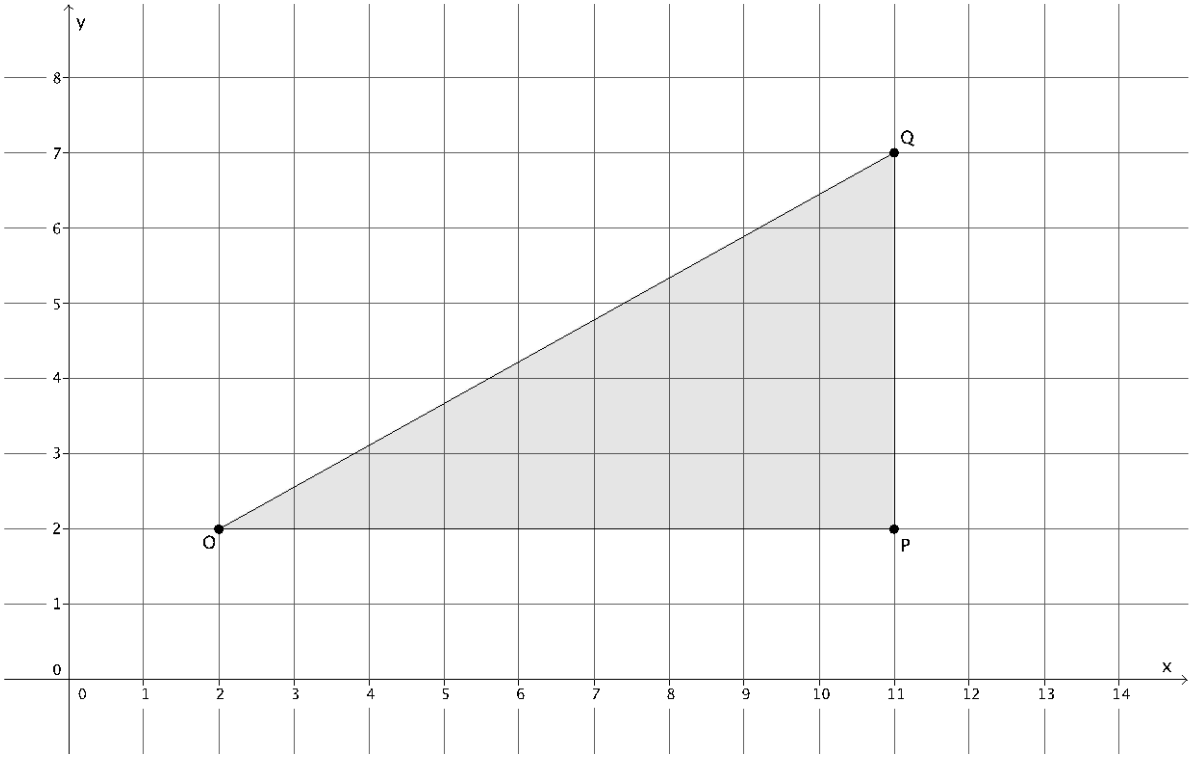
Name _____ Date _____

1. Use the diagram below to answer the questions that follow.



a. Dilate triangle ΔOPQ from center O and scale factor $r = \frac{4}{9}$. Label the image $\Delta OP'Q'$.

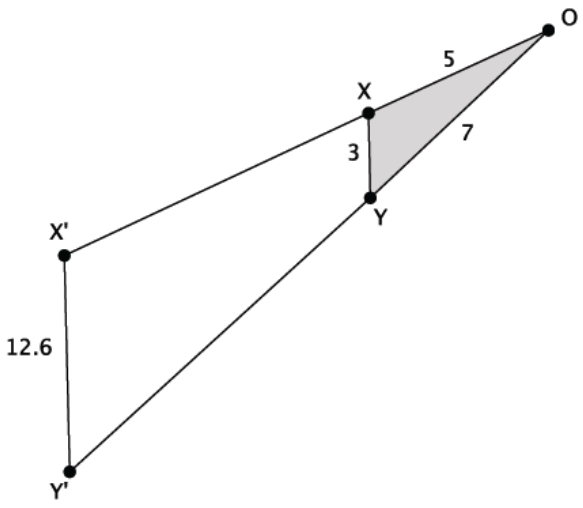
b. Find the coordinates of P' and Q'.

c. Are $\angle OQP$ and $\angle OQ'P'$ equal in measure? Explain.

d. What is the relationship between the lines PQ and $P'Q'$? Explain in terms of similar triangles.

e. If the length of segment $|OQ| = 9.8$ units, what is the length of segment $|OQ'|$? Explain in terms of similar triangles.

2. Use the diagram below to answer the questions that follow. The length of each segment is as shown: segment OX is 5 units, segment OY is 7 units, segment XY is 3 units, and segment $X'Y'$ is 12.6 units.

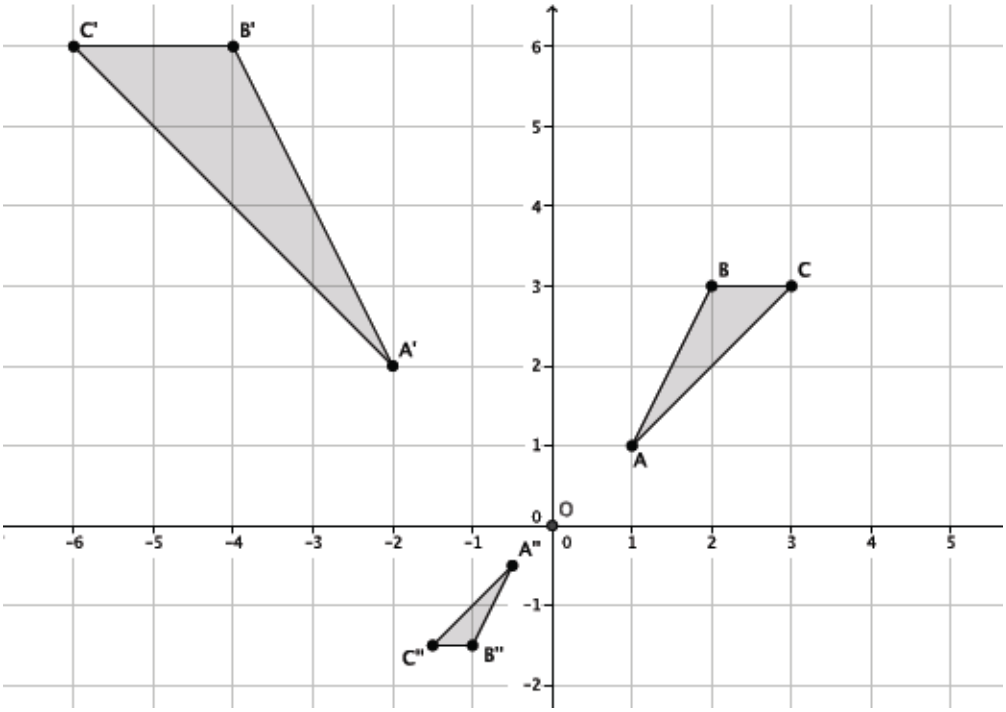


a. Suppose XY is parallel to $X'Y'$. Is triangle ΔOXY similar to triangle $\Delta OX'Y'$? Explain.

b. What is the length of segment OX'? Show your work.

c. What is the length of segment OY'? Show your work.

3. Given $\Delta ABC \sim \Delta A'B'C'$ and $\Delta ABC \sim \Delta A''B''C''$ in the diagram below, answer parts (a)–(c).



a. Describe the sequence that shows the similarity for ΔABC and $\Delta A'B'C'$.

b. Describe the sequence that shows the similarity for ΔABC and $\Delta A''B''C''$.

c. Is $\Delta A'B'C'$ similar to $\Delta A''B''C''$? How do you know?

A Progression Toward Mastery					
Assessment Task Item		**STEP 1** Missing or incorrect answer and little evidence of reasoning or application of mathematics to solve the problem.	**STEP 2** Missing or incorrect answer but evidence of some reasoning or application of mathematics to solve the problem.	**STEP 3** A correct answer with some evidence of reasoning or application of mathematics to solve the problem, or an incorrect answer with substantial evidence of solid reasoning or application of mathematics to solve the problem.	**STEP 4** A correct answer supported by substantial evidence of solid reasoning or application of mathematics to solve the problem.
1	a 8.G.A.4	Student does not mark any points on the drawing.	Student drew an arbitrary triangle that is not a dilation according to the scale factor and is not labeled.	Student drew a triangle $\Delta OQ'P'$ and labeled the points, but it was not a dilation according to the scale factor.	Student drew a triangle $\Delta OQ'P'$ according to the scale factor and labeled the points.
	b 8.G.A.4	Student does not attempt the problem or leaves the problem blank.	Student identifies both of the coordinates of P' or Q' incorrectly OR student may have transposed the coordinates of P' as $(2, 6)$.	Student identifies one of the coordinates of P' correctly and the x-coordinate of Q' correctly. A calculation error may have led to to an incorrect y-coordinate of Q'.	Student correctly identifies the coordinates of Q' as $\left(6, \frac{38}{9}\right)$. Student correctly identifies the coordinates of P' as $(6, 2)$.
	c 8.G.A.4	Student does not attempt the problem or leaves the problem blank. Student states that $\angle OQP \neq \angle OQ'P'$.	Student states that $\angle OQP = \angle OQ'P'$. Student does not attempt any explanation or reasoning. Explanation or reasoning is not mathematically based. For example, student may write: "it looks like they are the same."	Student states that $\angle OQP = \angle OQ'P'$. Student explanation includes mathematical language. Student explanation may not be complete, e.g., stating dilations are degree persevering without explaining $D(\angle OQP) = \angle OQ'P'$.	Student states that $\angle OQP = \angle OQ'P'$. Student explanation includes mathematical language. Reasoning includes that $D(\angle OQP) = \angle OQ'P'$, and dilations are degree preserving.

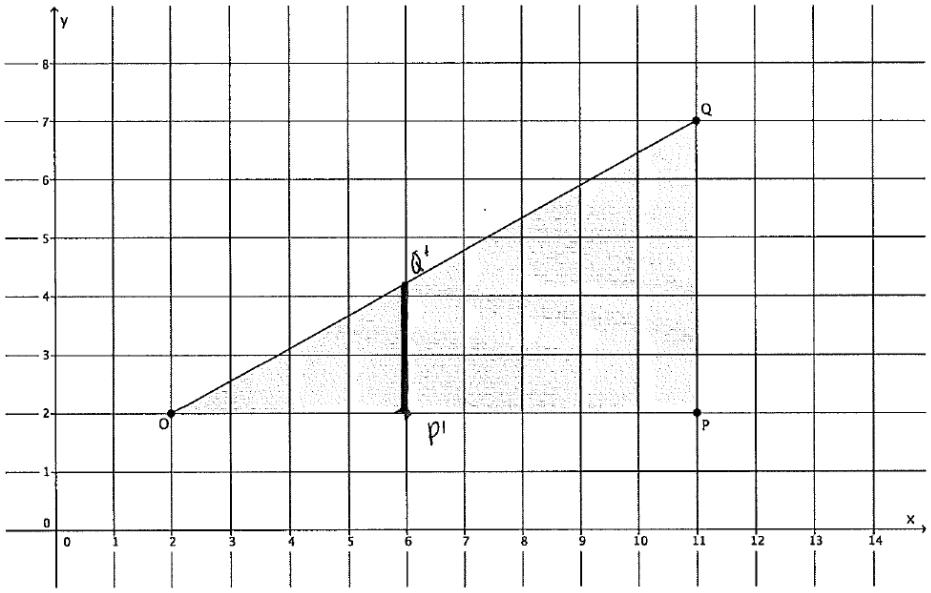
	d **8.G.A.5**	Student does not attempt the problem or leaves the problem blank. Student may state that $PQ \parallel P'Q'$. Student does not attempt any explanation or reasoning.	Student may state that $PQ \parallel P'Q'$. Student may not use mathematical language in explanation or reasoning. For example, student may write: "they look like they won't touch," or "the angles are the same." Reasoning may include some facts. Reasoning may not be complete. There are significant gaps in explanation.	Student states that $PQ \parallel P'Q'$. Student uses some mathematical language in explanation or reasoning. Reasoning includes some of the following facts: $\angle O = \angle O$, $\angle OQP = \angle OQ'P'$ and $\angle OPQ = \angle OP'Q'$, then by AA criterion for similarity, $\Delta OPQ \sim \Delta OP'Q'$. Then, by FTS $PQ \parallel P'Q'$. Reasoning may not be complete.	Student states that $PQ \parallel P'Q'$. Student uses mathematical language in explanation or reasoning. Reasoning includes the following facts: At least two pairs of corresponding angles are equal, e.g., $\angle O = \angle O$ and/or $\angle OQP = \angle OQ'P'$ and/or $\angle OPQ = \angle OP'Q'$, then by AA criterion for similarity, $\Delta OPQ \sim \Delta OP'Q'$. Then, by FTS $PQ \parallel P'Q'$. Reasoning is thorough and complete.
	e **8.G.A.5**	Student does not attempt the problem or leaves the problem blank.	Student answers incorrectly. Student may not use mathematical language in explanation or reasoning. Student reasoning does not include a reference to similar triangles. Student reasoning may or may not include that the ratio of lengths are equal to scale factor. There are significant gaps in explanation.	Student answers correctly that $OQ' \approx 4.4$ units. Student uses some mathematical language in explanation or reasoning. Student may or may not have referenced similar triangles in reasoning. Student reasoning includes that the ratio of lengths are equal to scale factor. Explanation or reasoning may not be complete.	Student answers correctly that $OQ' \approx 4.4$ units. Student uses mathematical language in explanation or reasoning. Student referenced similar triangles in reasoning. Student reasoning includes that the ratio of lengths are equal to scale factor. Reasoning is thorough and complete.
2	**a** **8.G.A.5**	Student does not attempt the problem or leaves the problem blank. Student answers yes or no only. Student does not attempt to explain reasoning.	Student may or may not answer correctly. Student may use some mathematical language in explanation or reasoning. Explanation or reasoning is not mathematically based, e.g., "they look like they are." There are significant gaps in explanation.	Student answers yes correctly. Student uses some mathematical language in explanation or reasoning. Explanation includes some of the following facts: Since $XY \| \| X'Y'$, then corresponding angles of parallel lines are congruent by AA criterion for similar triangles; therefore,	Student answers yes correctly. Student uses mathematical language in explanation or reasoning. Explanation includes the following facts: Since $XY \| \| X'Y'$, then corresponding angles of parallel lines are congruent by AA criterion for similar triangles; therefore, $\Delta OXY \sim \Delta OX'Y'$. Reasoning is thorough

				$\Delta OXY \sim \Delta OX'Y'$. Reasoning may not be complete.	and complete.
	b **8.G.A.5**	Student does not attempt the problem or leaves the problem blank.	Student may or may not have answered correctly. Student uses some method other than proportion to solve problems, e.g., guessing. Student may have made calculation errors.	Student may or may not have answered correctly. Student uses a proportion to solve problem. Student may have set up proportion incorrectly. Student may have made calculation errors.	Student answers correctly with length of $OX' = 21$ units. Student uses a proportion to solve problem.
	c **8.G.A.5**	Student does not attempt the problem or leaves the problem blank.	Student may or may not have answered correctly. Student uses some method other than proportion to solve problems, e.g., guessing. Student may have made calculation errors.	Student may or may not have answered correctly. Student uses a proportion to solve problem. Student may have set up proportion incorrectly. Student may have made calculation errors.	Student answers correctly with length of $OY' = 29.4$ units. Student uses a proportion to solve problem.
3	**a** **8.G.A.5**	Student does not attempt the problem or leaves the problem blank.	Student does not attempt any explanation or reasoning. Student may or may not have stated dilation and does not give any center or scale factor. Student may or may not have stated the congruence. Student may have stated the incorrect congruence. Explanation or reasoning is not mathematically based, e.g., "one looks about three times bigger than the other."	Student states dilation. Student states dilation is centered at origin, but does not give scale factor, $r > 1$, or states scale factor of $r > 1$, but does not give center. Student may or may not have stated the congruence. Student may have stated the incorrect congruence. Student uses some mathematical language in explanation or reasoning.	Student states correctly there is a dilation with center at the origin and has a scale factor, $r = 2$. Student states correctly there is a congruence of reflection across the y-axis. Student uses mathematical language in explanation or reasoning such as $\Lambda(D(\Delta ABC)) = \Delta A'B'C'$ and $\Delta ABC \sim \Delta A'B'C'$. Reasoning is thorough and complete.
	b **8.G.A.5**	Student does not attempt the problem or leaves the problem blank.	Student does not attempt any explanation or reasoning. Student may or may not have stated dilation and does not give any center or scale factor. Student may or may not have stated the congruence. Student may have stated the incorrect	Student states dilation. Student states dilation is centered at origin, but does not give scale factor, $0 < r < 1$ or states scale factor of $0 < r < 1$, but does not give center. Student may or may not have stated the congruence. Student may have stated	Student states correctly there is a dilation with center at the origin and has a scale factor, $0 < r < 1$. Student states correctly there is a congruence of rotation of $180°$ centered at the origin. Student uses mathematical language in explanation or

			congruence. Explanation or reasoning is not mathematically based, e.g., "one looks about half the size of the other."	the incorrect congruence. Student uses some mathematical language in explanation or reasoning.	reasoning, such as: $R(D(\Delta ABC)) = \Delta A''B''C''$ and $\Delta ABC \sim \Delta A''B''C''$. Reasoning is thorough and complete.
	c **8.G.5**	Student does not attempt the problem or leaves the problem blank.	Student may or may not have answered correctly. Student does not attempt any explanation or reasoning. Student does not reference the AA Criterion for similarity. Explanation or reasoning is not mathematically based, e.g., "They don't look like they are the same."	Student may or may not have answered correctly. Student states that only one set of angles is congruent. Student uses some mathematical language in explanation or reasoning. Student may or may not reference the AA Criterion for similarity.	Student answers correctly that yes, $\Delta A'B'C' \sim \Delta A''B''C''$. Student states that dilations are angle preserving. Student shows that since $\Delta ABC \sim \Delta A'B'C'$ and $\Delta ABC \sim \Delta A''B''C''$, at least two corresponding angles are congruent (i.e., $\angle A \cong \angle A' \cong \angle A''$). Student references the AA Criterion for similarity. Student uses mathematical language in explanation or reasoning. Reasoning is thorough and complete.

Name _____ Date _____

1. Use the diagram below to answer the questions that follow.



a. Dilate triangle ΔOPQ from center O and scale factor $r = \frac{4}{9}$. Label the image $\Delta OP'Q'$.

b. Find the coordinates of P' and Q'.

$P' = (6, 2)$

$Q' = \left(6, \frac{38}{9}\right)$

$\frac{|P'Q'|}{|PQ|} = \frac{4}{9}$

$\frac{|P'Q'|}{5} = \frac{4}{9}$

$|P'Q'| = \frac{20}{9}$

$\frac{20}{9} + 2 = \frac{20}{9} + \frac{18}{9}$

$= \frac{38}{9}$

c. Are $\angle OQP$ and $\angle OQ'P'$ equal in measure? Explain.

YES $\angle OQP = \angle OQ'P'$. SINCE $D(\Delta OQP) = \Delta OQ'P'$ AND DILATIONS ARE DEGREE PRESERVING, THEN $\angle OQP = \angle OQ'P'$.

$\angle OQP$ & $\angle OQ'P'$ ARE CORRESPONDING ANGLES OF PARALLEL LINES PQ & P'Q', THEREFORE $\angle OQP = \angle OQ'P'$.

d. What is the relationship between the lines PQ and $P'Q'$? Explain in terms of similar triangles.

THE LINES PQ AND P'Q' ARE PARALLEL. $\triangle OPQ \sim \triangle OP'Q'$ BY THE AA CRITERION ($\angle O = \angle O$, $\angle OPQ = \angle OP'Q'$), THEREFORE BY THE FUNDAMENTAL THEOREM OF SIMILARITY $PQ \parallel P'Q'$.

e. If the length of segment $|OQ| = 9.8$ units, what is the length of segment $|OQ'|$? Explain in terms of similar triangles.

SINCE $\triangle OPQ \sim \triangle OP'Q'$, THEN THE RATIOS OF LENGTHS OF CORRESPONDING SIDES WILL BE EQUAL TO THE SCALE FACTOR. THEN

$$\frac{|OP'|}{|OP|} = \frac{|OQ'|}{|OQ|} = \frac{4}{9}$$

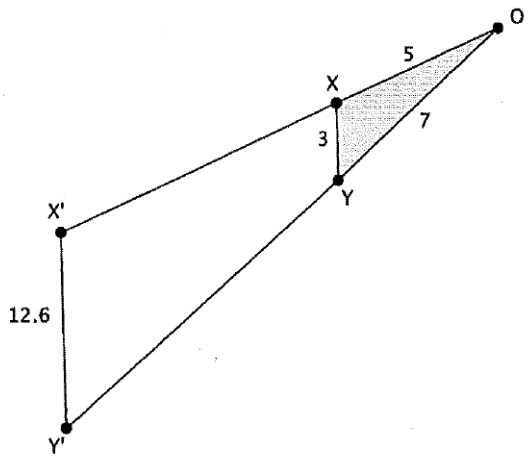
$$\frac{4}{9} = \frac{|OQ'|}{9.8}$$

$$39.2 = 9(|OQ'|)$$

$$4.36 = |OQ'|$$

THE LENGTH OF $|OQ'|$ IS APPROXIMATELY 4.4 UNITS.

2. Use the diagram below to answer the questions that follow. The length of each segment is as shown: segment OX is 5 units, segment OY is 7 units, segment XY is 3 units, and segment $X'Y'$ is 12.6 units.



a. Suppose XY is parallel to $X'Y'$. Is triangle ΔOXY similar to triangle $\Delta OX'Y'$? Explain.

YES, $\Delta OXY \sim OX'Y'$. SINCE $XY \parallel X'Y'$ THEN $\angle OXY = \angle OX'Y'$ AND $\angle OYX = \angle OY'X'$. BECAUSE CORRESPONDING ANGLES OF PARALLEL LINES ARE EQUAL, BY AA $\Delta OXY \sim \Delta OX'Y'$.

b. What is the length of segment OX'? Show your work.

$\frac{12.6}{3} = \frac{|OX'|}{5}$ $5(12.6) = 3(|OX'|)$ $63 = 3(|OX'|)$ $21 = |OX'|$

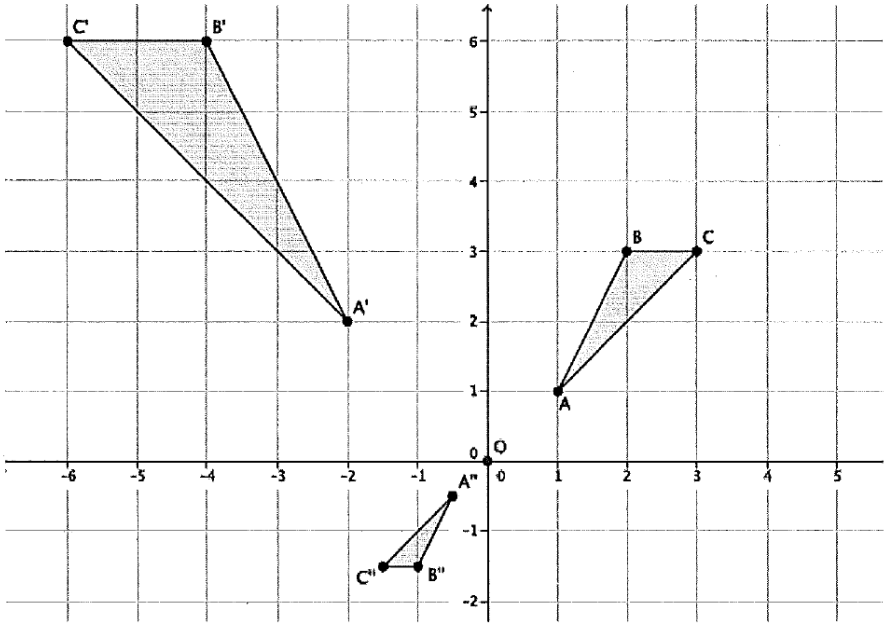
THE LENGTH OF $|OX'|$ IS 21 UNITS.

c. What is the length of segment OY'? Show your work.

$\frac{12.6}{3} = \frac{|OY'|}{7}$ $12.6(7) = 3(|OY'|)$ $88.2 = 3(|OY'|)$ $29.4 = |OY'|$

THE LENGTH OF $|OY'|$ IS 29.4 UNITS.

3. Given $\Delta ABC \sim \Delta A'B'C'$ and $\Delta ABC \sim \Delta A''B''C''$ in the diagram below, answer parts (a)-(c).



a. Describe the sequence that shows the similarity for ΔABC and $\Delta A'B'C'$. $\frac{B'C'}{BC} = \frac{2}{1} = 2 = r$

LET D BE THE DILATION FROM CENTER O AND SCALE FACTOR r=2. LET THERE BE A REFLECTION ACROSS THE Y-AXIS. THEN THE DILATION FOLLOWED BY THE REFLECTION MAPS ΔABC ONTO $\Delta A'B'C'$.

b. Describe the sequence that shows the similarity for ΔABC and $\Delta A''B''C''$.

LET D BE THE DILATION FROM CENTER O AND SCALE FACTOR $0 < r < 1$. LET THERE BE A ROTATION OF 180° AROUND CENTER D. THEN THE DILATION FOLLOWED BY THE ROTATION MAPS ΔABC ONTO $\Delta A''B''C''$.

c. Is $\Delta A'B'C'$ similar to $\Delta A''B''C''$? How do you know?

YES $\Delta A'B'C' \sim \Delta A''B''C''$. DILATIONS PRESERVE ANGLE MEASURES AND SINCE $\Delta ABC \sim \Delta A'B'C$ AND $\Delta ABC \sim \Delta A''B''C''$, WE KNOW $\angle A = \angle A' = \angle A''$, $\angle B = \angle B' = \angle B''$. BY AA CRITERION FOR SIMILARITY $\Delta A'B'C' \sim \Delta A''B''C''$. ALSO, SIMILARITY IS TRANSITIVE.