Lesson 4: Simplifying Square Roots

Student Outcomes

- Students use factors of a number to simplify a square root.

Lesson Notes

This lesson is optional. In this lesson, students learn to simplify square roots by examining the factors of a number and looking specifically for perfect squares. Students must learn how to work with square roots in Grade 8 in preparation for their work in Grade 9 and the quadratic formula. Though this lesson is optional, it is strongly recommended that students learn how to work with numbers in radical form in preparation for the work that they will do in Algebra I. Throughout the remaining lessons of this module students will work with dimensions in the form of a simplified square root and learn to express answers as a simplified square root to increase their fluency in working with numbers in this form.

Classwork

Opening Exercises 1–6 (5 minutes)

> **Opening Exercises 1–6**
>
> **MP.8**
>
> 1.
>   a.   What does $\sqrt{16}$ equal?
>   b.   What does 4×4 equal?
>   c.   Does $\sqrt{16} = \sqrt{4 \times 4}$?
>
> 2.
>   a.   What does $\sqrt{36}$ equal?
>   b.   What does 6×6 equal?
>   c.   Does $\sqrt{36} = \sqrt{6 \times 6}$?
>
> 3.
>   a.   What does $\sqrt{121}$ equal?
>   b.   What does 11×11 equal?
>   c.   Does $\sqrt{121} = \sqrt{11 \times 11}$?
>
> 4.
>   a.   What does $\sqrt{81}$ equal?
>   b.   What does 9×9 equal?
>   c.   Does $\sqrt{81} = \sqrt{9 \times 9}$?
>
> 5.   What is another way to write $\sqrt{20}$?
> $$\sqrt{20} = \sqrt{4 \times 5}$$
>
> 6.   What is another way to write $\sqrt{28}$?
> $$\sqrt{28} = \sqrt{4 \times 7}$$

Discussion (7 minutes)

- We know from the last lesson that square roots can be simplified to a whole number when they are perfect squares. That is,

$$\sqrt{9} = \sqrt{3 \times 3} = \sqrt{3^2} = 3$$

- Given x^2 (x is a positive integer and x squared is a perfect square), it is easy to see that when $C = \sqrt{x^2}$ and $D = x$ that $C = D$, where C and D are positive numbers. In terms of the previous example, when $C = \sqrt{9} = \sqrt{3^2}$ and $D = 3$, then $3 = 3$.

- We can show that this is true even when we do not have perfect squares. All we need to show is that when C and D are positive numbers and n is a positive integer, that $C^n = D^n$. If we can show that $C^n = D^n$, then we know that $C = D$.

Scaffolding:
- If necessary, remind students about numbers and their squares using the visual below.
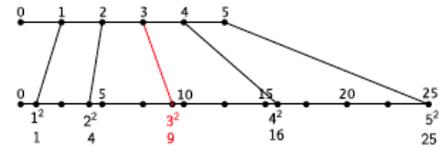
Ask students to explain why $C^n = D^n$ implies $C = D$. They should reference the definition of exponential notation that they learned in Module 1. For example, since $C^n = \underbrace{C \times C \times \cdots \times C}_{n}$ and $D^n = \underbrace{D \times D \times \cdots \times D}_{n}$, and we are given that

$\underbrace{C \times C \times \cdots \times C}_{n} = \underbrace{D \times D \times \cdots \times D}_{n}$, then C must be the same number as D.

- Now, for the proof that the nth root of a number can be expressed as a product of the nth root of its factors:

Let $C = \sqrt[n]{ab}$ and $D = \sqrt[n]{a} \times \sqrt[n]{b}$, where a and b are positive integers and n is a positive integer greater than or equal to 2. We want to show that $C^n = D^n$.

Scaffolding:
- Students may know the term *product* in other contexts. For that reason, student may need to be reminded that it refers to multiplication in this context.

$$C^n = \left(\sqrt[n]{ab}\right)^n$$
$$= \underbrace{\sqrt[n]{ab} \times \sqrt[n]{ab} \times \cdots \times \sqrt[n]{ab}}_{n \text{ times}}$$
$$= ab$$

$$D^n = \left(\sqrt[n]{a} \times \sqrt[n]{b}\right)^n$$
$$= \underbrace{\left(\sqrt[n]{a} \times \sqrt[n]{b}\right) \times \left(\sqrt[n]{a} \times \sqrt[n]{b}\right) \times \cdots \times \left(\sqrt[n]{a} \times \sqrt[n]{b}\right)}_{n}$$
$$= \underbrace{\sqrt[n]{a} \times \sqrt[n]{a} \times \cdots \times \sqrt[n]{a}}_{n \text{ times}} \times \underbrace{\sqrt[n]{b} \times \sqrt[n]{b} \times \cdots \times \sqrt[n]{b}}_{n \text{ times}}$$
$$= ab$$

- Since $C^n = D^n$ implies $C = D$, then $\sqrt[n]{ab} = \sqrt[n]{a} \times \sqrt[n]{b}$.

- Let's look again at some concrete numbers. What is $\sqrt{36}$?

  - $\sqrt{36} = 6$.

- Now consider the factors of 36. Specifically those that are perfect squares. We want to rewrite $\sqrt{36}$ as a product of perfect squares. What will that be?

  - $\sqrt{36} = \sqrt{4 \times 9}$

- Based on what we just learned, we can write $\sqrt{36} = \sqrt{4 \times 9} = \sqrt{4} \times \sqrt{9}$. What does the last expression simplify to? How does it compare to our original statement that $\sqrt{36} = 6$.
  - □ *$\sqrt{4} \times \sqrt{9} = 2 \times 3 = 6$. The answers are the same so $\sqrt{36} = \sqrt{4} \times \sqrt{9}$.*
- Simplify $\sqrt{64}$ two different ways. Explain your work to a partner.
  - □ *$\sqrt{64} = \sqrt{8 \times 8} = \sqrt{8^2} = 8$ The number 64 is a product of 8 multiplied by itself, which is the same as 8^2. Since the square root symbol asks for the number that when multiplied by itself is 64, then $\sqrt{64} = 8$.*
  - □ *$\sqrt{64} = \sqrt{16 \times 4} = \sqrt{16} \times \sqrt{4} = \sqrt{4^2} \times \sqrt{2^2} = 4 \times 2 = 8$ The number 64 is a product of 16 and 4. We can rewrite $\sqrt{64}$ as a product of its factors, $\sqrt{16 \times 4}$, then as $\sqrt{16} \times \sqrt{4}$. Each of the numbers 16 and 4 are perfect squares that can be simplified as before, so $\sqrt{16} \times \sqrt{4} = \sqrt{4^2} \times \sqrt{2^2} = 4 \times 2 = 8$. Therefore, $\sqrt{64} = 8$. This means that $\sqrt{64} = \sqrt{16} \times \sqrt{4}$.*

Example 1 (4 minutes)

> **Example 1**
>
> **Simplify the square root as much as possible.**
>
> $\sqrt{50} =$

- Is the number 50 a perfect square? Explain.
  - □ *The number 50 not a perfect square because there is no integer squared that equals 50.*
- Since 50 is not a perfect square, when we need to simplify $\sqrt{50}$, we write the factors of the number 50 looking specifically for those that are perfect squares. What are the factors of 50?
  - □ $50 = 2 \times 5^2$
- Since $50 = 2 \times 5^2$, then $\sqrt{50} = \sqrt{2 \times 5^2}$. We can rewrite $\sqrt{50}$ as a product of its factors:

$$\sqrt{50} = \sqrt{2} \times \sqrt{5^2}$$

Obviously, 5^2 is a perfect square. Therefore $\sqrt{5^2} = 5$, so $\sqrt{50} = 5 \times \sqrt{2} = 5\sqrt{2}$. Since $\sqrt{2}$ is not a perfect square we will leave it as it is.

- The number $\sqrt{50}$ is said to be in its simplified form when all perfect square factors have been simplified. Therefore, $5\sqrt{2}$ is the simplified form of $\sqrt{50}$.
- Now that we know $\sqrt{50}$ can be expressed as a product of its factors, we also know that we can multiply expressions containing square roots. For example, if we are given $\sqrt{2} \times \sqrt{5^2}$ we can rewrite the expression as $\sqrt{2 \times 5^2} = \sqrt{50}$.

Example 2 (3 minutes)

Example 2

Simplify the square root as much as possible.

$\sqrt{28} =$

- Is the number 28 a perfect square? Explain.
  - *The number* 28 *is not a perfect square because there is no integer squared that equals* 28.
- What are the factors of 28?
  - $28 = 2^2 \times 7$
- Since $28 = 2^2 \times 7$, then $\sqrt{28} = \sqrt{2^2 \times 7}$. We can rewrite $\sqrt{28}$ as a product of its factors:

$$\sqrt{28} = \sqrt{2^2} \times \sqrt{7}$$

Obviously, 2^2 is a perfect square. Therefore, $\sqrt{2^2} = 2$, and $\sqrt{28} = 2 \times \sqrt{7} = 2\sqrt{7}$. Since $\sqrt{7}$ is not a perfect square we will leave it as it is.

- The number $\sqrt{28}$ is said to be in its simplified form when all perfect square factors have been simplified. Therefore, $2\sqrt{7}$ is the simplified form of $\sqrt{28}$.

Exercises 7–10 (5 minutes)

Students complete Exercises 7–10 independently.

Exercises 7–10

Simplify the square roots as much as possible.

7. $\sqrt{18}$

$$\sqrt{18} = \sqrt{2 \times 3^2}$$
$$= \sqrt{2} \times \sqrt{3^2}$$
$$= 3\sqrt{2}$$

8. $\sqrt{44}$

$$\sqrt{44} = \sqrt{2^2 \times 11}$$
$$= \sqrt{2^2} \times \sqrt{11}$$
$$= 2\sqrt{11}$$

9. $\sqrt{169}$

$$\sqrt{169} = \sqrt{13^2}$$
$$= 13$$

10. $\sqrt{75}$

$$\sqrt{75} = \sqrt{3 \times 5^2}$$
$$= \sqrt{3} \times \sqrt{5^2}$$
$$= 5\sqrt{3}$$

Example 3 (4 minutes)

> **Example 3**
>
> **Simplify the square root as much as possible.**
>
> $\sqrt{128} =$

In this example, students may or may not recognize 128 as 64×2. The work below assumes that they do not. Consider showing students the solution below, as well as this alternative solution: $\sqrt{128} = \sqrt{64 \times 2} = \sqrt{64} \times \sqrt{2} = 8 \times \sqrt{2} = 8\sqrt{2}$.

- Is the number 128 a perfect square? Explain.
  - *The number* 128 *is not a perfect square because there is no integer squared that equals* 128.
- What are the factors of 128?
  - $128 = 2^7$

Since $128 = 2^7$, then $\sqrt{128} = \sqrt{2^7}$. We know that we can simplify perfect squares so we can rewrite 2^7 as $2^2 \times 2^2 \times 2^2 \times 2$ because of what we know about the Laws of Exponents. Then
$\sqrt{128} = \sqrt{2^2} \times \sqrt{2^2} \times \sqrt{2^2} \times \sqrt{2}$
Each 2^2 is a perfect square. Therefore, $\sqrt{128} = 2 \times 2 \times 2 \times \sqrt{2} = 8\sqrt{2}$.

Example 4 (4 minutes)

> **Example 4**
>
> **Simplify the square root as much as possible.**
>
> $\sqrt{288} =$

In this example, students may or may not recognize 288 as 144×2. The work below assumes that they do not. Consider showing students the solution below, as well as this alternative solution: $\sqrt{288} = \sqrt{144 \times 2} = \sqrt{144} \times \sqrt{2} = 12 \times \sqrt{2} = 12\sqrt{2}$.

- Is the number 288 a perfect square? Explain.
  - *The number* 288 *is not a perfect square because there is no integer squared that equals* 288.
- What are the factors of 288?
  - $288 = 2^5 \times 3^2$
- Since $288 = 2^5 \times 3^2$, then $\sqrt{288} = \sqrt{2^5 \times 3^2}$. What do we do next?
  - *Use the Laws of Exponents to rewrite* 2^5 *as* $2^2 \times 2^2 \times 2$.
- Then $\sqrt{288}$ is equivalent to
  - $\sqrt{288} = \sqrt{2^2} \times \sqrt{2^2} \times \sqrt{2} \times \sqrt{3^2}$
- What does this simplify to?
  - $\sqrt{288} = \sqrt{2^2} \times \sqrt{2^2} \times \sqrt{2} \times \sqrt{3^2} = \sqrt{2^2} \times \sqrt{2^2} \times \sqrt{3^2} \times \sqrt{2} = 2 \times 2 \times 3 \times \sqrt{2} = 12\sqrt{2}$

Exercises 11–14 (5 minutes)

Students work independently or in pairs to complete Exercises 11–14.

Exercises 11–14

11. Simplify $\sqrt{108}$.

$$\sqrt{108} = \sqrt{2^2 \times 3^3}$$
$$= \sqrt{2^2} \times \sqrt{3^2} \times \sqrt{3}$$
$$= 2 \times 3\sqrt{3}$$
$$= 6\sqrt{3}$$

12. Simplify $\sqrt{250}$.

$$\sqrt{250} = \sqrt{2 \times 5^3}$$
$$= \sqrt{2} \times \sqrt{5^2} \times \sqrt{5}$$
$$= 5\sqrt{2} \times \sqrt{5}$$
$$= 5\sqrt{10}$$

13. Simplify $\sqrt{200}$.

$$\sqrt{200} = \sqrt{2^3 \times 5^2}$$
$$= \sqrt{2^2} \times \sqrt{2} \times \sqrt{5^2}$$
$$= 2 \times 5\sqrt{2}$$
$$= 10\sqrt{2}$$

14. Simplify $\sqrt{504}$.

$$\sqrt{504} = \sqrt{2^3 \times 3^2 \times 7}$$
$$= \sqrt{2^2} \times \sqrt{2} \times \sqrt{3^2} \times \sqrt{7}$$
$$= 2 \times 3 \times \sqrt{2} \times \sqrt{7}$$
$$= 6\sqrt{14}$$

Scaffolding:

▪ Some simpler problems are included here:

Simplify $\sqrt{12}$.

$$\sqrt{12} = \sqrt{2^2 \times 3}$$
$$= \sqrt{2^2} \times \sqrt{3}$$
$$= 2 \times \sqrt{3}$$
$$= 2\sqrt{3}$$

Simplify $\sqrt{48}$.

$$\sqrt{48} = \sqrt{2^4 \times 3}$$
$$= \sqrt{2^2} \times \sqrt{2^2} \times \sqrt{3}$$
$$= 2 \times 2 \times \sqrt{3}$$
$$= 4\sqrt{3}$$

Simplify $\sqrt{350}$.

$$\sqrt{350} = \sqrt{5^2 \times 2 \times 7}$$
$$= \sqrt{5^2} \times \sqrt{2} \times \sqrt{7}$$
$$= 5 \times \sqrt{2} \times \sqrt{7}$$
$$= 5\sqrt{14}$$

Closing (3 minutes)

Summarize, or ask students to summarize, the main points from the lesson:

▪ We know how to simplify a square root by using the factors of a given number, and then simplifying the perfect squares.

Lesson Summary

Square roots of non-perfect squares can be simplified by using the factors of the number. Any perfect square factors of a number can be simplified.

For example:

$$\sqrt{72} = \sqrt{36 \times 2}$$
$$= \sqrt{36} \times \sqrt{2}$$
$$= \sqrt{6^2} \times \sqrt{2}$$
$$= 6 \times \sqrt{2}$$
$$= 6\sqrt{2}$$

Exit Ticket (5 minutes)

Name _____   Date _____

Lesson 4: Simplifying Square Roots

Exit Ticket

Simplify the square roots as much as possible.

1. $\sqrt{24}$

2. $\sqrt{338}$

3. $\sqrt{196}$

4. $\sqrt{2,420}$

Exit Ticket Sample Solutions

**Simplify the square roots as much as possible.**

1. $\sqrt{24}$

$$\begin{aligned}\sqrt{24} &= \sqrt{2^2 \times 6} \\ &= \sqrt{2^2} \times \sqrt{6} \\ &= 2\sqrt{6}\end{aligned}$$

2. $\sqrt{338}$

$$\begin{aligned}\sqrt{338} &= \sqrt{13^2 \times 2} \\ &= \sqrt{13^2} \times \sqrt{2} \\ &= 13\sqrt{2}\end{aligned}$$

3. $\sqrt{196}$

$$\begin{aligned}\sqrt{196} &= \sqrt{14^2} \\ &= 14\end{aligned}$$

4. $\sqrt{2,420}$

$$\begin{aligned}\sqrt{2,420} &= \sqrt{2^2 \times 11^2 \times 5} \\ &= \sqrt{2^2} \times \sqrt{11^2} \times \sqrt{5} \\ &= 2 \times 11 \times \sqrt{2} \\ &= 22\sqrt{5}\end{aligned}$$

Problem Set Sample Solutions

Simplify each of the square roots in Problems 1–5 as much as possible.

1. $\sqrt{98}$

$$\begin{aligned}\sqrt{98} &= \sqrt{2 \times 7^2} \\ &= \sqrt{2} \times \sqrt{7^2} \\ &= 7\sqrt{2}\end{aligned}$$

2. $\sqrt{54}$

$$\begin{aligned}\sqrt{54} &= \sqrt{2 \times 3^3} \\ &= \sqrt{2} \times \sqrt{3^2} \times \sqrt{3}\end{aligned}$$

3. $\sqrt{144}$

$$\begin{aligned}\sqrt{144} &= \sqrt{12^2} \\ &= 12\end{aligned}$$

4. $\sqrt{512}$

$$\begin{aligned}\sqrt{512} &= \sqrt{2^9} \\ &= \sqrt{2^2} \times \sqrt{2^2} \times \sqrt{2^2} \times \sqrt{2^2} \times \sqrt{2} \\ &= 2 \times 2 \times 2 \times 2\sqrt{2} \\ &= 16\sqrt{2}\end{aligned}$$

5. $\sqrt{756}$

$$\begin{aligned}\sqrt{756} &= \sqrt{2^2 \times 3^3 \times 7} \\ &= \sqrt{2^2} \times \sqrt{3^2} \times \sqrt{3} \times \sqrt{7} \\ &= 2 \times 3 \times \sqrt{21} \\ &= 6\sqrt{21}\end{aligned}$$
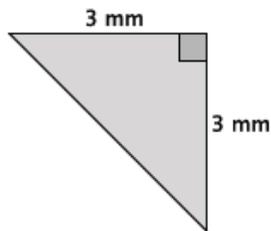
6. What is the length of the unknown side of the right triangle? Simplify your answer.

3 cm

8 cm
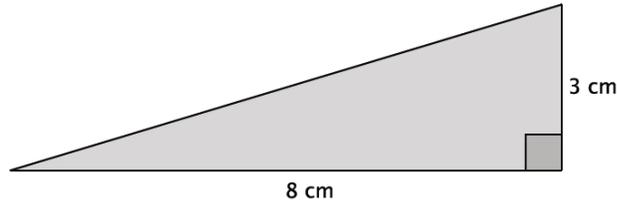
Let c represent the length of the hypotenuse.

$$3^2 + 8^2 = c^2$$
$$9 + 64 = c^2$$
$$75 = c^2$$
$$\sqrt{75} = \sqrt{c^2}$$
$$\sqrt{75} = c$$
$$\sqrt{5^2 \times 3} = c$$
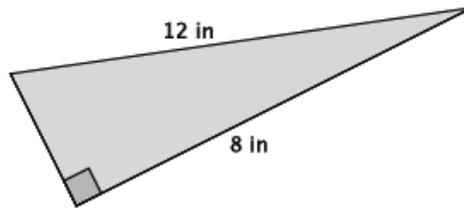$$\sqrt{5^2} \times \sqrt{3} = c$$
$$5\sqrt{3} = c$$

7. What is the length of the unknown side of the right triangle? Simplify your answer.

3 mm

3 mm

Let c represent the length of the hypotenuse.

$$3^2 + 3^2 = c^2$$
$$9 + 9 = c^2$$
$$18 = c^2$$
$$\sqrt{18} = \sqrt{c^2}$$
$$\sqrt{18} = c$$
$$\sqrt{3^2} \times \sqrt{2} = c$$
$$3\sqrt{2} = c$$

8. What is the length of the unknown side of the right triangle? Simplify your answer.

Let x represent the unknown length.

$$x^2 + 8^2 = 12^2$$
$$x^2 + 64 = 144$$
$$x^2 + 64 - 64 = 144 - 64$$
$$x^2 = 80$$
$$\sqrt{x^2} = \sqrt{80}$$
$$x = \sqrt{80}$$
$$x = \sqrt{2^4 \times 5}$$
$$x = \sqrt{2^2} \times \sqrt{2^2} \times \sqrt{5}$$
$$x = 2 \times 2\sqrt{5}$$
$$x = 4\sqrt{5}$$

9. Josue simplified $\sqrt{450}$ as $15\sqrt{2}$. Is he correct? Explain why or why not.

$$\sqrt{450} = \sqrt{2 \times 3^2 \times 5^2}$$
$$= \sqrt{2} \times \sqrt{3^2} \times \sqrt{5^2}$$
$$= 3 \times 5 \times \sqrt{2}$$
$$= 15\sqrt{2}$$

Yes, Josue is correct, because the number $450 = 2 \times 3^2 \times 5^2$. The factors that are perfect squares simplify to 15 leaving just the factor of 2 that cannot be simplified. Therefore, $\sqrt{450} = 15\sqrt{2}$.

10. Tiah was absent from school the day that you learned how to simplify a square root. Using $\sqrt{360}$, write Tiah an explanation for simplifying square roots.

To simplify $\sqrt{360}$, first write the factors of 360. The number $360 = 2^3 \times 3^2 \times 5$. Now we can use the factors to write $\sqrt{360} = \sqrt{2^3 \times 3^2 \times 5}$, which can then be expressed as $\sqrt{360} = \sqrt{2^3} \times \sqrt{3^2} \times \sqrt{5}$. Because we want to simplify square roots, we can rewrite the factor $\sqrt{2^3}$ as $\sqrt{2^2} \times \sqrt{2}$ because of the Laws of Exponents. Now we have:

$$\sqrt{360} = \sqrt{2^2} \times \sqrt{2} \times \sqrt{3^2} \times \sqrt{5}$$

Each perfect square can be simplified:

$$\sqrt{360} = 2 \times \sqrt{2} \times 3 \times \sqrt{5}$$
$$= 2 \times 3 \times \sqrt{2} \times \sqrt{5}$$
$$= 6\sqrt{10}$$

The simplified version of $\sqrt{360} = 6\sqrt{10}$.