Lesson 1: Exponential Notation

Student Outcomes

- Students know what it means for a number to be raised to a power and how to represent the repeated multiplication symbolically.
- Students know the reason for some bases requiring parentheses.

Lesson Notes

This lesson is foundational for the topic of properties of integer exponents. However, if your students have already mastered the skills in this lesson it is your option to move forward and begin with Lesson 2.

Classwork

Socratic Discussion (15 minutes)

MP.2 & MP.7

When we add 5 copies of 3, we devise an abbreviation – a new notation, for this purpose:

$$3 + 3 + 3 + 3 + 3 = 5 \times 3$$

Now if we multiply the same number, 3, with itself 5 times, how should we abbreviate this?

$$3 \times 3 \times 3 \times 3 \times 3 = ?$$

Allow students to make suggestions, see sidebar for scaffolds.

$$3 \times 3 \times 3 \times 3 \times 3 = 3^5$$

Similarly, we also write $3^3 = 3 \times 3 \times 3$; $3^4 = 3 \times 3 \times 3 \times 3$; etc.

Scaffolding:

Remind students of their previous experiences:

- The square of a number, e.g., 3×3 is denoted by 3^2
- From the expanded form of a whole number, we also learned, e.g., 10^3 stands for $10 \times 10 \times 10$.

We see that when we add 5 copies of 3, we write 5×3, but when we multiply 5 copies of 3, we write 3^5. Thus, the "multiplication by 5" in the context of addition corresponds exactly to the superscript 5 in the context of multiplication.

Make students aware of the correspondence between addition and multiplication because what they know about *repeated addition* will help them learn exponents as *repeated multiplication* as we go forward.

5^6 means $5 \times 5 \times 5 \times 5 \times 5 \times 5$ and $\left(\frac{9}{7}\right)^4$ means $\frac{9}{7} \times \frac{9}{7} \times \frac{9}{7} \times \frac{9}{7}$.

You have seen this kind of notation before, it is called exponential notation. In general, for any number x and any positive integer n,

$$x^n = \underbrace{(x \cdot x \cdots x)}_{n \text{ times}}$$

The number x^n is called x raised to the n-th power, n is the exponent of x in x^n and x is the base of x^n.

Examples 1–5

Work through Examples 1–5 as a group, supplement with additional examples if needed.

Example 1

$5 \times 5 \times 5 \times 5 \times 5 \times 5 = 5^6$

Example 2

$\frac{9}{7} \times \frac{9}{7} \times \frac{9}{7} \times \frac{9}{7} = \left(\frac{9}{7}\right)^4$

Example 3

$\left(-\frac{4}{11}\right)^3 = \left(-\frac{4}{11}\right) \times \left(-\frac{4}{11}\right) \times \left(-\frac{4}{11}\right)$

Example 4

$(-2)^6 = (-2) \times (-2) \times (-2) \times (-2) \times (-2) \times (-2)$

Example 5

$3.8^4 = 3.8 \times 3.8 \times 3.8 \times 3.8$

- Notice the use of parentheses in Examples 2, 3, and 4. Do you know why?
  - *In cases where the base is either fractional or negative, it prevents ambiguity about which portion of the expression is going to be multiplied repeatedly.*
- Suppose n is a fixed **positive integer**, then 3^n, by definition, is $3^n = \underbrace{(3 \times \cdots \times 3)}_{n\ times}$.
- Again, if n is a fixed positive integer, then by definition,

$$7^n = \underbrace{(7 \times \cdots \times 7)}_{n\ times}$$

$$\left(\frac{4}{5}\right)^n = \underbrace{\left(\frac{4}{5} \times \cdots \times \frac{4}{5}\right)}_{n\ times}$$

$$(-2.3)^n = \underbrace{\big((-2.3) \times \cdots \times (-2.3)\big)}_{n\ times}$$

MP.6

- In general, for any number x, $x^1 = x$, and for any positive integer $n > 1$, x^n is by definition,

$$x^n = \underbrace{(x \cdot x \cdots x)}_{n\ times}$$

Note to Teacher: If students ask about values of n that are not positive integers, let them know that positive and negative fractional exponents will be introduced in Algebra II and that negative *integer* exponents will be discussed in Lesson 4 of this module.

- The number x^n is called **x raised to the n-th power**, n is the **exponent** of x in x^n and x is the **base** of x^n
- x^2 is called the **square** of x, and x^3 is its **cube**.
- You have seen this kind of notation before when you gave the expanded form of a whole number for powers of 10, it is called **exponential notation**.

Exercises 1–10 (5 minutes)

Students complete independently and check answers before moving on.

Exercise 1

$$\underbrace{4 \times \cdots \times 4}_{7\ times} = 4^7$$

Exercise 2

$$\underbrace{3.6 \times \cdots \times 3.6}_{\_\_\_\_\ times} = 3.6^{47}$$

$47\ times$

Exercise 3

$$\underbrace{(-11.63) \times \cdots \times (-11.63)}_{34\ times} = (-11.63)^{34}$$

Exercise 4

$$\underbrace{12 \times \cdots \times 12}_{\_\_\_\_\ times} = 12^{15}$$

$15\ times$

Exercise 5

$$\underbrace{(-5) \times \cdots \times (-5)}_{10\ times} = (-5)^{10}$$

Exercise 6

$$\underbrace{\frac{7}{2} \times \cdots \times \frac{7}{2}}_{21\ times} = \left(\frac{7}{2}\right)^{21}$$

Exercise 7

$$\underbrace{(-13) \times \cdots \times (-13)}_{6\ times} = (-13)^6$$

Exercise 8

$$\underbrace{\left(-\frac{1}{14}\right) \times \cdots \times \left(-\frac{1}{14}\right)}_{10\ times} = \left(-\frac{1}{14}\right)^{10}$$

Exercise 9

$$\underbrace{x \cdot x \cdots x}_{185\ times} = x^{185}$$

Exercise 10

$$\underbrace{x \cdot x \cdots x}_{\_\_\_\_\ times} = x^n$$

$n\ times$

Exercises 11–14 (15 minutes)

Allow students to complete Exercises 11–14 individually or in a small group.

- When a negative number is raised to an odd power, what is the sign of the result?
- When a negative number is raised to an even power, what is the sign of the result?

Make the point that when a negative number is raised to an odd power, the sign of the answer is negative. Conversely, if a negative number is raised to an even power, the sign of the answer is positive.

Exercise 11

Will these products be positive or negative? How do you know?

$$\underbrace{(-1)\times(-1)\times\cdots\times(-1)}_{12\ times}=(-1)^{12}$$

This product will be positive. Students may state that they computed the product and it was positive, but if they say that, let them show their work. Students may say that the answer is positive because the exponent is positive; this would not be acceptable in view of the next example.

$$\underbrace{(-1)\times(-1)\times\cdots\times(-1)}_{13\ times}=(-1)^{13}$$

This product will be negative. Students may state that they computed the product and it was negative; if so, they must show their work. Based on the discussion that occurred during the last problem, you may need to point out that a positive exponent does not always result in a positive product.

Exercise 12

Is it necessary to do all of the calculations to determine the sign of the product? Why or why not?

These two problems force the students to think beyond the computation level. If students have trouble, go back to the previous two problems and have them discuss in small groups what an even number of negative factors yields, and what an odd number of negative factors yields.

$$\underbrace{(-5)\times(-5)\times\cdots\times(-5)}_{95\ times}=(-5)^{95}$$

Students should state that an odd number of negative factors yield a negative product.

$$\underbrace{(-1.8)\times(-1.8)\times\cdots\times(-1.8)}_{122\ times}=(-1.8)^{122}$$

Students should state that an even number of negative factors yields a positive product.

Exercise 13

Fill in the blanks about whether the number is positive or negative.

If n is a positive even number, then $(-55)^n$ is *positive*.

If n is a positive odd number, then $(-72.4)^n$ is *negative*.

Exercise 14

Josie says that $\underbrace{(-15)\times\cdots\times(-15)}_{6\ times}=-15^6$. Is she correct? How do you know?

Students should state that Josie is not correct for the following two reasons: 1) They just stated that an even number of factors yields a positive product and this conflicts with the answer Josie provided, and 2) the notation is used incorrectly because, as is, the answer is the negative of 15^6, instead of the product of 6 copies of -15. The base is (-15). Recalling the discussion at the beginning of the lesson, when the base is negative it should be written clearly through the use of parentheses. Have students write the answer correctly.

Closing (5 minutes)

- Why should we bother with exponential notation? Why not just write out the multiplication?
- Engage the class in discussion, but make sure that they get to know at least the following two reasons:
  - Like all good notation, exponential notation saves writing.
  - Exponential notation is used for recording scientific measurements of very large and very small quantities. It is indispensable for the clear indication of a number's magnitude (see Lessons 10–13).
- Here is an example of the labor saving aspect of the exponential notation: Suppose a colony of bacteria doubles in size every 8 hours for a few days under tight laboratory conditions. If the initial size is B, what is the size of the colony after 2 days?
  - *Answer: in 2 days there are six 8-hour periods, so the size will be* 2^6B.
- Give more examples if time allows as a lead in to Lesson 2. Example situations: exponential decay with respect to heat transfer, vibrations, ripples in a pond, or interest on a bank deposit after some years have passed.

Exit Ticket (5 minutes)

Name ____ Date____________________

Lesson 1: Exponential Notation

Exit Ticket

1.

a. Express the following in exponential notation:

$$\underbrace{(-13) \times \cdots \times (-13)}_{35\ times}$$

b. Will the product be positive or negative?

2. Fill in the blank:

$$\underbrace{\frac{2}{3} \times \cdots \times \frac{2}{3}}_{\_\_\_\_\_ times} = \left(\frac{2}{3}\right)^4$$

3. Arnie wrote:

$$\underbrace{(-3.1) \times \cdots \times (-3.1)}_{4\ times} = -3.1^4$$

Is Arnie correct in his notation? Why or why not?

Exit Ticket Sample Solutions

The following responses indicate an understanding of the objectives of this lesson:

1.

a. Express the following in exponential notation:

$$\underbrace{(-13) \times \cdots \times (-13)}_{35\ times}$$

$(-13)^{35}$

b. Will the product be positive or negative?

The product will be negative.

2. Fill in the blank:

$$\underbrace{\frac{2}{3} \times \cdots \times \frac{2}{3}}_{\_\_\_\_ times} = \left(\frac{2}{3}\right)^4$$

$4\ times$

3. Arnie wrote:

$$\underbrace{(-3.1) \times \cdots \times (-3.1)}_{4\ times} = -3.1^4$$

Is Arnie correct in his notation? Why or why not?

Arnie is not correct. The base, -3.1, should be in parentheses to prevent ambiguity, at present the notation is not correct.

Problem Set Sample Solutions

1. Use what you know about exponential notation to complete the expressions below.

$$\underbrace{(-5) \times \cdots \times (-5)}_{17\ times} = (-5)^{17}$$

$$\underbrace{3.7 \times \cdots \times 3.7}_{\_\_\_ times} = 3.7^{19}$$

$19\ times$

$$\underbrace{7 \times \cdots \times 7}_{\_\_\_ times} = 7^{45}$$

$45\ times$

$$\underbrace{6 \times \cdots \times 6}_{4\ times} =$$

6^4

$$\underbrace{4.3 \times \cdots \times 4.3}_{13\ times} = 4.3^{13}$$

$$\underbrace{(-1.1) \times \cdots \times (-1.1)}_{9\ times} = (-1.1)^9$$

$$\underbrace{\left(\frac{2}{3}\right) \times \cdots \times \left(\frac{2}{3}\right)}_{19\ times} = \left(\frac{2}{3}\right)^{19}$$

$$\underbrace{\left(-\frac{11}{5}\right) \times \cdots \times \left(-\frac{11}{5}\right)}_{\_\_\_ times} = \left(-\frac{11}{5}\right)^x$$

$x\ times$

$$\underbrace{(-12) \times \cdots \times (-12)}_{\_\_\_ times} = (-12)^{15}$$

$15\ times$

$$\underbrace{a \times \cdots \times a}_{m\ times} = a^m$$

2. **Write an expression with (-1) as its base that will produce a positive product.**

Accept any answer with (-1) to an exponent that is even.

3. **Write an expression with (-1) as its base that will produce a negative product.**

Accept any answer with (-1) to an exponent that is odd.

4. **Rewrite each number in exponential notation using 2 as the base.**

$8 = 2^3$	$16 = 2^4$	$32 = 2^5$
$64 = 2^6$	$128 = 2^7$	$256 = 2^8$

5. **Tim wrote 16 as $(-2)^4$. Is he correct?**

Tim is correct that $16 = (-2)^4$.

6. **Could -2 be used as a base to rewrite 32? 64? Why or why not?**

A base of -2 cannot be used to rewrite 32 because $(-2)^5 = -32$. A base of -2 can be used to rewrite 64 because $(-2)^6 = 64$. If the exponent, n, is even, $(-2)^n$ will be positive. If the exponent, n, is odd, $(-2)^n$ cannot be a positive number.