CONCEPT DEVELOPMENT

Mathematics Assessment Project

CLASSROOM CHALLENGES

A Formative Assessment Lesson

Using Coordinates to Interpret and Represent Data

Mathematics Assessment Resource Service
University of Nottingham & UC Berkeley
Beta Version

Using Coordinates to Interpret and Represent Data

MATHEMATICAL GOALS

This lesson unit is intended to help you assess how well students are able to work in the coordinate plane to solve a problem in a real-world context. In particular this unit aims to identify and help students who have difficulty measuring and interpreting horizontal or vertical intervals on graphs. This is needed later, when interpreting slope.

COMMON CORE STATE STANDARDS

This lesson relates to the following *Standards for Mathematical Content* in the *Common Core State Standards for Mathematics*:

6-G: Use coordinates to find the length of a side (here an interval) joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.

This lesson also relates to the following *Mathematical Practices* in the *Common Core State Standards for Mathematics*:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
7. Look for and make use of structure.

INTRODUCTION

This lesson unit is structured in the following way:

- Before the lesson, students work individually on an assessment task designed to reveal their current understanding and difficulties. You review their responses and create questions for them to consider when improving their work.
- After a brief introduction, students work collaboratively in small groups interpreting a graph. They then exchange their interpretation with a group that worked on a different graph. Each group sketches the other group's interpretation as a graph.
- Students then share their work.
- In a whole-class discussion, students review what they have learned.
- In a follow-up lesson students use their learning and your questions to review their work.

MATERIALS REQUIRED

- Each student will need a copy of the assessment tasks *A Growth Graph* and *Another Growth Graph*.
- Each small group of students will need a pair of scissors and at least one sheet of graph paper; either a copy of the sheets *Vending Machine Graph: Monday* and *Vending Machine Cards: Monday,* or the equivalent for *Tuesday*; the sheets *Vending Machine Graph For _ _ _ _ _ (day)* and *Blank Vending Machine Cards*; a few cut-up *Completed Cards: Monday* and *Completed Cards: Tuesday* sheets. These may be cut up or left whole, as you prefer.
- There is a projector resource to support whole-class discussions.

TIME NEEDED

20 minutes before the lesson, a 60-minute lesson and 20 minutes in a follow-up lesson (or for homework). Timings are approximate and will depend on the needs of the class.

BEFORE THE LESSON

Assessment task: *A Growth Graph* (20 minutes)

Have students complete this task, in class or for homework, a few days before the formative assessment lesson. This will give you an opportunity to assess the work, and to find out the kinds of difficulties students have with it. You should then be able to target your help more effectively in the follow-up lesson.

Give each student a copy of the assessment task: *A Growth Graph*.

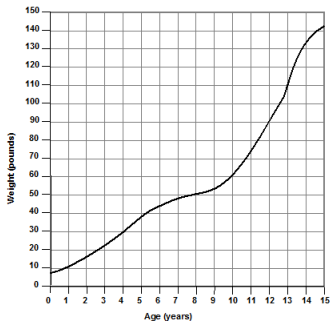
> *Read the task carefully and then answer the questions.*

It is important that, as far as possible, students are allowed to answer the questions without assistance. Some students may find it difficult to get started: be aware that if you offer help too quickly, students will merely do what you say and will not think for themselves. If, after several minutes, students are still struggling, try to help them understand what is required.

Students should not worry too much if they cannot understand or do everything, because in the next lesson they will engage in a similar task, which should help them. Explain to students that by the end of the next lesson, they should expect to answer questions such as these confidently. This is their goal.

A Growth Graph

An unusual animal, called Dek, is carefully monitored between birth and fifteen years old and its weight is represented by the graph below.



To answer the questions below you may want to write on the graph.

1. When does Dek weigh 50 pounds?
2. What does Dek weigh when it is 5½ years old?
3. In what year does Dek gain 20 pounds?
4. During which five-year period does Dek gain the least weight:
   ages 0 to 5; 5 to 10; or 10 to 15?
   Explain how you know.
5. In what two-year period does Dek gain the most weight?
   Explain how you know.
6. Another unusual animal, called Martha, is discovered.
   Martha's weight between birth and 15 years old is also recorded.
   Use the information below to sketch on Dek's graph, the growth curve for Martha.
   - Martha weighs the same as Dek from birth to the age of 4.
     The only other times Martha and Dek weigh the same are when they are 11 and 15 years old.
   - At 13 Martha weighs 20 pounds more than Dek.
     The difference between their weights is never more than this.
   - Only between the age of 9 and 13 does Martha gain more weight each year than Dek.

Assessing students' responses

Collect students' responses to the task. Make some notes on what their work reveals about their current levels of understanding and their different problem-solving approaches. We suggest that you do not score students' work. Research shows that this will be counter-productive as it will encourage students to compare their scores and distract their attention from what they can do to improve their mathematics.

Instead, help students to make further progress by summarizing their difficulties as a series of questions. Some suggestions for these are given in the *Common issues* table on the next page. These have been drawn from common difficulties observed in trials of this unit.

You may use questions from the list on page T-3 or make a list of your own questions, based on your students' work. We recommend that you either:

- Write one or two questions on each student's work, or
- Give each student a printed version of your list of questions and highlight the questions for each individual student.

If you do not have time to do this, you could select a few questions that will be of help to the majority of students and write these on the board when you return the work to the students at the end of the lesson.

Students receive your comments on their work in the follow-up lesson.

Common issues	Suggested questions and prompts
Student misreads a value on an axis	• Check your answers.
Student confuses value at a point with the length of an interval For example, The student figures out the age when Dek weighs 20 pounds not the year in which his weight increases by 20 pounds (Q3) Or: Student assumes that Dek puts on the most weight when he is heaviest, that is between the age of 13 and 15 (Q5)	• For the age you have written down, what is Dek's weight at the start of the year? What is his weight at the end of the year? • When does Dek weigh the most? • How much weight did he put on when he was 14? 13? 12?
Student has difficulty comparing the length of two intervals. For example, Student assumes that Dek puts on the least weight between 0 and 5 years (Q4)	• To answer the question what figures do you need? • How much does Dek weigh at the start and end of each five-year period? How can you use these figures to answer the question?
Student overlooks one or more constraints For example: The student does not take into account that the difference in weight is never more than 20 pounds. Or: The student draws a line that crosses Dek's line more or less than twice. Or: The student draws Martha's growth line with an incorrect gradient.	• Use the two lines on the graph to figure out the maximum difference between Dek's and Martha's weight. Does this match the information you are given? • How many times should the two lines cross? How do you know? • For **each** year between 9 and 13, who puts on the most weight? Does this match the information you are given?

SUGGESTED LESSON OUTLINE

Whole-class introduction (5 minutes)

The purpose of the introduction is to make sure students understand the context of the task. At this stage, there is no need to pre-empt any math issues, instead allow them to arise naturally as students collaboratively work on the task. You may find students can work together to overcome these issues.

Display Slide P-1 of the projector resource:

Explain to the class that office workers can purchase sodas from the vending machine.

For the task, students will use the 24-hour clock. You may need to check that they understand it:

Do you know what the 24-hour clock is?

What time is 14-o'clock?

Students may know the 24-hour clock as 'military time'. A time such as 2 pm may be given as 1400 (just four digits) or as 14:00, with a colon separating two pairs of digits. Students should be able to understand either notation.

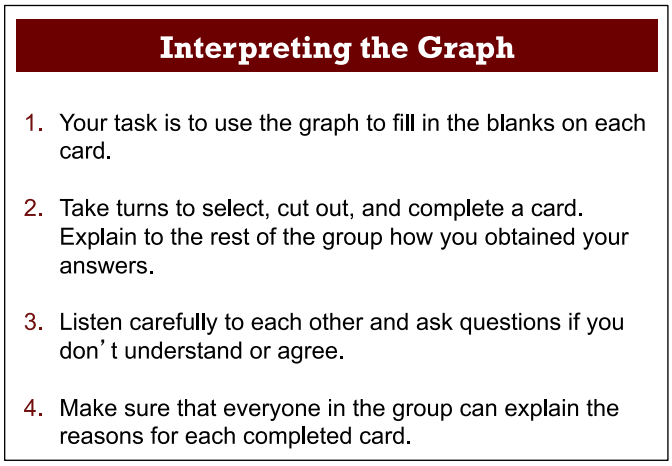
Collaborative small-group work: Interpreting the Data (15 minutes)

Organize students into pairs or threes. If you have not already cut out the cards, give each group a pair of scissors so that they can do it.

Give one half of the class a copy of the sheets *Vending Machine Graph: Monday* and *Vending Machine Cards: Monday* and the other half of the class a copy of the sheets *Vending Machine Graph: Tuesday* and *Vending Machine Cards: Tuesday*. The idea of doing this is that groups will later exchange completed sets of cards and use them to try to reconstruct the original graphs.

Each graph represents the number of sodas in a machine throughout a particular day.

Show and explain to students Slide P-2 of the projector resource:



Some details are missing on the cards. Use the graph to work out what to write on the cards.

When you have completed the cards, you will exchange them with a group that is working on a different set of cards for a different day.

The other group will use your completed cards to reconstruct the graph representing the number of sodas in the machine.

While students work in small groups you have two tasks: to note different student approaches to the task and to support student problem solving.

Note different student approaches to the task

Listen and watch students carefully. Note different approaches to the task. In particular, note any common mistakes. When figuring out the number of sodas bought in a specified period do students count horizontal or vertical intervals (lines), or do they subtract y-coordinate values? You can then use this information to focus a whole-class discussion towards the end of the lesson.

Support student problem solving

Try not to make suggestions that move students towards a particular approach to the task. Instead, ask questions that help students to clarify their thinking and encourage them to justify their answers.

You may want to use similar questions to the ones in the *Common issues* table to support your questioning. If a student has great difficulty getting started you could ask:

Could you sort the cards, where possible, from earliest to latest times?

If the whole class is struggling on the same issue, you could write one or two relevant questions on the board or hold a brief whole-class discussion.

What does a horizontal line on the graph represent?
[A time when there is no change in the number of sodas in the machine.]

What does a nearly vertical line represent?
[A sudden decrease (or increase) in the number of sodas in the machine.]

What is the quickest way to figure out the number of sodas bought in this time period (provide a particular time period).

Collaborative small-group work: Representing the Data (15 minutes)

As students complete the cards, collect the *Vending Machine Graph* and ask them to exchange their cards with another group that has completed a set of cards for a different day. They do not exchange graphs, as these have all been collected in.

If you find you are short of time, for groups that receive sets of *Vending Machine Cards* with few completed, you may want to give out some or all of the *Completed Cards* from the relevant day.

Give each group *Vending Machine Graph For _ _ _ _ _ _ _ (day)*.

Your task is to use the information on each card to sketch a graph of the number of sodas in the machine.

For a few minutes just look at the cards that you have been given and try to make sense of them.

Then show and explain to students Slide P-3 of the projector resource:

Representing the Data

1. Take turns to use the cards to draw part of the graph. Explain your reasoning to the rest of the group.
2. Listen carefully to each other and ask questions if you don't understand or agree.
3. Make sure that everyone in the group can explain all parts of the sketched graph.

Emphasize to students that the cards may not provide enough information to draw a complete, accurate graph, just a sketch. Support the students as in the first collaborative activity.

Sharing Work (15 minutes)

As students finish sketching a graph, ask students to share their solutions with the group they originally received the cards from.

Give each large group some *Blank Vending Machine Cards.*

Return to the students *Vending Machine Graph: Monday* and *Vending Machine Graph: Tuesday*.

Show and explain to students Slide P-4 of the projector resource:

Sharing Work

1. Check your sketched graphs against the printed ones.
2. Notice any differences and take turns to explain each of the differences.
3. If anything is unclear, ask for clarification. If you do not agree with the explanation, then explain why.
4. Together, consider if you should change any parts of your sketched graphs or answers on the cards.
5. Also consider what further information would have helped improve the accuracy of the sketched graph. Add this information to the blank cards.

Why do you think your sketched graph is different from the original one?
Do you think the graph or a card needs to be revised?

Encourage them to think carefully about what specific information would help significantly improve the accuracy of the sketched graph.

Whole-class discussion: Reviewing learning (10 minutes)

Hold a brief whole-class discussion of what has been learned.

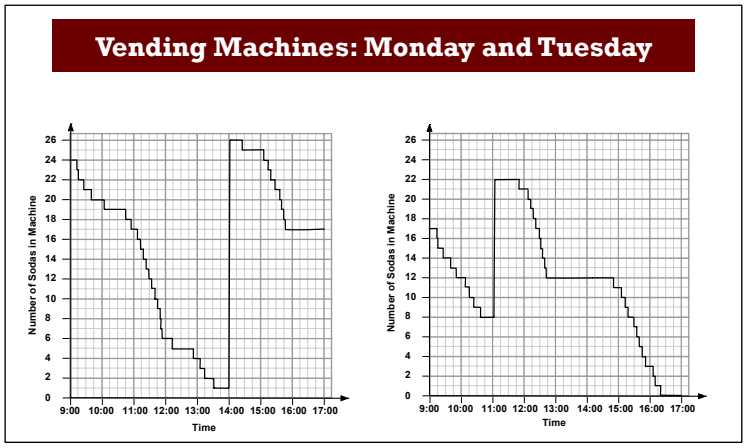
What did you find easy about the task?

What did you find difficult about the task?

What helped you to make progress with the task?

Ask students to compare graphs.

Slide P-5 of the projector resource shows both graphs.



Encourage students to speculate why the graphs are as they are; what could be the story behind the graphs?

What are the big differences between the two graphs?
[Students may notice the times the machine is filled up, the times when there are no purchases or the times when there are a lot of purchases etc.]

Can you think of a reason for each of these differences?

Are there any similarities between the graphs? [The total number of sodas bought each day, the number of sodas bought just after 9am, the period before 5pm when no sodas were bought, etc.] Can you think of a reason for each of these similarities?

What do you think the long vertical lines are telling us? Why?

How many people do you think worked in the office? Please explain.

Follow-up lesson: reviewing the assessment task (20 minutes)

Give each student a copy of the review task, *Another Growth Graph* and their original papers from the assessment task, *A Growth Graph*. If you have not added questions to individual pieces of work then write your list of questions on the board. Students should select from this list only those questions they think are appropriate to their own work.

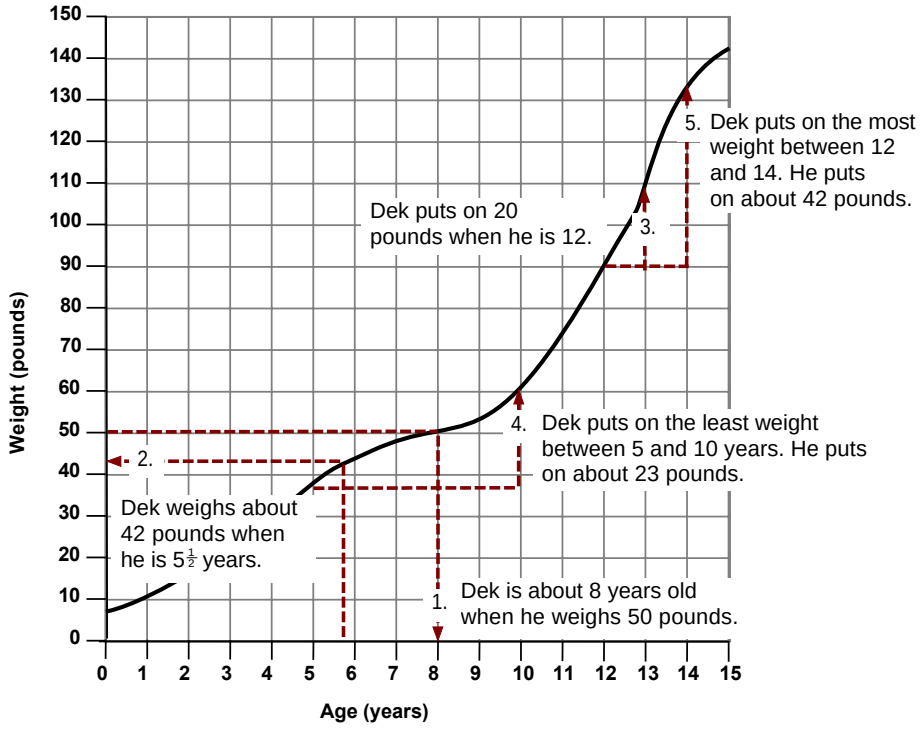
Look at your original responses and the questions [on the board/written on your paper.] Think about what you have learned.

Now look at the new task sheet, Another Growth Graph. Can you use what you have learned to answer these questions?

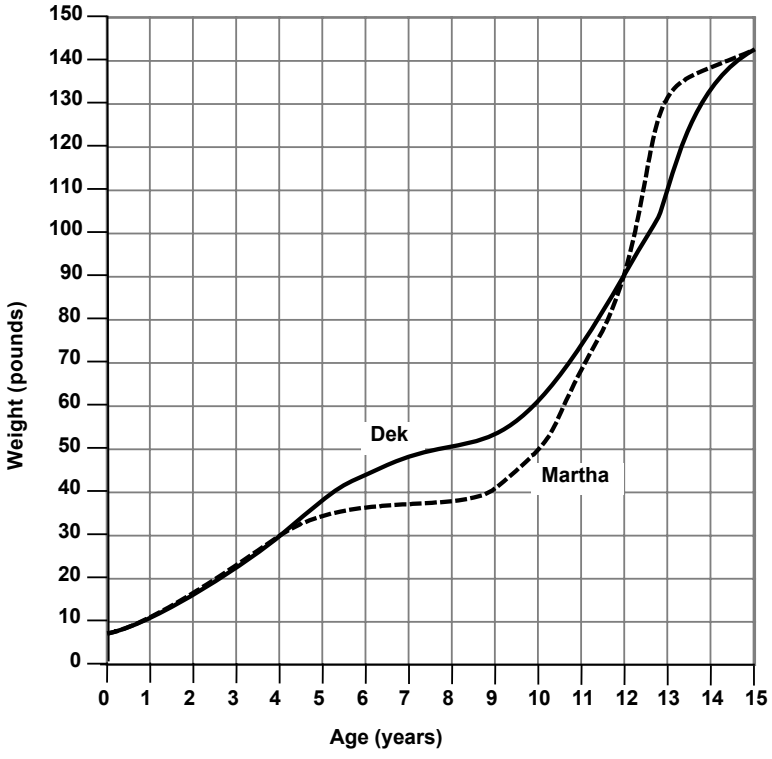
Some teachers give this as homework.

SOLUTIONS

Assessment task: *A Growth Graph*



Below is a possible sketch of Martha's growth curve:



Martha's curve will not necessarily look exactly like this, and the differences may still be consistent with the information given.

Lesson Task: *Vending Machines*

Monday

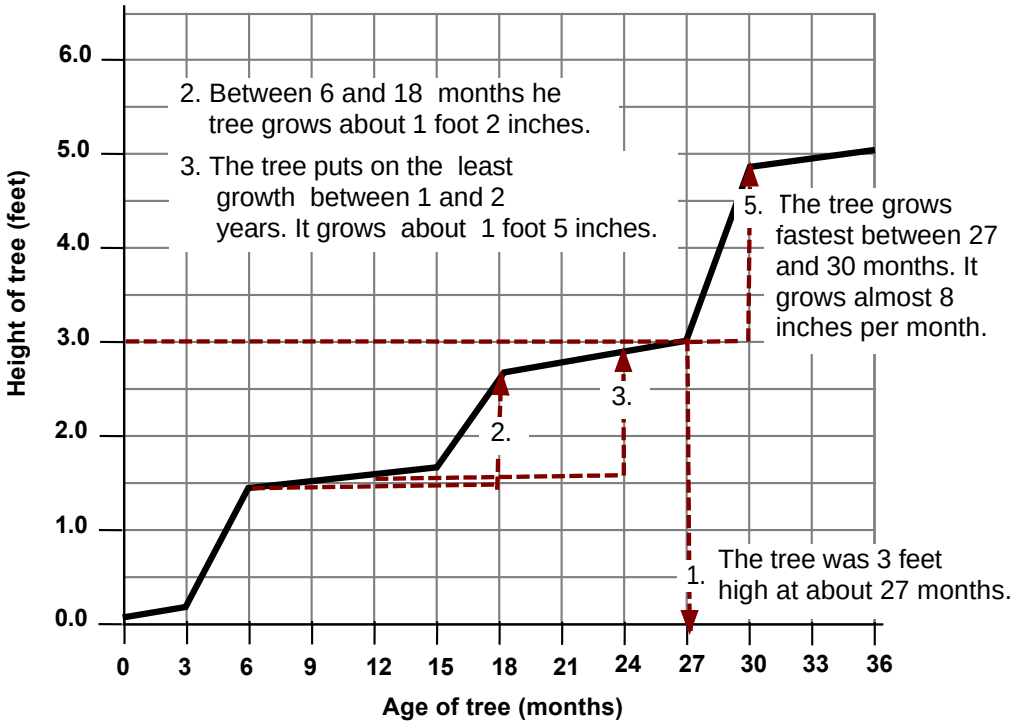
M1	The number of sodas bought between 13:00 and 15:00 was **4**	M2	The machine was filled up once, just after **14**:00. The number of sodas added to the machine was **25**
M3	The greatest number of sodas bought in one hour was between **11**:00 and **12**:00. In this hour **11** sodas were bought.	M4	The longest time when no sodas were bought was between **15:45 and 17:00**
M5	The number of sodas bought between 9:00 and 13:00 was **20**	M6	Between 15:00 and 16:00 twice the number of sodas were bought than between **9:00 and 10:00**
M7	There were just 24 sodas in the machine between **9:00 and 9:15 and 15:05 and 15:15**	M8	The least number of sodas in the machine was **1** This was between **13:30 and 14:00**
M9	There were less than 18 sodas in the machine. between **10:55 and 14:00 and 15:50 and 17:00**	M10	The number of sodas bought during the day was **32**

Tuesday

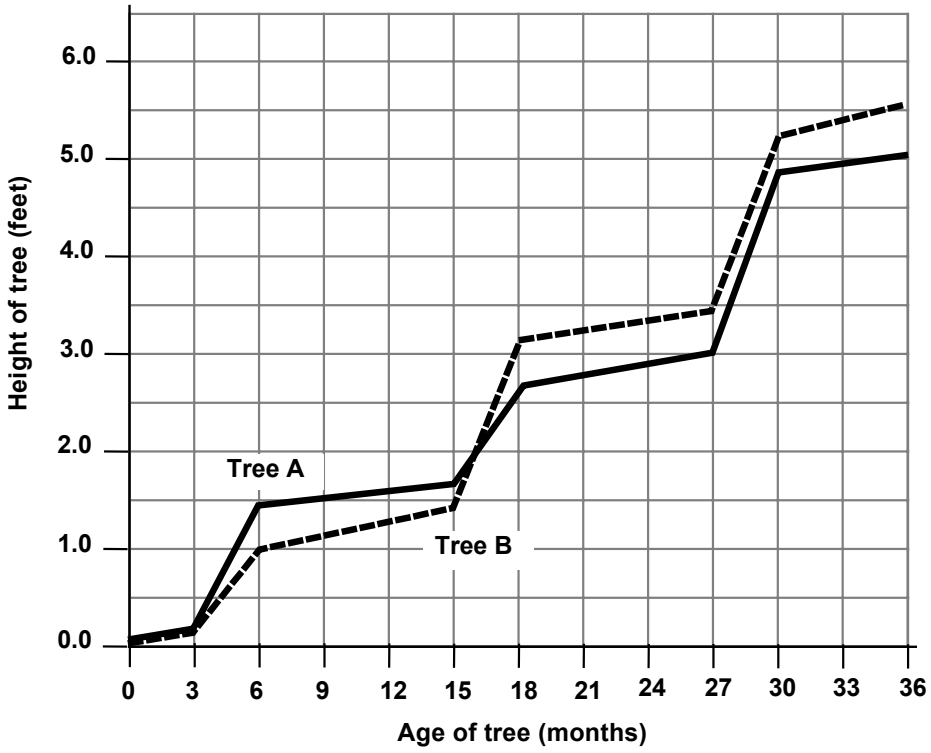
T1	The number of sodas bought between 10:00 and 12:00 was **5**	T2	The machine was filled up once, just after **11:00**. The number of sodas added to the machine was **14**
T3	There were less than 5 sodas in the machine between **15:45 and 17:00**	T4	The longest time when no sodas were bought was between **12:40 and 14:50** The number of sodas in the machine was **12**
T5	There were just 17 sodas in the machine between **9:00 and 9:15 and 12:20 and 12:30**	T6	Between 15:00 and 16:00 twice the number of sodas were bought than between **10:00 and 11:00**.
T7	The number of sodas bought between 11:00 and 17:00 was **22**	T8	The least number of sodas in the machine was **0** The time was between **16:20 and 17:00**
T9	At 10:00 the number of sodas in the machine was **12**	T10	The number of sodas bought during the day was **31**

Assessment task: *Another Growth Graph*

4. A possible reason for steps in the graph is that trees do most of their growing during spring.

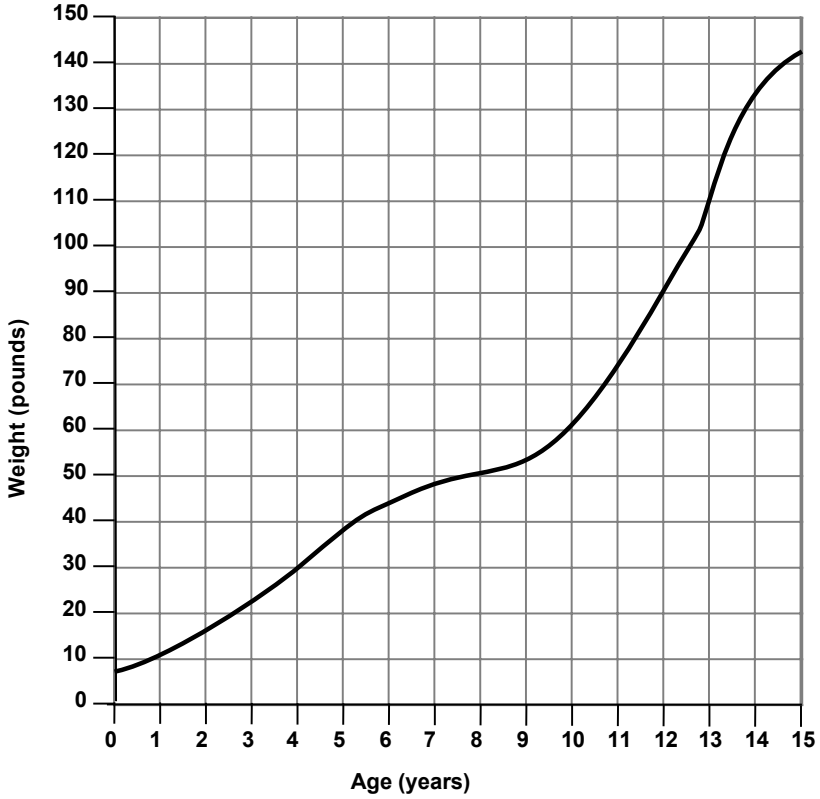


Below is a possible sketch of Tree B's growth:



A Growth Graph

An unusual animal, called Dek, is carefully monitored between birth and fifteen years old and its weight is represented by the graph below.



To answer the questions below you may want to write on the graph.

1. When does Dek weigh 50 pounds?

2. What does Dek weigh when it is 5½ years old?

3. In what year does Dek gain 20 pounds?

4. During which five-year period does Dek gain the least weight:
   ages 0 to 5; 5 to 10; or 10 to 15?
   Explain how you know.

5. In what two-year period does Dek gain the most weight?
   Explain how you know.

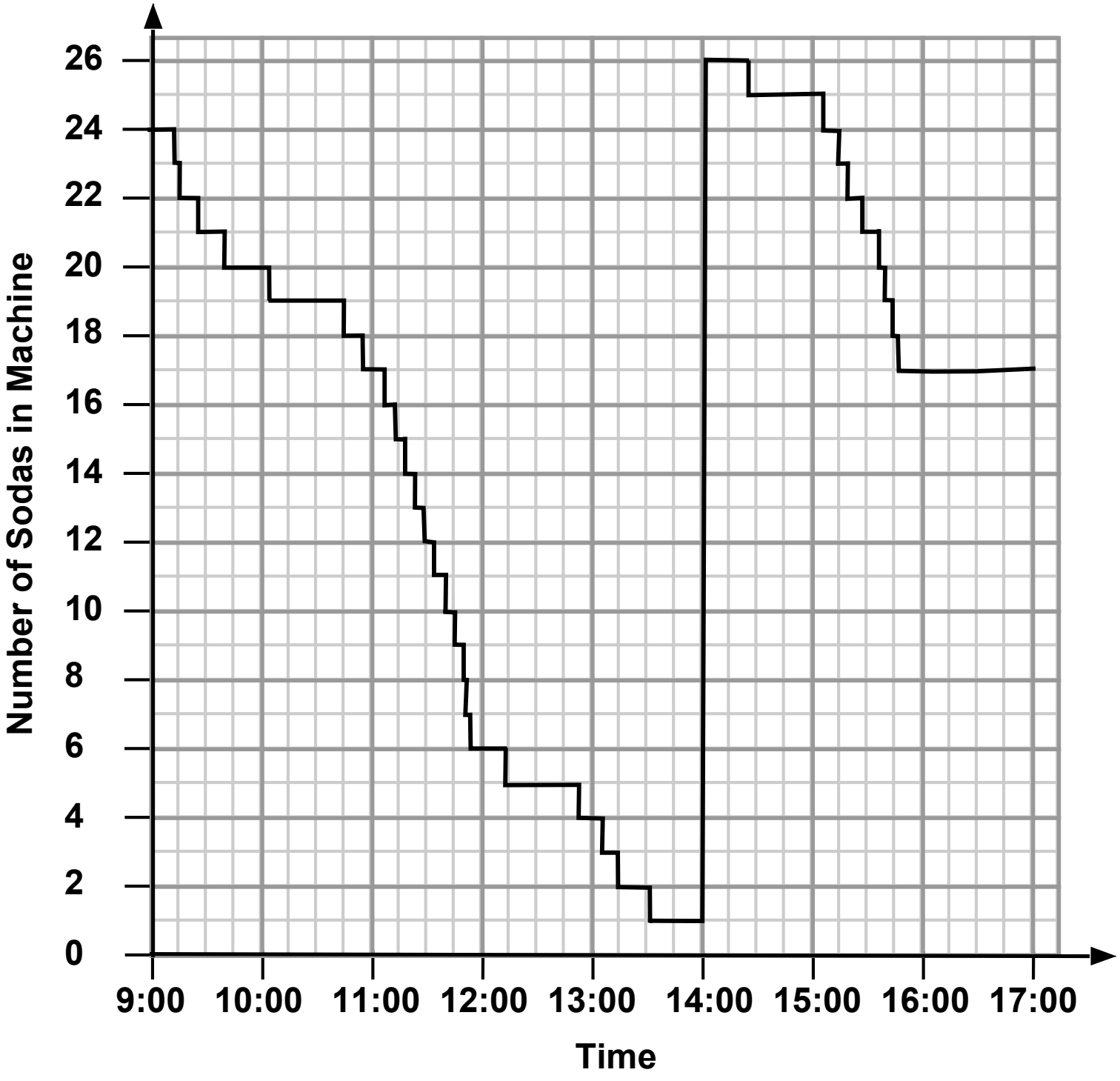
6. Another unusual animal, called Martha, is discovered.
   Martha's weight between birth and 15 years old is also recorded.
   Use the information below to sketch on Dek's graph the growth curve for Martha.

   - Martha weighs the same as Dek from birth to the age of 4.
     The only other times Martha and Dek weigh the same are when they are 12 and 15 years old.

   - At 13 Martha weighs 20 pounds more than Dek.
     The difference between their weights is never more than this.

   - Only between the age of 9 and 13 does Martha gain more weight each year than Dek.

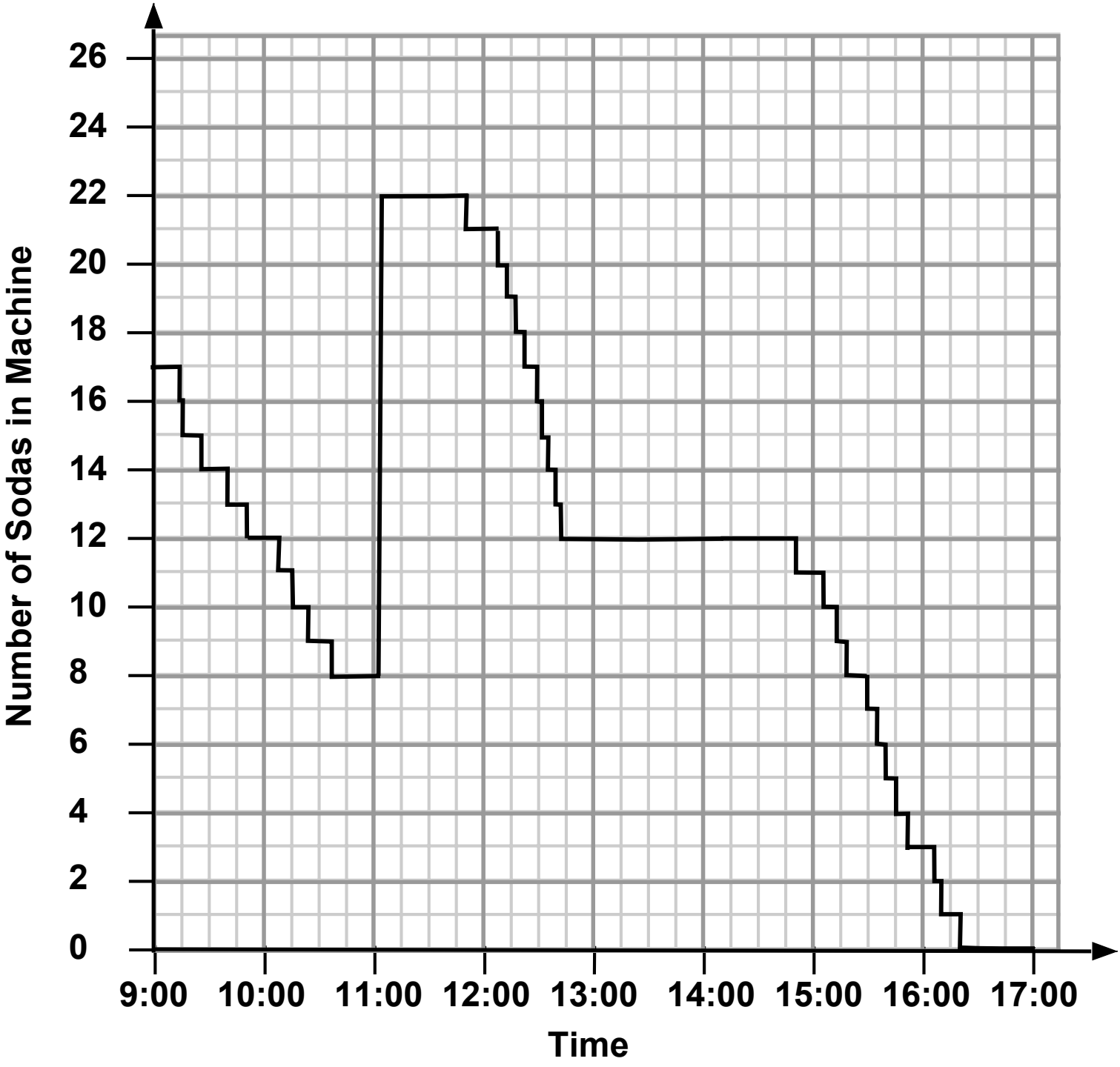
Vending Machine Graph: Monday



Vending Machine Cards: Monday

M1 The number of sodas bought between 13:00 and 15:00 was _ _ _	M2 The machine was filled up once, just after _ _ :00. The number of sodas added to the machine was _ _ _
M3 The greatest number of sodas bought in one hour was between _ _ :00 and _ _ :00. In this hour _ _ sodas were bought.	M4 The longest time when no sodas were bought was between _
M5 The number of sodas bought between 9:00 and 13:00 was _ _ _	M6 Between 15:00 and 16:00 twice the number of sodas were bought than between _ _ :00 and _ _:00
M7 There were just 24 sodas in the machine between _	M8 The least number of sodas in the machine was _ _ _ This was between _
M9 There were less than 18 sodas in the machine between _	M10 The number of sodas bought during the day was _ _ _

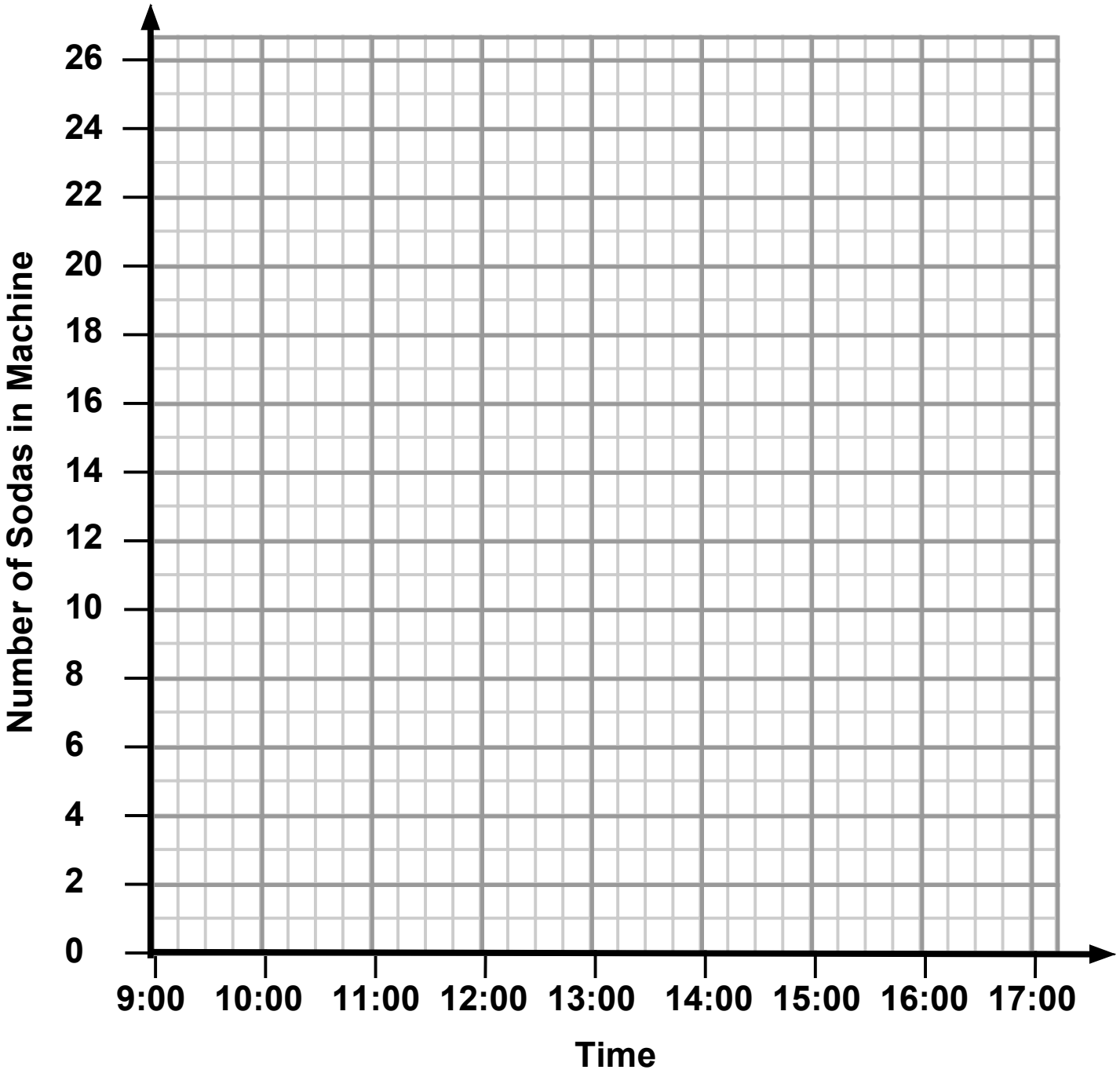
Vending Machine Graph: Tuesday



Vending Machine Cards: Tuesday

T1 The number of sodas bought between 10:00 and 12:00 was _ _ _	T2 The machine was filled up once, just after _ _ :00. The number of sodas added to the machine was _ _ _
T3 There were less than 5 sodas in the machine between _	T4 The longest time when no sodas were bought was between _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ The number of sodas in the machine was _ _ _
T5 There were just 17 sodas in the machine between _	T6 Between 15:00 and 16:00 twice the number of sodas were bought than between _ _ :00 and _ _:00.
T7 The number of sodas bought between 11:00 and 17:00 was _ _ _	T8 The least number of sodas in the machine was _ _ _ The time was between_ _
T9 At 10:00 the number of sodas in the machine was _ _ _	T10 The number of sodas bought during the day was _ _ _

Vending Machine Graph for _ _ _ _ _ _ _ *(day)*



Blank Vending Machine Cards

M11	M12
T11	T12

Completed Cards: Monday

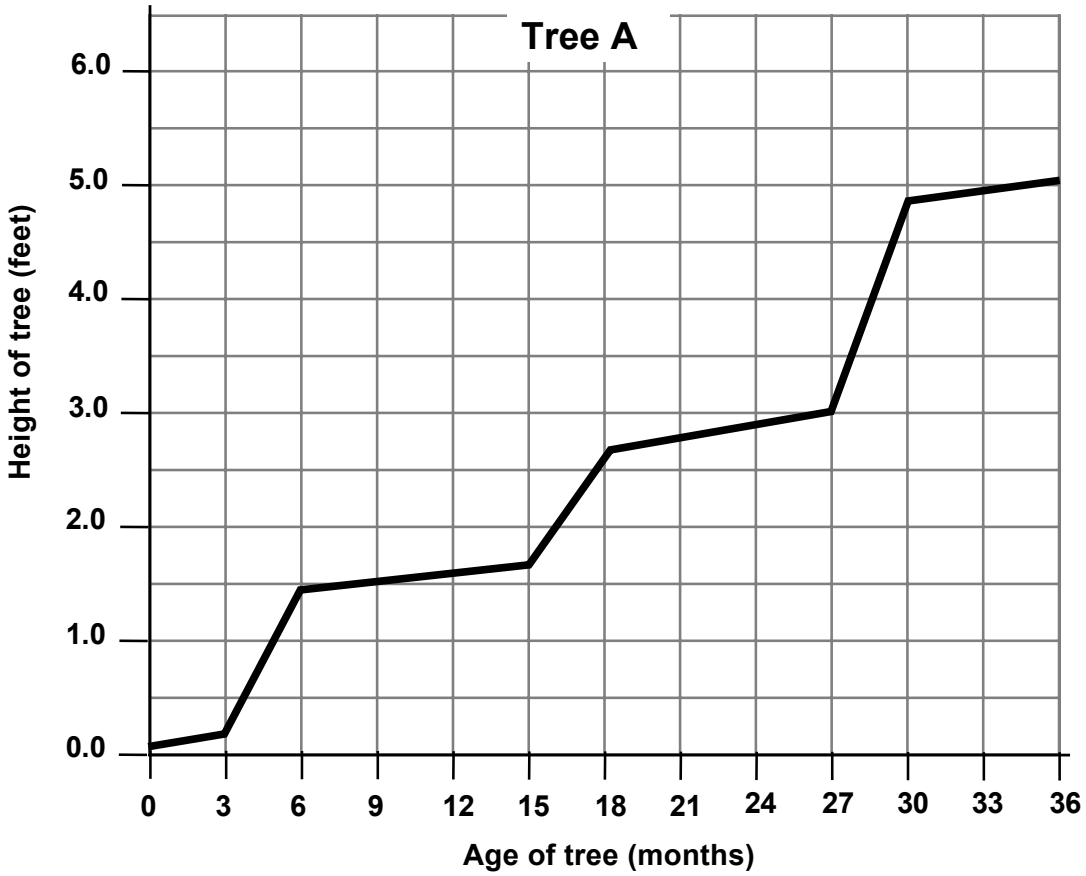
M1 The number of sodas bought between 13:00 and 15:00 was **4**	M2 The machine was filled up once, just after **14**:00. The number of sodas added to the machine was **25**
M3 The greatest number of sodas bought in one hour was between **11**:00 and **12**:00. In this hour **11** sodas were bought.	M4 The longest time when no sodas were bought was between **15:45 and 17:00**
M5 The number of sodas bought between 9:00 and 13:00 was **20**	M6 Between 15:00 and 16:00 twice the number of sodas were bought than between **9:00 and 10:00**
M7 There were just 24 sodas in the machine between **9:00 and 9:15** **and 15:05 and 15:15**	M8 The least number of sodas in the machine was **1** This was between **13:30 and 14:00**
M9 There were less than 18 sodas in the machine between **10:55 and 14:00** **and 15:50 and 17:00**	M10 The number of sodas bought during the day was **32**

Completed Cards: Tuesday

T1 The number of sodas bought between 10:00 and 12:00 was **5**	T2 The machine was filled up once, just after **11:00**. The number of sodas added to the machine was **14**
T3 There were less than 5 sodas in the machine between **15:45 and 17:00**	T4 The longest time when no sodas were bought was between **12:40 and 14:50** The number of sodas in the machine was **12**
T5 There were just 17 sodas in the machine between **9:00 and 9:15** **and 12:20 and 12:30**	T6 Between 15:00 and 16:00 twice the number of sodas were bought than between **10:00 and 11:00**.
T7 The number of sodas bought between 11:00 and 17:00 was **22**	T8 The least number of sodas in the machine was **0** The time was between **16:20 and 17:00**
T9 At 10:00 the number of sodas in the machine was **12**	T10 The number of sodas bought during the day was **31**

Another Growth Graph

A tree's growth over 36 months is represented by the graph below:



To answer the questions below you may want to write on the graph.

1. After how many months is the tree 3 feet high?

2. How much does the tree grow between six and eighteen months?

3. During which year does the tree grow the least?
   How much does it grow over this period?

4. What possible reason could there be for the steps in the graph?

5. Between which months does the tree grow the fastest? Explain how you know.

6. The height of another tree (**Tree B**) is also recorded.
Use the information below to sketch on the graph for **Tree A**, changes in height for **Tree B** over the same 36-month period:

   - For the first 3 months the two trees have the same height as each other.
   The only other time the two trees have the same height is at 16 months.
   - At 6 months **Tree A** is 6 inches taller than **Tree B**.
   The difference between their heights is never more than this.
   - At 27 months **Tree B** is 3½ feet tall.
   - For each month between 18 and 30 months the two trees put on the same height as each other.
   - Between 6 and 15 months and between 30 and 36 months **Tree B** grows more quickly than **Tree A**.

Office Vending Machine

Interpreting the Graph

1. Your task is to use the graph to fill in the blanks on each card.

2. Take turns to select, cut out, and complete a card. Explain to the rest of the group how you obtained your answers.

3. Listen carefully to each other and ask questions if you don’t understand or agree.

4. Make sure that everyone in the group can explain the reasons for each completed card.

Representing the Data

1. Take turns to use the cards to draw part of the graph. Explain your reasoning to the rest of the group.

2. Listen carefully to each other and ask questions if you don't understand or agree.

3. Make sure that everyone in the group can explain all parts of the sketched graph.

Sharing Work

1. Check your sketched graphs against the printed ones.

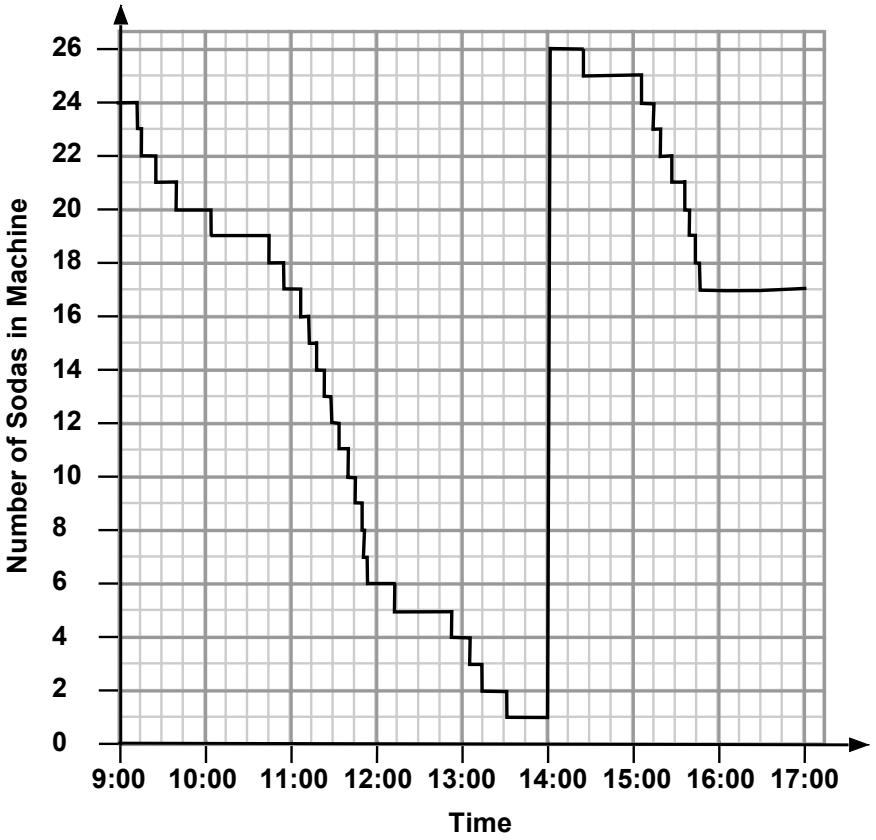
2. Notice any differences and take turns to explain each of the differences.

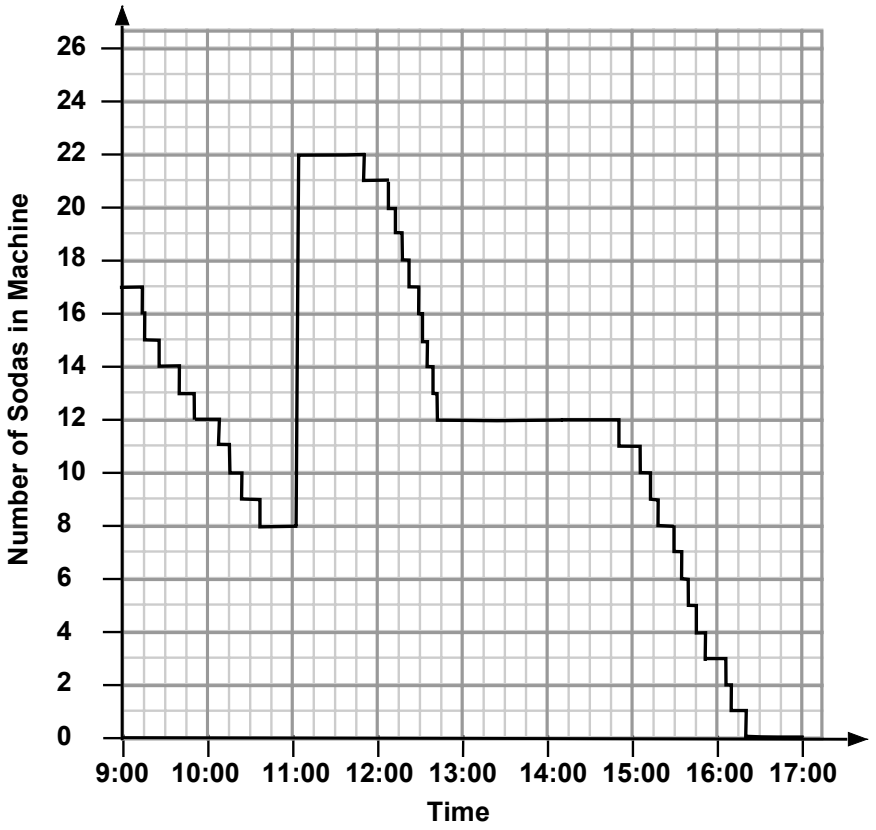
3. If anything is unclear, ask for clarification. If you do not agree with the explanation, then explain why.

4. Together, consider if you should change any parts of your sketched graphs or answers on the cards.

5. Also consider what further information would have helped improve the accuracy of the sketched graph. Add this information to the blank cards.

Vending Machines: Monday and Tuesday





Mathematics Assessment Project

CLASSROOM CHALLENGES

This lesson was designed and developed by the
Shell Center Team
at the
University of Nottingham
Malcolm Swan, Clare Dawson, Sheila Evans,
Marie Joubert and Colin Foster
with
Hugh Burkhardt, Rita Crust, Andy Noyes, and Daniel Pead

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David Foster, Mary Bouck, and Diane Schaefer
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Alan Schoenfeld, Hugh Burkhardt, Daniel Pead, and Malcolm Swan
and based at the University of California, Berkeley

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