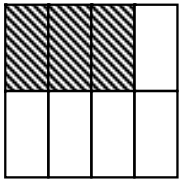
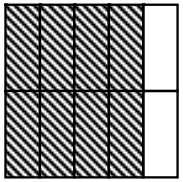
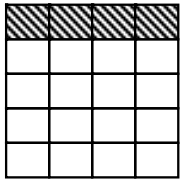
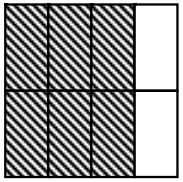
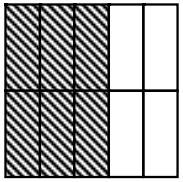
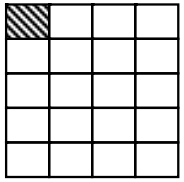
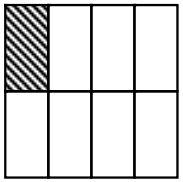
3. Explain your thinking to your group. The other members of your group must check and challenge your explanation if they disagree.

4. Place your cards in order, from smallest on the left to largest on the right. Check that you all agree about the order. Move any cards you need to, until you are all happy with the order.

Decimals and Percents



Areas

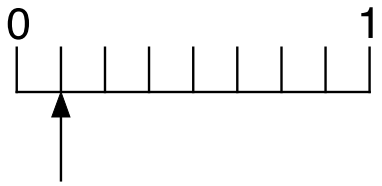
Area A 	Area B 	Area C 
Area D 	Area E 	Area F 
Area G 	Area H	Area I

Fractions

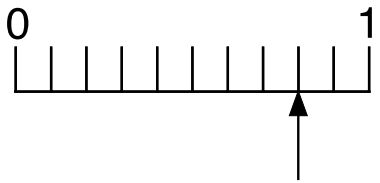
$\frac{3}{8}$	$\frac{4}{5}$	$\frac{1}{2}$
$\frac{3}{4}$	$\frac{6}{10}$	$\frac{5}{4}$
$\frac{1}{8}$		

Scales

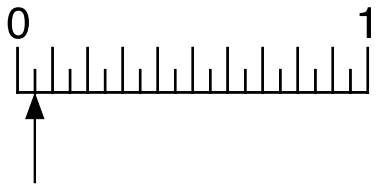
Scale A



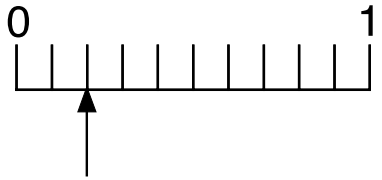
Scale B



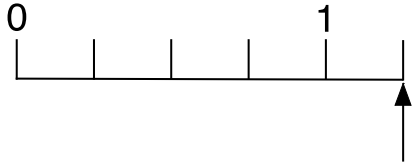
Scale C



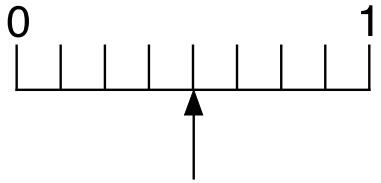
Scale D



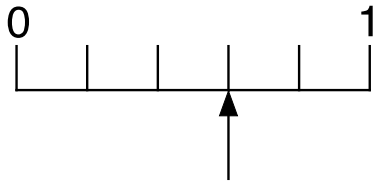
Scale E



Scale F



Scale G



Scale H

Scale I

Mathematics Assessment Project

CLASSROOM CHALLENGES

This lesson was designed and developed by the
Shell Center Team
at the
University of Nottingham
Malcolm Swan, Clare Dawson, Sheila Evans,
Marie Joubert and Colin Foster
with
Hugh Burkhardt, Rita Crust, Andy Noyes, and Daniel Pead

It was refined on the basis of reports from teams of observers led by
David Foster, Mary Bouck, and Diane Schaefer
based on their observation of trials in US classrooms
along with comments from teachers and other users.

This project was conceived and directed for
MARS: Mathematics Assessment Resource Service
by
Alan Schoenfeld, Hugh Burkhardt, Daniel Pead, and Malcolm Swan
and based at the University of California, Berkeley

We are grateful to the many teachers, in the UK and the US, who trialed earlier versions
of these materials in their classrooms, to their students, and to
Judith Mills, Mathew Crosier, Nick Orchard and Alvaro Villanueva who contributed to the design.

This development would not have been possible without the support of
Bill & Melinda Gates Foundation
We are particularly grateful to
Carina Wong, Melissa Chabran, and Jamie McKee