Lesson 24: Surface Area

Student Outcomes

- Students determine the surface area of three-dimensional figures, those that are composite figures and those that have missing sections.

Lesson Notes

This lesson is a continuation of Lesson 23. Students will continue to work on surface area advancing to figures with missing sections.

Classwork

Example 1 (8 minutes)

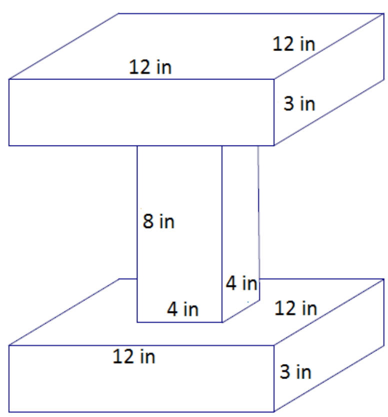
MP.7 & MP.8

Students should solve this problem on their own.

> *Scaffolding:*
> - As in Lesson 23, students can draw nets of the figures to help them visualize the area of the faces. They could determine the area of these without the holes first and subtract the surface area of the holes.

Example 1

Determine the surface area of the image.



Surface area of top and bottom prisms

Lateral sides $= 4(12\ in. \times 3\ in.)$
$= 144\ in^2$

Base face $= 12\ in. \times 12\ in.$
$= 144\ in^2$

Base face with hole $= 12\ in. \times 12\ in. - 4\ in. \times 4\ in.$
$= 128\ in^2$

There are two of these, making up $832\ in^2$.

Surface area of middle prism

Lateral Sides $= 4(4\ in. \times 8\ in.)$
$= 128\ in^2$

Surface area: $832\ in^2 + 128\ in^2 = 960\ in^2$

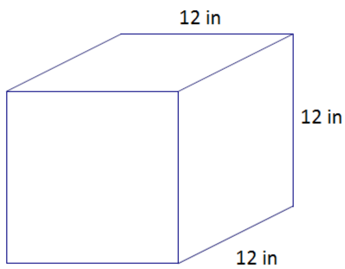
- Describe the method you used to determine the surface area.
  - *Answers will vary: I determined the surface area of each prism separately and added them together. Then I subtracted the area of the sections that were covered by another prism.*
- If all three prisms were separate, would the sum of their surface areas be the same as the surface area you determined in this example?
  - *No, if the prisms were separate, there would be more surfaces shown. The three separate prisms would have a greater surface area than this example. The area would be greater by the area of four* 4 *in.* $\times$ 4 *in. squares (*$64\ in^2$*).*

Example 2 (5 minutes)

> *Scaffolding:*
>
> - As in Lesson 23, students can draw nets of the figures to help them visualize the area of the faces. They could determine the area of these without the holes first and subtract the surface area of the holes.

Example 2

a. Determine the surface area of the cube.



$Surface\ area = 6s^2$

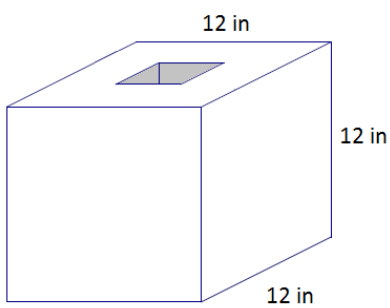
$SA = 6(12\ in)^2$

$SA = 6(144\ in^2)$

$SA = 864\ in^2$

- Explain how $6(12\text{ in.})^2$ represents the surface area of the cube.
  - *The area of one face, one square with side length of* 12 *in., is* $(12\ in.)^2$*, and so a total area of all six faces is* $6(12\ in.)^2$.

b. A square hole with a side length of 4 inches is drilled through the cube. Determine the new surface area.



Area of interior lateral sides

$= 4(12\ in. \times 4\ in.)$

$= 192\ in^2$

Surface Area of cube with holes

$= 6(12\ in.)^2 - 2(4\ in. \times\ 4\ in.) + 4(12\ in. \times 4\ in.)$

$= 864\ in^2 - 32\ in^2 + 192\ in^2$

$= 1,024\ in^2$

- How does drilling a hole in the cube change the surface area?
  - *We have to subtract the area of the square at the surface from each end.*
- What happens to the surfaces that now show inside the cube?
  - *These are now part of the surface area.*
- What is the shape of the piece that was removed from the cube?
  - *A rectangular prism was drilled out of the cube with the following dimensions:* 4 *in.* $\times$ 4 *in.* $\times$ 12 *in.*
- How can we use this to help us determine the new total surface area?
  - *We can find the surface area of the cube and the surface area of the rectangular prism, but we will have to subtract the area of the square bases from the cube and also exclude these bases in the area of the rectangular prism.*
- Why is the surface area larger when holes have been cut into the cube?
  - *There are more surfaces showing now. All of the surfaces need to be included in the surface area.*

- Explain how the expression $6(12\text{ in.})^2 - 2(4\text{ in.} \times 4\text{ in.}) + 4(12\text{ in.} \times 4\text{ in.})$ represents the surface area of the cube with the hole.
  - *From the total surface area of a whole (uncut) cube, $6(12\ in.)^2$, the area of the bases (the cuts made to the surface of the cube) are subtracted: $6(12\ in.)^2 - 2(4\ in. \times 4\ in.)$. To this expression we add the area of the four lateral faces of the cut out prism, $4(12\ in. \times 4\ in.)$. The complete expression then is $6(12\ in.)^2 - 2(4\ in. \times 4\ in.) + 4(12\ in. \times 4\ in.)$.*

Example 3 (5 minutes)

Example 3

A right rectangular pyramid has a square base with a side length of 10 inches. The surface area of the pyramid is 260 in^2. Find the height of the four lateral triangular faces.

$$\textit{Area of base} = 10\ in. \times 10\ in. = 100\ in^2$$

$$\textit{Area of the four faces} = 260\ in^2 - 100\ in^2 = 160\ in^2$$

The total area of the four faces is $160\ in^2$. Therefore, the area of each triangular faces is $40\ in^2$.

$\textit{Area of lateral side} = \frac{1}{2}bh$

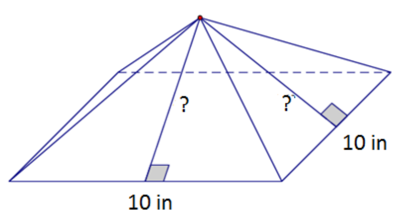
$40\ in^2 = \frac{1}{2}(10\ in.)h$

$40\ in^2 = (5\ in.)h$

$h = 8\ in.$

The height of each lateral triangular face is 8 inches.

- What strategies could you use to help you solve this problem?
  - *I could draw a picture of the pyramid and label the sides so that I can visualize what the problem is asking me to do.*



- What information have we been given? How can we use the information?
  - *We know the total surface area, and we know the length of the sides of the square.*
  - *We can use the length of the sides of the square to give us the area of the square base.*
- How will the area of the base help us determine the slant height?
  - *First, we can subtract the area of the base from the total surface area in order to determine what is left for the lateral sides.*
  - *Now we can divide the remaining area by 4 to get the area of just one triangular face.*
  - *Finally, we can work backwards. We have the area of the triangle, and we know the base is 10 in., so we can solve for the height.*

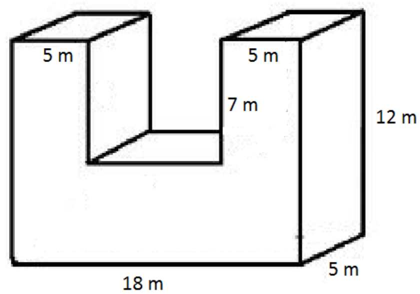
Exercises 1–8 (20 minutes)

Students work in pairs to complete the exercises.

Exercises 1–8

Determine the surface area of each figure. Assume all faces are rectangles unless it is indicated otherwise.

1.



Top and bottom $= 2(18\ m \times 5\ m) = 180\ m^2$

Extra interior sides $= 2(5\ m \times 7\ m) = 70\ m^2$

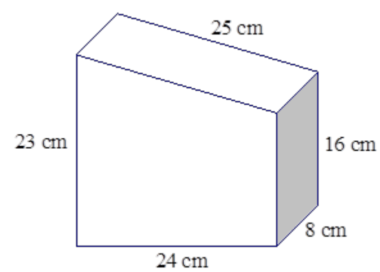
Left and right sides $= 2(5\ m \times 12\ m) = 120\ m^2$

Front and back sides $= 2\big((18\ m \times 12\ m) - (8\ m \times 7\ m)\big)$
$= 2(216\ m^2 - 56\ m^2)$
$= 2(160\ m^2)$
$= 320\ m^2$

Surface area $= 180\ m^2 + 70\ m^2 + 120\ m^2 + 320\ m^2$
$= 690\ m^2$

2. In addition to your calculation, explain how the surface area was determined.

The surface area of the prism is found by taking the sum of the areas of the trapezoidal front and back areas of the four different sized rectangles that make up the lateral faces.



Area top $= 25\ cm \times 8\ cm$
$= 200\ cm^2$

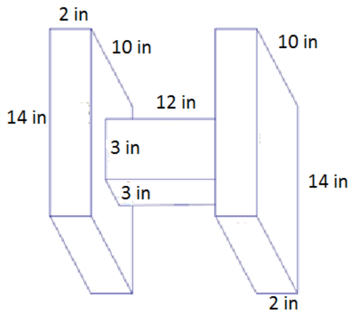
Area bottom $= 24\ cm \times 8\ cm$
$= 192\ cm^2$

Area sides $= (23\ cm \times 8\ cm) + (16\ cm\ 8\ cm)$
$= 312\ cm^2$

Area front and back $= 2\left(\frac{1}{2}(16\ cm + 23\ cm)(24\ cm)\right)$
$= 2(468\ cm^2)$
$= 936\ cm^2$

Surface Area $= 200\ cm^2 + 192\ cm^2 + 312\ cm^2 + 936\ cm^2$
$= 1,640\ cm^2$

3.



Surface Area of Prisms on the Sides:

Area of front and back $= 2(2\ in. \times 14\ in.)$
$= 56\ in^2$

Area of top and bottom $= 2(2\ in. \times 10\ in.)$
$= 40\ in^2$

Area of side $= 14\ in. \times 10\ in. = 140\ in^2$

Area of side with hole $= 14\ in. \times 10\ in. - 3\ in. \times 3\ in.$
$= 131\ in^2$

There are two such rectangular prisms, so the surface area of both is $734\ in^2$.

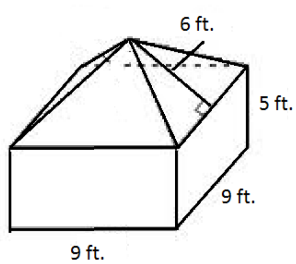
Surface Area of Middle Prism:

Area of front and back $= 2(3\ in. \times 12\ in.) = 72\ in^2$

Area of sides $= 2(3\ in. \times 12\ in.) = 72\ in^2$

Surface area $= 734\ in^2 + 144\ in^2 = 878\ in^2$

4. In addition to your calculation, explain how the surface area was determined.



The surface area of the prism is found by taking the area of the base of the rectangular prism and the area of its four lateral faces and adding it to the area of the four lateral faces of the pyramid.

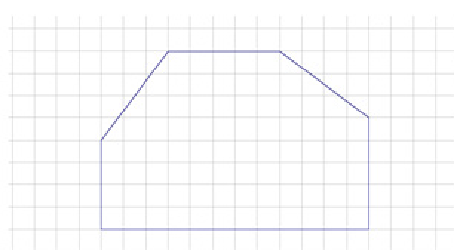
Area of base $= 9\ ft. \times 9\ ft.$
$= 81\ ft^2$

Area of rectangular sides $= 4(9\ ft. \times 5\ ft.)$
$= 180\ ft^2$

Area of triangular sides $= 4\left(\frac{1}{2}(9\ ft.)(6\ ft.)\right)$
$= 108\ ft^2$

Surface Area $= 81\ ft^2 + 180\ ft^2 + 108\ ft^2$
$= 369\ ft^2$

5. A hexagonal prism has the following base and has a height of 8 units. Determine the surface area of the prism.



Area of bases $= 2(48 + 6 + 20 + 10)\ units^2$
$= 168\ units^2$

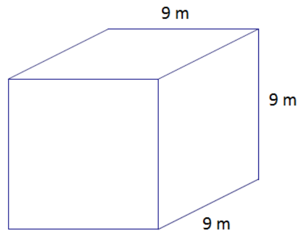
Area of 5 unit sides $= 4(5 \times 8)\ units^2$
$= 160\ units^2$

Area of other sides $= (4 \times 8)\ units^2 + (12 \times 8)\ units^2$
$= 128\ units^2$

Surface Area $= 168\ units^2 + 160\ units^2 + 128\ units^2$
$= 456\ units^2$

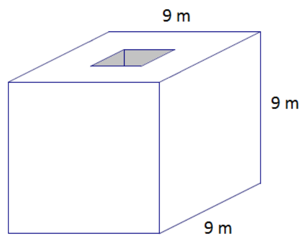
6. **Determine the surface area of each figure.**

   a.



$SA = 6s^2$
$= 6(9\ m)^2$
$= 6(81\ m^2)$
$= 486\ m^2$

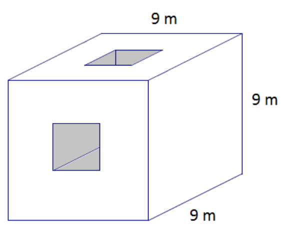
   b. **A cube with a square hole with 3 m side lengths has been drilled through the cube.**



Lateral sides of the hole $= 4(9\ m \times 3\ m) = 108\ m^2$

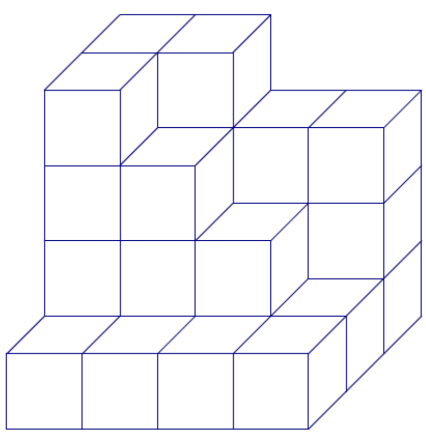
Surface area of cube with holes $= 486\ m^2 - 2(3\ m \times 3\ m) + 108\ m^2$
$= 576\ m^2$

   c. **A second square hole with 3 m side lengths has been drilled through the cube.**



Surface Area $= 576\ m^2 - 4(3\ m \times 3\ m) + 2(4(3\ m \times 3\ m))$
$= 612\ m^2$

7. **The figure below shows 28 cubes with an edge length of 1 unit. Determine the surface area.**



Area top and bottom $= 24\ units^2$

Area sides $= 18\ units^2$

Area front and back $= 28\ units^2$

Surface Area $= 24 + 18 + 28$
$= 70\ units^2$

8. **The base rectangle of a right rectangular prism is 4 ft. $\times$ 6 ft. The surface area is 288 ft^2. Find the height.**

Let h be the height in feet.

Area of one base: 4 *ft.* $\times$ 6 *ft.* $= 24$ *ft*^{2}

Area of two bases: $2(24$ *ft*$^2) = 48$ *ft*^{2}

Numeric area of four lateral faces: 288 *ft*$^2 - 48$ *ft*$^2 = 240$ *ft*^{2}

Algebraic area of four lateral faces: $2(6h + 4h)$

Solve for h

$$2(6h + 4h) = 240$$
$$10h = 120$$
$$h = 12$$

The height is 12 *feet.*

Closing (2 minutes)

- Write down three tips that you would give a friend that is trying to calculate surface area.

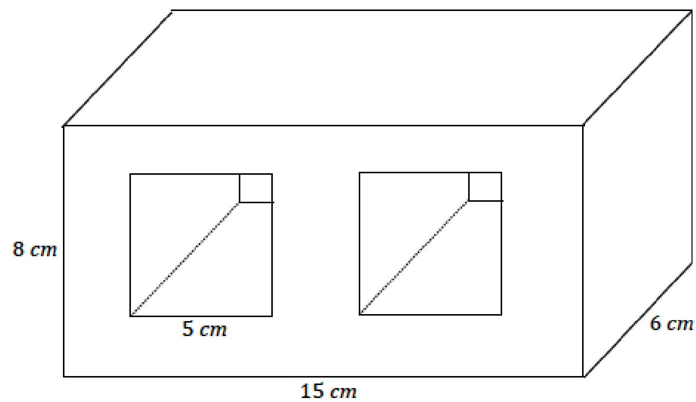
Exit Ticket (5 minutes)

Name ____ Date ________________

Lesson 24: Surface Area

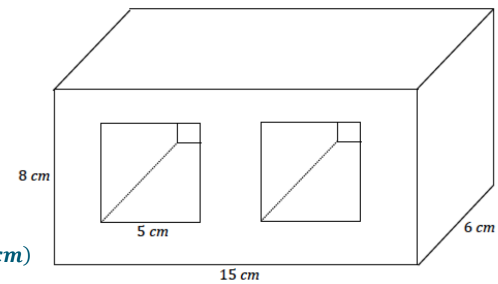
Exit Ticket

Determine the surface area of the right rectangular prism after the two square holes have been drilled. Explain how you determined the surface area.



Exit Ticket Sample Solutions

Determine the surface area of the right rectangular prism after the two square holes have been drilled. Explain how you determined the surface area.



Area of top and bottom $= 2(15\ cm \times 6\ cm)$
$= 180\ cm^2$

Area of sides $= 2(6\ cm \times 8\ cm)$
$= 96\ cm^2$

Area of front and back $= 2(15\ cm \times 8\ cm) - 4(5\ cm \times 5\ cm)$
$= 140\ cm^2$

Area inside $= 8(5\ cm \times 6\ cm)$
$= 240\ cm^2$

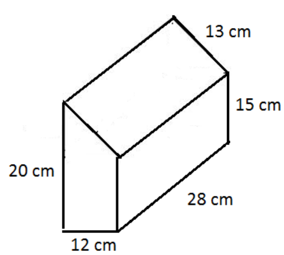
Surface Area $= 180\ cm^2 + 96\ cm^2 + 140\ cm^2 + 240\ cm^2$
$= 656\ cm^2$

Take the sum of the areas of the four lateral faces of the main rectangular prism, and subtract the areas of the four square cuts from the area of the bases of the main rectangular prism. Finally, add the lateral faces of the prisms that were cut out of the main prism.

Problem Set Sample Solutions

Determine the surface area of each figure.

1. **In addition to the calculation of the surface area, describe how you found the surface area.**



Area of top $= 28\ cm \times 13\ cm$
$= 364\ cm^2$

Area of bottom $= 28\ cm \times 12\ cm$
$= 336\ cm^2$

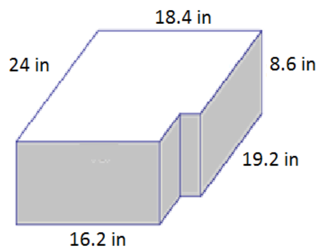
Area of left and right sides $= 28\ cm \times 20\ cm + 15\ cm \times 28\ cm$
$= 980\ cm^2$

Area of front and back sides $= 2\left((12\ cm \times 15\ cm) + \frac{1}{2}(5\ cm \times 12\ cm)\right)$
$= 2(180\ cm^2 + 30\ cm^2)$
$= 2(210\ cm^2)$
$= 420\ cm^2$

Surface area $= 364\ cm^2 + 336\ cm^2 + 980\ cm^2 + 420\ cm^2$
$= 2,100\ cm^2$

Split the area of the two trapezoidal bases, take the sum of the areas, and then add the areas of the four different sized rectangles that make up the lateral faces.

2.



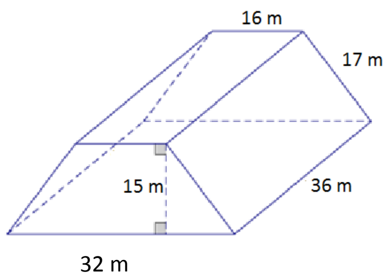
Area of front and back $= 2(18.4\ in. \times 8.6\ in.) = 316.48\ in^2$

Area of sides $= 2(8.6\ in. \times\ 24\ in.) = 412.8\ in^2$

Area of top and bottom $= 2((18.4\ in. \times 24\ in.) - 4.8\ in. \times\ 2.2\ in.)$
$= 2(441.6\ in^2\ - 10.56\ in^2)$
$= 2(431.04\ in^2)$
$= 862.08\ in^2$

Surface Area $= 316.48\ in^2 + 412.8\ in^2 + 862.08\ in^2$
$= 1,591.36\ in^2$

3.



Area of front and back $= 2\left(\frac{1}{2}(32\ m + 16\ m)15\ m\right)$
$= 720\ m^2$

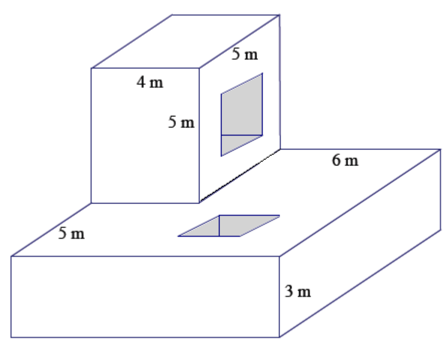
Area of top $=\ 16\ m\ \times\ 36\ m$
$=\ 576\ m^2$

Area of left and right sides $=\ 2(17\ m \times\ 36\ m)$
$=\ 2(612\ m^2)$
$=\ 1,224\ m^2$

Area of bottom $=\ 32\ m\ \times\ 36\ m$
$= 1,152\ \ m^2$

Surface Area $=\ 720\ \ m^2 + 1,152\ \ m^2 + 1,224\ \ m^2\ \ + 576\ m^2$
$=\ 3,672\ m^2$

4. **Determine the surface area after two square holes with a side length of 2 m are drilled through the solid figure composed of two rectangular prisms.**



Surface Area of Top Prism Before the Hole is Drilled:

Area of top $= 4\ m\ \times\ 5\ m$
$=\ 20\ m^2$

Area of front and back $=\ 2(4\ m\ \times\ 5\ m)$
$=\ 40\ m^2$

Area of sides $=\ 2(5\ m\ \times\ 5\ m)$
$=\ 50\ m^2$

Surface Area of Bottom Prism Before the Hole is Drilled:

Area of top $= 10\ m\ \times 10\ m - 20\ m^2$
$=\ 80\ m^2$

Area of bottom $=\ 10\ m\ \times\ 10\ m$
$=\ 100\ m^2$

Area of front and back $=\ 2(10\ m\ \times\ 3\ m)$
$=\ 60\ m^2$

Area of sides $=\ 2(10\ m\ \times\ 3\ m)$
$=\ 60\ m^2$

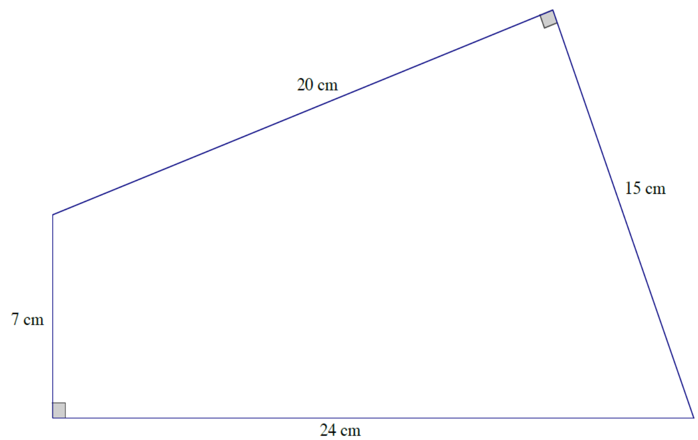
Surface Area of Interiors:

Area of Interiors $= 4(2\ m\ \times 4\ m) + 4(2\ m\ \times\ 3\ m)$
$=\ 56\ m^2$

Surface area $=\ 110\ m^2 +\ 300\ m^2 + 56\ m^2 - 16\ m^2 =\ 450\ m^2$

5. The base of a right prism is shown below. Determine the surface area if the height of the prism is 10 cm. Explain how you determined the surface area.

Take the sum of the areas of the two bases made up of two right triangles, and add to it the sum of the areas of the lateral faces made up by rectangles of different sizes.



Area of sides $= (20\ cm \times 10\ cm) + (15\ cm \times 10\ cm) + (24\ cm \times 10\ cm) + (7\ cm \times 10\ cm)$
$= 200\ cm^2 + 150\ cm^2 + 240\ cm^2 + 70\ cm^2$
$= 660\ cm^2$

Area of bases $= 2\left(\frac{1}{2}(7\ cm \times 24\ cm) + \frac{1}{2}(20\ cm \times 15\ cm)\right)$
$= (7\ cm \times 24\ cm) + (20\ cm \times 15\ cm)$
$= 168\ cm^2 + 300\ cm^2$
$= 468\ cm^2$

Surface area $= 660\ cm^2 + 468\ cm^2$
$= 1,128\ cm^2$