



Lesson 10: True and False Equations

Student Outcomes

- Students understand that an equation is a statement of equality between two expressions. When values are substituted for the variables in an equation, the equation is either true or false. Students find values to assign to the variables in equations that make the equations true statements.

Classwork

Exercise 1 (5 minutes)

Give students a few minutes to reflect on Exercise 1. Then ask students to share their initial reactions and thoughts in answering the questions.

Exercise 1

- Consider the statement: "The President of the United States is a United States citizen."
Is the statement a grammatically correct sentence?
What is the subject of the sentence? What is the verb in the sentence? And what is the object of the sentence?
And finally, is the sentence true?
- Consider the statement: "The President of France is a United States citizen."
Is the statement a grammatically correct sentence?
What is the subject of the sentence? What is the verb in the sentence? And what is the object of the sentence?
And finally, is the sentence true?
- Consider the statement: " $2 + 3 = 1 + 4$."
This is a sentence. What is the verb of the sentence? What is the subject of the sentence and what is the object?
Is the sentence true?
- Consider the statement: " $2 + 3 = 9 + 4$."
Is this statement a sentence? And if so, is the sentence true or false?

Hold a general class discussion about parts (c) and (d) of the exercise. Be sure to raise the following points:

- One often hears the chime that "Mathematics is a language." And indeed it is. For us reading this text, that language is English. (And if this text were written in French that language would be French, or if this text were written in Korean, that language would be Korean.)

- A mathematical statement, such as $2 + 3 = 1 + 4$ is a grammatically correct sentence. The subject of the sentence is the numerical expression "2 + 3", its verb is "equals" or "is equal to," and its object is the numerical expression "1 + 4." We say that the statement is TRUE because these two numerical expressions evaluate to the same numerical value (namely, five).
- The mathematical statement $2 + 3 = 9 + 4$ is also a grammatically correct sentence, but we say it is FALSE because the numerical expression to the left (the subject of the sentence) and the numerical expression to the right (the object of the sentence) do not evaluate to the same numerical value.

(Perhaps remind students of parts (a) and (b) of the exercise: grammatically correct sentences can be false.)

- Recall the definition:

A *number sentence* is a statement of equality between two numerical expressions.

A number sentence is said to be *true* if both numerical expressions are equivalent (that is, both evaluate to the same number). It is said to be *false* otherwise. True and false are called *truth values*.

Exercise 2 (7 minutes)

Have students complete this exercise independently and then review the answers as a class.

Exercise 2

Determine whether the following number sentences are TRUE or FALSE.

a. $4 + 8 = 10 + 5$

FALSE

b. $\frac{1}{2} + \frac{5}{8} = 1.2 - 0.075$

TRUE

c. $(71 \cdot 603) \cdot 5876 = 603 \cdot (5876 \cdot 71)$

TRUE: The commutative and associative properties of multiplication demand these numerical expressions match.

d. $13 \times 175 = 13 \times 90 + 85 \times 13$

TRUE: Notice the right side equals $13 \times (90 + 85)$.

e. $(7 + 9)^2 = 7^2 + 9^2$

FALSE

f. $\pi = 3.141$

FALSE (The value of π is not exactly 3.141.)

g. $\sqrt{(4 + 9)} = \sqrt{4} + \sqrt{9}$

FALSE

h. $\frac{1}{2} + \frac{1}{3} = \frac{2}{5}$

FALSE

i. $\frac{1}{2} + \frac{1}{3} = \frac{2}{6}$

FALSE

j. $\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$

TRUE

k. $3^2 + 4^2 = 7^2$

FALSE

l. $3^2 \times 4^2 = 12^2$

TRUE

m. $3^2 \times 4^3 = 12^6$

FALSE

n. $3^2 \times 3^3 = 3^5$

TRUE

Exercise 3 (3 minutes)

Allow students to answer the questions in their student materials and then discuss with the class.

Exercise 3

- Could a number sentence be both TRUE and FALSE?
- Could a number sentence be neither TRUE nor FALSE?

A number sentence has a left-hand numerical expression that evaluates to a single number and has a right-hand numerical expression that also evaluates to a single numerical value. Either these two single values match or they do not. A numerical sentence is thus either TRUE or FALSE (and not both).

An *algebraic equation* is a statement of equality between two expressions.

Algebraic equations can be number sentences (when both expressions are numerical), but often they contain symbols whose values have not been determined.

Exercise 4 (6 minutes)

Exercise 4

a. Which of the following are algebraic equations?

i. $3.1x - 11.2 = 2.5x + 2.3$

ii. $10\pi^4 + 3 = 99\pi^2$

iii. $\pi + \pi = 2\pi$

iv. $\frac{1}{2} + \frac{1}{2} = \frac{2}{4}$

v. $79\pi^3 + 70\pi^2 - 56\pi + 87 = (60\pi + 29,928)/\pi^2$

All of them.

b. Which of them are also number sentences?

Numbers ii., iii., iv, and v. (Note that the symbol π has a value that has already stated / known.)

c. For each number sentence, state whether the number sentence is true or false.

ii. FALSE, iii. True, iv. False., v. False. Note that ii. And v. are both very close to evaluating to true. Some calculators may not be able to discern the difference. Wolfram Alpha's web based application can be used to reveal the differences.

Exercise 5 (9 minutes)

Discuss the three cases for algebraic equations given in the student materials, and based on the preparedness of your students, complete the exercise as a whole class, or in small groups, pairs, or individually.

Exercise 5

When algebraic equations contain a symbol whose value has not yet been determined, we use analysis to determine whether:

1. The equation is true for all the possible values of the variable(s), or
2. The equation is true for a certain set of the possible value(s) of the variable(s), or
3. The equation is never true for any of the possible values of the variable(s).

For each of the three cases, write an algebraic equation that would be correctly described by that case. Use only the variable, x , where x represents a real number.

1. $2(x + 3) = 2x + 6$; *By the distributive property, the two expressions on each side of the equal sign are algebraically equivalent; therefore, the equation is true for all possible real numbers, x .*
2. $x + 5 = 11$; *This equation is only a true number sentence if $x = 6$. Any other real number would make the equation a false number sentence.*
3. $x^2 = -1$; *There is no real number x that could make this equation a true number sentence.*

Share and discuss some possible answers for each.

Example 1 (4 minutes)

Example 1

Consider the following scenario.

Julie is 300 feet away from her friend's front porch and observes, "Someone is sitting on the porch."

Given that she didn't specify otherwise, we would assume that the "someone" Julie thinks she sees is a human. We can't guarantee that Julie's observatory statement is true. It could be that Julie's friend has something on the porch that merely looks like a human from far away. Julie assumes she is correct and moves closer to see if she can figure out who it is. As she nears the porch she declares, "Ah, it is our friend, John Berry."

- Often in mathematics, we observe a situation and make a statement we believe to be true. Just as Julie used the word "someone", in mathematics we use variables in our statements to represent quantities not yet known. Then, just as Julie did, we "get closer" to study the situation more carefully and find out if our, "someone" exists and if so "who" it is.
- Notice that we are comfortable assuming that the "someone" Julie referred to is a human, even though she didn't say so. In mathematics we have a similar assumption. If it is not stated otherwise, we assume that variable symbols represent a real number. But in some cases, we might say the variable represents an integer, or an even integer, or a positive integer, for example.
- Stating what type of number the variable symbol represents is called stating its **domain**.

Exercise 6 (6 minutes)

- In the sentence $w^2 = 4$, w can represent any real number we care to choose (its domain). If we choose to let w be 5, then the number sentence is false. If we let $w = 2$, then the sentence is true. Is there another value for w that would also make the sentence true?

▫ $w = -2$

Exercise 6

Name a value of the variable that would make each equation a true number sentence.

Here are several examples of how we can name the value of the variable:

Let $w = -2$, then $w^2 = 4$ is true.

Or:

$w^2 = 4$ is true when $w = -2$

Or:

$w^2 = 4$ is true if $w = -2$

Or:

$w^2 = 4$ is true for $w = -2$ and $w = 2$.

There might be more than one option for what numerical values to write. (And feel free to write more than one possibility.)

Warning: Some of these are tricky. Keep your wits about you!

- a. Let _____, then $7 + x = 12$ is true.
 $x = 5$
- b. Let _____, then $3r + 0.5 = \frac{37}{2}$ is true.
 $r = 6$
- c. $m^3 = -125$ is true for _____.
 $m = -5$
- d. A number x and its square, x^2 have the same value when _____.
 $x = 1$ or when $x = 0$.
- e. The average of 7 and n is -8 if _____.
 $n = -23$.
- f. Let _____, then $2a = a + a$ is true.
 $a = \text{any real number}$
- g. $q + 67 = q + 68$ is true for _____.
There is no value one can assign to q to turn this equation into a true statement.

Exit Ticket (5 minutes)

Name _____

Date _____

Lesson 10: True and False Equations

Exit Ticket

1. Consider the following equation, where a represents a real number: $\sqrt{a+1} = \sqrt{a} + 1$.

Is this statement a number sentence? If so, is the sentence TRUE or FALSE?

2. Suppose we are told that b has the value 4. Can we determine whether the equation below is TRUE or FALSE? If so, say which it is, if not, state that it cannot be determined. Justify your answer.

$$\sqrt{b+1} = \sqrt{b} + 1$$

3. For what value of c is the following equation TRUE?

$$\sqrt{c+1} = \sqrt{c} + 1$$

Exit Ticket Sample Solutions

1. Consider the following equation, where a represents a real number: $\sqrt{a+1} = \sqrt{a} + 1$.

Is this statement a number sentence? If so, is the sentence TRUE or FALSE?

No, it is not a number sentence because no value has been assigned to a . Thus, it is neither TRUE nor FALSE.

2. Suppose we are told that b has the value 4. Can we determine whether the equation below is TRUE or FALSE? If so, say which it is, if not, state that it cannot be determined. Justify your answer.

$$\sqrt{b+1} = \sqrt{b} + 1$$

FALSE, the left-hand expression has value $\sqrt{4+1} = \sqrt{5}$ and the right-hand expression has value $2 + 1 = 3$. These are not the same value.

3. For what value of c is the following equation TRUE?

$$\sqrt{c+1} = \sqrt{c} + 1$$

$\sqrt{c+1} = \sqrt{c} + 1$, if we let $c = 0$.

Problem Set Sample Solutions

Determine whether the following number sentences are true or false.

1. $18 + 7 = \frac{50}{2}$

TRUE

2. $3.123 = 9.369 \cdot \frac{1}{3}$

TRUE

3. $(123 + 54) \cdot 4 = 123 + (54 \cdot 4)$

FALSE

4. $5^2 + 12^2 = 13^2$

TRUE

5. $(2x2)^2 = \sqrt{256}$

TRUE

6. $\frac{4}{3} = 1.333$

FALSE

In the following equations, let $x = -3$ and $y = 2/3$. Determine whether the following equations are true, false, or neither true nor false.

7. $xy = -2$

TRUE

8. $x + 3y = -1$

TRUE

9. $x + z = 4$

Neither TRUE nor FALSE

10. $9y = -2x$

TRUE

11. $y/x = -2$

FALSE

12. $(-2/x)/y = -1$

FALSE

For each of the following, assign a value to the variable, x , to make the equation a true statement.

13. $(x^2 + 5)(3 + x^4)(100x^2 - 10)(100x^2 + 10) = 0$ for _____.

$x = \frac{1}{\sqrt{10}}$ or $x = -\frac{1}{\sqrt{10}}$.

14. $\sqrt{(x+1)(x+2)} = \sqrt{20}$ for _____.

$x = 3$ or $x = -6$

15. $(d + 5)^2 = 36$ for _____.

$d = 1$ or $d = -11$.

16. $(2z + 2)(z^5 - 3) + 6 = 0$ for _____.

$z = 0$ seems the easiest answer.

17. $\frac{1+x}{1+x^2} = \frac{3}{5}$ for _____.

$x = 2$ works.

18. $\frac{1+x}{1+x^2} = \frac{2}{5}$ for _____.

$x = 3$ works. So does $x = -\frac{1}{2}$

19. The diagonal of a square of side length L is 2 inches long when _____.

$L = \sqrt{2}$ inches.

20. $(T - \sqrt{3})^2 = T^2 + 3$ for _____.

$T = 0$

21. $\frac{1}{x} = \frac{x}{1}$ if _____.

$x = 1$ and also if $x = -1$

22. $\left(2 + \left(2 - \left(2 + \left(2 - (2 + r)\right)\right)\right)\right) = 1$ for _____.

$r = -1$

23. $x + 2 = 9$
for $x = 7$

24. $x + 2^2 = -9$
for $x = -13$

25. $-12t = 12$
for $t = -1$

26. $12t = 24$
for $t = 2$

27. $\frac{1}{b-2} = \frac{1}{4}$
for $b = 6$

28. $\frac{1}{2b-2} = -\frac{1}{4}$
for $b = -1$

29. $\sqrt{x} + \sqrt{5} = \sqrt{x+5}$
for $x = 0$

30. $(x - 3)^2 = x^2 + (-3)^2$
for $x = 0$

31. $x^2 = -49$
No real number will make the equation true

32. $\frac{2}{3} + \frac{1}{5} = \frac{3}{x}$
for $x = 45/13$

Fill in the blank with a variable term so that the given value of the variable will make the equation true.

33. $\underline{\hspace{1cm}} + 4 = 12$; $x = 8$
 x

34. $\underline{\hspace{1cm}} + 4 = 12$; $x = 4$
 $2x$

Fill in the blank with a constant term so that the given value of the variable will make the equation true.

35. $4y - \underline{\quad} = 100$; $y = 25$
0

36. $4y - \underline{\quad} = 0$; $y = 6$
24

37. $r + \underline{\quad} = r$; r is any real number
0

38. $r \times \underline{\quad} = r$; r is any real number
1

Generate the following:

39. An equation that is always true

40. An equation that is true when $x = 0$

41. An equation that is never true

42. An equation that is true when $t = 1$ or $t = -1$

43. An equation that is true when $y = -0.5$

44. An equation that is true when $z = \pi$

Problems 39 – 44: Answers vary