



Lesson 25: Volume of Right Prisms

Student Outcomes

- Students use the formula $V = bh$ to determine the volume of a right prism. Students identify the base and compute the area of the base by decomposing it into pieces.

Lesson Notes

In Module 3 (Lesson 23), students learned that a polygonal base of a given right prism can be decomposed into triangular and rectangular regions that are bases of a set of right prisms that have heights equal to the height of the given right prism. By extension, the volume of any right prism can be found by multiplying the area of its base times the height of the prism. Students revisit this concept in the Opening Exercise before moving onto examples.

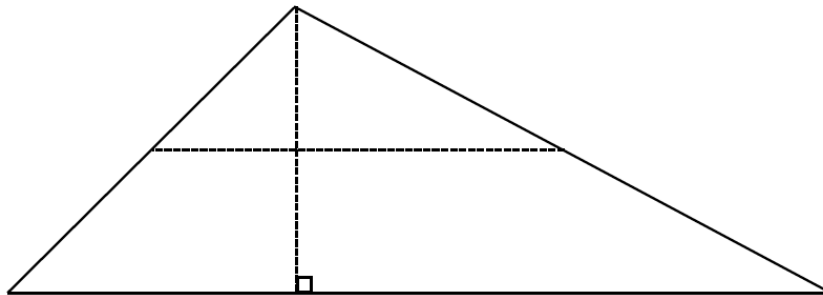
Classwork

Opening Exercise (3 minutes)

Opening Exercise

Take your copy of the following figure, and cut it into four pieces along the dotted lines (the vertical line is the altitude, and the horizontal line joins the midpoints of the two sides of the triangle).

Arrange the four pieces so that they fit together to form a rectangle.

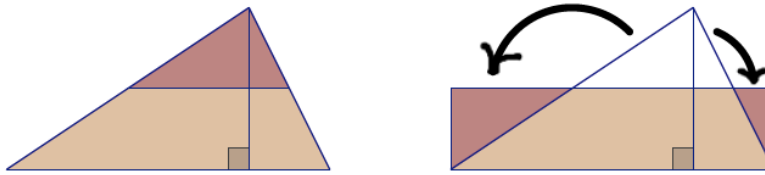


If a prism were formed out of each shape, the original triangle and your newly rearranged rectangle, and both prisms had the same height, would they have the same volume? Discuss with a partner.

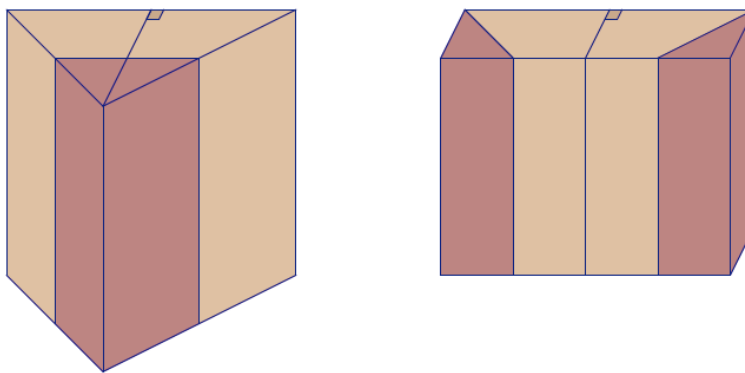
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Discussion (5 minutes)

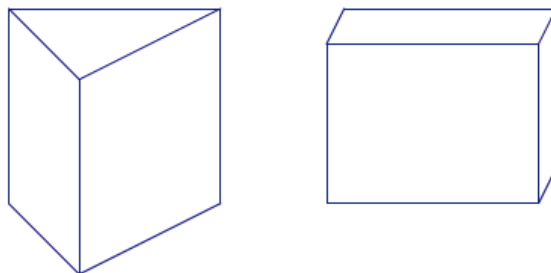
Any triangle can be cut and rearranged to form a rectangle by using the altitude from a vertex to the longest side and a line segment parallel to the longest side halfway between the vertex and the longest side.



Vertical slices through a right triangular prism show that it can be cut and rearranged as a right rectangular prism.

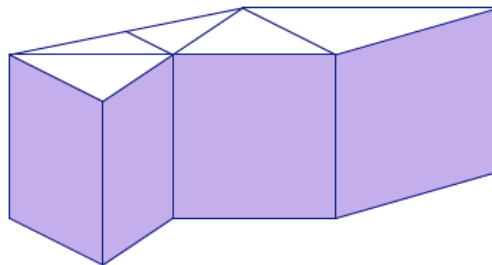


The triangular prism and the right rectangular prism have the same base area, same height, and the same volume:



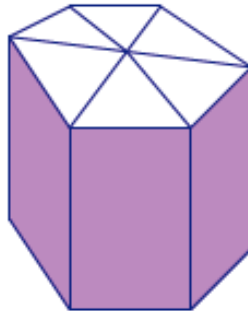
Volume of right triangular prism = volume of right rectangular prism = area of base \times height.

The union of right triangular prisms with the same height:



$$\text{Volume} = \text{sum of volumes} = \text{sum of area of base} \times \text{height} = \text{total area of base} \times \text{height}$$

Any right polygonal prism can be packed without gaps or overlaps by right triangular prisms of the same height:



For any right polygonal prism: Volume = area of base \times height

Exercise 1 (3 minutes)

Exercise 1

a. Show that the following figures have equal volumes.

Volume of triangular prism:

$$\frac{1}{2}(8\text{ cm} \times 9\text{ cm}) \times 8\text{ cm} = 288\text{ cm}^3$$

Volume of rectangular prism:

$$8\text{ cm} \times 8\text{ cm} \times 4.5\text{ cm} = 288\text{ cm}^3$$

b. How can it be shown that the prisms will have equal volumes without completing the entire calculation?

If one base can be cut up and rearranged to form the other base, then the bases have the same area. If the prisms have the same height, then the cutting and rearranging of the bases can show how to slice and rearrange one prism so that it looks like the other prism. Since slicing and rearranging does not change volume, the volumes are the same.

Example 1 (5 minutes)

Example 1

Calculate the volume of the following prism.

- What is the initial difficulty of determining the volume of this prism?
 - *The base is not in the shape of a rectangle or triangle.*
- Do we have a way of finding the area of a kite in one step? If not, how can we find the area of a kite?
 - *We do not have a way to find the area of a kite in one step. We can break the kite up into smaller shapes. One way would be to break it into two triangles, but it can also be broken into four triangles.*

Once students understand that the base must be decomposed into triangles, allow them time to solve the problem.

- Provide a numeric expression that determines the area of the kite-shaped base.
 - *Area of the base:* $\frac{1}{2}(20 \text{ in.} \cdot 18 \text{ in.}) + \frac{1}{2}(4 \text{ in.} \cdot 18 \text{ in.})$
- Find the volume of the prism.
 - *Volume of the prism:* $\left(\frac{1}{2}(20 \text{ in.} \cdot 18 \text{ in.}) + \frac{1}{2}(4 \text{ in.} \cdot 18 \text{ in.})\right)(3 \text{ in.}) = 648 \text{ in}^3$

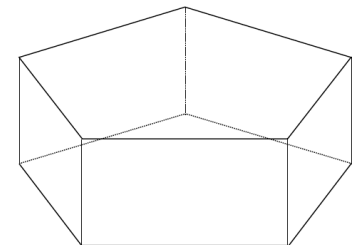
Example 2 (7 minutes)

Example 2

A container is shaped like a right pentagonal prism with an open top. When a cubic foot of water is dumped into the container, the depth of the water is 8 inches. Find the area of the pentagonal base.

Scaffolding:

- Provide an image of a pentagonal prism as needed:



- How can we use volume to solve this problem?
 - *We know that volume = bh, where b represents the area of the pentagonal base and h is the height of the prism.*
- What information do we know from reading the problem?
 - *The volume is 1 cubic foot and the height of the water is 8 inches.*
- Knowing the volume formula, can we use this information to solve the problem?
 - *We can use this information to solve the problem, but the information is given using two different dimensions.*
- How can we fix this problem?
 - *We have to change inches into feet so we have the same units.*
- How can we change inches into feet?
 - *Because 12 inches make 1 foot, we will have to divide the 8 inches by 12 to get the height in feet.*
- Convert 8 inches into feet.
 - $\frac{8}{12} \text{ ft.} = \frac{2}{3} \text{ ft.}$
- Now that we know the volume of the water is 1 cubic foot and the height of the water is $\frac{2}{3}$ feet, how can we determine the area of the pentagonal base?
 - *Use the volume formula.*
- Use the information we know to find the area of the base.
 - $1 \text{ ft}^3 = b \left(\frac{2}{3}\right) \text{ ft.}$ Therefore, the area of the pentagonal base is $\frac{3}{2} \text{ ft}^2$ or $1\frac{1}{2} \text{ ft}^2$.

Example 3 (10 minutes)

Example 3

Two containers are shaped like right triangular prisms each with the same height. The base area of the larger container is 200% more than the base area of the smaller container. How many times must the smaller container be filled with water and poured into the larger container in order to fill the larger container?

Solution by manipulating the equation of the volume of the smaller prism:

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- Let us call the area of the base of the smaller prism, b . Write an expression for the area of the larger base and explain how it models the situation.
 - *The base area of the larger prism is $3b$ because 200% more means that its area is 300% of b . Both prisms have the same height, h .*
- Compute the volume of the smaller prism.
 - *The volume of the smaller prism is $V_S = bh$.*
- What is the volume of the larger prism?
 - *The volume of the larger prism is $V_L = 3bh$.*
- How many times greater is the volume of larger prism relative to the smaller prism?
 - $\frac{3bh}{bh} = 3$. The smaller container must be filled three times in order to fill the larger container.

Scaffolding:

- Students may feel more comfortable manipulating actual values for the dimensions of the prisms as done in the second solution to Example 2. Using sample values may be a necessary stepping stone to understanding how to manipulate the base area of one prism in terms of the other.

Solution by substituting values for the smaller prism's dimensions:

- In order to solve this problem, create two right triangular prisms. What dimensions should we use for the smaller container?
 - *Answers will vary, but for this example we will use a triangle that has a base of 10 inches and a height of 5 inches. The prism will have a height of 2 inches.*
- What is the area of the base for the smaller container? Explain.
 - *The area of the base of the smaller container is 25 in^2 because $A = \frac{1}{2}(10 \text{ in.})(5 \text{ in.}) = 25 \text{ in}^2$.*
- What is the volume of the smaller container? Explain.
 - *The volume of the smaller container is 50 in^3 because $V = 25 \text{ in}^2 \times 2 \text{ in.} = 50 \text{ in}^3$.*
- What do we know about the larger container?
 - *The area of the larger container's base is 200% more than the area of the smaller container's base.*
 - *The height of the larger container is the same as the height of the smaller container.*
- If the area of the larger container's base is 200% more than the area of the smaller container's base, what is the area of the larger container's base?
 - *The area of the larger container's base would be $25 \text{ in}^2 + 2(25 \text{ in}^2) = 75 \text{ in}^2$.*
- What is the volume of the larger container? Explain.
 - *The volume of the larger container is 150 in^3 because $V = 75 \text{ in}^2 \times 2 \text{ in.} = 150 \text{ in}^3$.*

- How many times must the smaller container be filled with water and poured into the larger container in order to fill the larger container? Explain.
 - *The smaller container would have to be filled three times in order to fill the larger container. Each time you fill the smaller container you will have 50 in^3 ; therefore, you will need to fill the smaller container three times to get a volume of 150 in^3 .*
- Would your answer be different if we used different dimensions for the containers? Why or why not?
 - *Our answer would not change if we used different dimensions. Because the area of the base of the larger container is triple the area of the base of the smaller container and the two heights are the same, the volume of the larger container is triple that of the smaller container.*

Exercise 3 (6 minutes)

Exercise 3

Two aquariums are shaped like right rectangular prisms. The ratio of the dimensions of the larger aquarium to the dimensions of the smaller aquarium is 3:2.

Addie says the larger aquarium holds 50% more water than the smaller aquarium.

Berry says that the larger aquarium holds 150% more water.

Cathy says that the larger aquarium holds over 200% more water.

Are any of the girls correct? Explain your reasoning.

Cathy is correct. If the ratio of the dimensions of the larger aquarium to the dimensions of the smaller aquarium is 3:2, then the volume must be 1.5^3 or 3.375 times greater than the smaller aquarium. Therefore, the larger aquarium's capacity is 337.5% times as much water, which is 237.5% more than the smaller aquarium. Cathy said that the larger aquarium holds over 200% more water than the smaller aquarium, so she is correct.

MP.3

Closing (1 minute)

- The formula for the volume of a prism is $V = bh$, where b is the area of the base of the prism. A base that is neither a rectangle nor a triangle must be decomposed into rectangles and triangles in order to find the area of the base.

Exit Ticket (5 minutes)

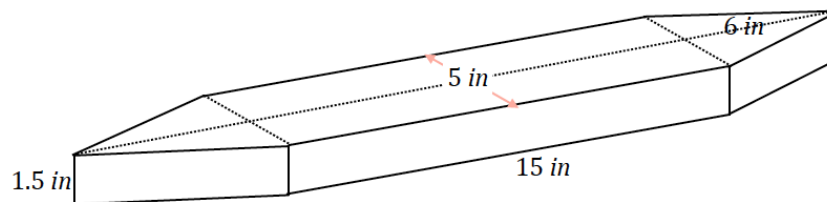
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Lesson 25: Volume of Right Prisms

Exit Ticket

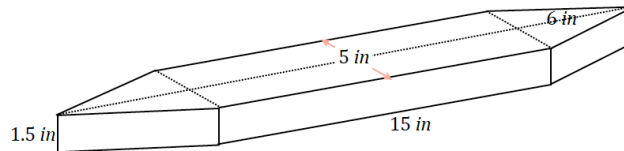
Determine the volume of the following prism. Explain how you found the volume.



Exit Ticket Sample Solutions

Determine the volume of the following prism. Explain how you found the volume.

To find the volume of the prism, the base must be decomposed into triangles and rectangles, since there is no way to find the area of the base as is. The base can be decomposed into two triangles and a rectangle, and their areas must be summed to find the area of the base. Once the area of the base is determined, it should be multiplied by the height to find the volume of the entire prism.



Area of both triangles: $2 \left(\frac{1}{2} (5 \text{ in.} \times 6 \text{ in.}) \right) = 30 \text{ in}^2$

Area of the rectangle: $15 \text{ in.} \times 5 \text{ in.} = 75 \text{ in}^2$

Total area of the base: $(30 + 75) \text{ in}^2 = 105 \text{ in}^2$

Volume of the prism: $(105 \text{ in}^2)(1.5 \text{ in.}) = 157.5 \text{ in}^3$

Problem Set Sample Solutions

1. The pieces in Figure 1 are rearranged and put together to form Figure 2.

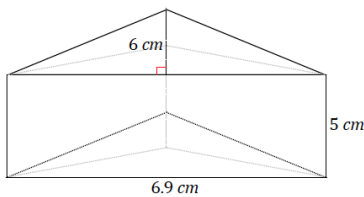


Figure 1

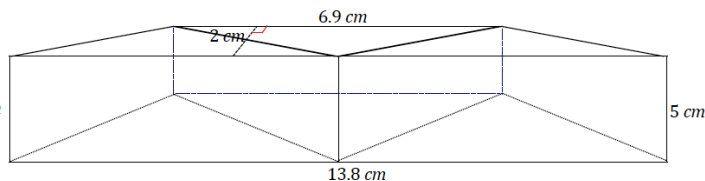


Figure 2

a. Use the information in Figure 1 to determine the volume of the prism.

Volume: $\frac{1}{2} (6.9 \text{ cm} \cdot 6 \text{ cm})(5 \text{ cm}) = 103.5 \text{ cm}^3$

b. Use the information in Figure 2 to determine the volume of the prism.

Volume: $\left(\frac{1}{2} (6.9 \text{ cm} + 13.8 \text{ cm}) \cdot 2 \text{ cm} \right) (5 \text{ cm}) = 103.5 \text{ cm}^3$

c. If we were not told that the pieces of Figure 1 were rearranged to create Figure 2, would it be possible to determine whether the volumes of the prisms were equal without completing the entire calculation for each?

Both prisms have the same height, so as long as it can be shown that both bases have the same area both prisms must have equal volumes. We could calculate the area of the triangle base and the trapezoid base and find that they are equal in area and be sure that both volumes are equal.

2. Each of two right prism containers is filled with 60 gallons of water. The depth of the water in the first container is 20 inches. The depth of the water in the second container is 30 inches. If the area of the base in the first container is 6 ft^2 , find the area of the base in the second container. Explain your reasoning.

We know that the volume of each of the two containers is 60 gallons; therefore, the volumes must be equal. In order to find the volume of the first container, we could multiply the area of the base (6 ft^2) by the height (20 inches). To find the volume of the second container we would also multiply the area of its base, which we will call A (area in ft^2) and the height (30 inches). These two expressions must equal each other since both containers have the same volume.

$$\begin{aligned}6 \times 20 &= A \times 30 \\120 &= 30A \\120 \div 30 &= 30A \div 30 \\4 &= A\end{aligned}$$

Therefore, the area of the second container will be 4 ft^2 . Note: The units for the volume are $1 \text{ ft.} \times 1 \text{ ft.} \times 1 \text{ in.}$ in this computation. Converting the inches to feet would make the computation in ft^3 , but it will not change the answer for A .

3. Two containers are shaped like right rectangular prisms. Each has the same height, but the base of the larger container is 50% more in each direction. If the smaller container holds 8 gallons when full, how many gallons does the larger container hold? Explain your reasoning.

The larger container will hold 18 gallons because each side length of the base is 1.5 times larger than the smaller container's dimensions. Therefore, the area of the larger container's base is 1.5^2 or 2.25 times larger than the smaller container. Because the height is the same in both containers, the volume of the larger container must be 2.25 times larger than the smaller container. $8 \text{ gal.} \times 2.25 = 18 \text{ gal.}$

4. A right prism container with the base area of 4 ft^2 and height of 5 ft. is filled with water until it is 3 ft. deep. If a solid cube with edge length 1 ft. is dropped to the bottom of the container, how much will the water rise?

The volume of the cube is 1 ft^3 . Let the number of feet the water will rise be x . Then the volume of the water over the 3 ft. mark is $4x \text{ ft}^3$ because this represents the area of the base (4 ft^2) times the height (x). Because the volume of the cube is 1 ft^3 , $4x \text{ ft}^3$ must equal 1 ft^3 .

$$\begin{aligned}4x &= 1 \\4x \div 4 &= 1 \div 4 \\x &= \frac{1}{4}\end{aligned}$$

Therefore, the water will rise $\frac{1}{4}$ ft. or 3 inches.

5. A right prism container with a base area of 10 ft^2 and height 9 ft. is filled with water until it is 6 ft. deep. A large boulder is dropped to the bottom of the container, and the water rises to the top completely submerging the boulder and without causing overflow. Find the volume of the boulder.

The increase in volume is the same as the volume of the boulder. The height of the water increases 3 ft. Therefore, the increase in volume is 10 ft^2 (area of the base) multiplied by 3 ft. (the change in height).

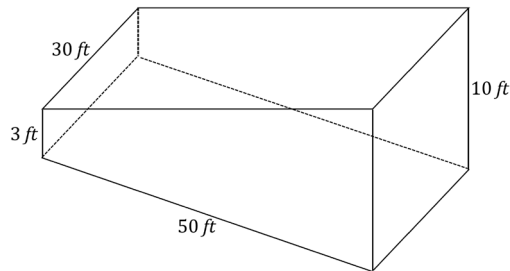
$$V = 10 \text{ ft}^2 \times 3 \text{ ft.} = 30 \text{ ft}^3$$

Because the increase in volume is 30 ft^3 , the volume of the boulder is 30 ft^3 .

6. A right prism container with a base area of 8 ft^2 and height 6 ft. is filled with water until it is 5 ft. deep. A solid cube is dropped to the bottom of the container and the water rises to the top. Find the length of the cube.

When the cube is dropped into the container, the water rises 1 foot, which means the volume increase 8 cubic feet. Therefore, the volume of the cube must be 8 cubic feet. We know that the length, width, and height of a cube are equal, so the length of the cube is 2 feet because $2 \text{ ft.} \times 2 \text{ ft.} \times 2 \text{ ft.} = 8 \text{ ft}^3$, which is the volume of the cube.

7. A rectangular swimming pool is 30 feet wide and 50 feet long. The rectangular floor of the swimming pool is 30 feet wide, 3 feet deep at one end, and 10 feet deep at the other.
- a. Sketch the swimming pool as a right prism.



- b. What kind of right prism is the swimming pool?

The swimming pool is a right trapezoidal prism.

- c. What is the volume of the swimming pool in cubic feet?

$$\text{Area of base} = \frac{50 \text{ ft.} (10 \text{ ft.} + 3 \text{ ft.})}{2} = 325 \text{ ft}^2$$

$$\text{Volume of pool} = 325 \text{ ft.} \times 30 \text{ ft.} = 9,750 \text{ ft}^3$$

- d. How many gallons will the swimming pool hold if each cubic feet of water is about 7.5 gallons?

$$9,750(7.5) \text{ gal.} = 73,125 \text{ gal.} \text{ The pool will hold } 73,125 \text{ gal.}$$

8. A milliliter (mL) has volume of 1 cm^3 . A 250 mL measuring cup is filled to 200 mL. A small stone is placed in the measuring cup. The stone is completely submerged and the water level rises to 250 mL.

- a. What is the volume of the stone in cm^3 ?

When the stone is dropped into the measuring cup, the increase in volume is 50 mL (250 – 200). We know that 1 mL has a volume of 1 cm^3 ; therefore, the stone has a volume of 50 cm^3 .

- b. Describe a right rectangular prism that has the same volume as the stone.

Answers will vary. Possible answers are listed below.

$$1 \text{ cm} \times 1 \text{ cm} \times 50 \text{ cm}$$

$$1 \text{ cm} \times 2 \text{ cm} \times 25 \text{ cm}$$

$$1 \text{ cm} \times 5 \text{ cm} \times 10 \text{ cm}$$

$$2 \text{ cm} \times 5 \text{ cm} \times 5 \text{ cm}$$

Opening Exercise

