Lesson 4: Solving for Unknown Angles Using Equations

Student Outcomes

- Students solve for unknown angles in word problems and in diagrams involving all learned angle facts.

Classwork

Opening Exercise (5 minutes)

Opening Exercise

Four times the measurement of an angle is the complement of the angle. Find the measurement of the angle and its complement.

$x + 4x = 90$ *complementary* $\angle s$

$5x = 90$

$\left(\frac{1}{5}\right)5x = \left(\frac{1}{5}\right)90$

$x = 18$

Measurement of the angle $= 18°$.

Measurement of the complement of the angle $= 72°$.

Scaffolding:

As in earlier lessons, tasks such as the Opening Exercise can be scaffolded into parts as follows:

- Explain the angle relationships in the diagram. Write the equation. Explain how the equation represents the diagram, including particular parts. Solve the equation. Interpret the solution and determine if it is reasonable.

In the following examples and exercises, students set up and solve an equation for the unknown angle based on the relevant angle relationships in the diagram. Encourage students to list the appropriate angle fact abbreviation for any step that depends on an angle relationship.

Example 1 (4 minutes)

Two options are provided here for Example 1. The second is more challenging than the first.

Example 1

Find the measurement of $\angle FAE$ and $\angle CAD$.

$2x + 6x + 90 = 170$ *vert.* $\angle s$

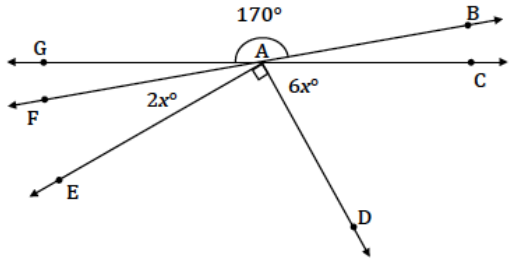
$8x + 90 - 90 = 170 - 90$

$\left(\frac{1}{8}\right)8x = \left(\frac{1}{8}\right)80$

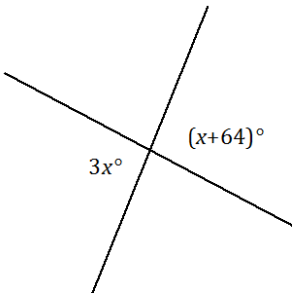
$x = 10$

The measurement of $\angle FAE = 2(10°) = 20°$.

The measurement of $\angle CAD = 6(10°) = 60°$.



Two lines meet at a point. List the relevant angle relationship in the diagram. Set up and solve an equation to find the value of x. Find the measurement of one of the vertical angles.



Students use information in the figure and a protractor to solve for x.

i) Students will measure a $64°$ angle as shown; the remaining portion of the angle must be $x°$ ($\angle s$ add).

ii) Students can use their protractor to find the measurement of x and use this measurement to partition the other angle in the vertical pair.

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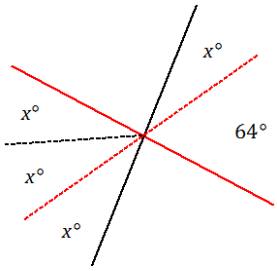
As a check, students should substitute the measured x value into each expression and evaluate; each angle of the vertical pair should be equal to the other. Students can also use their protractor to measure each angle of the vertical angle pair.



With a modified figure, students can write an algebraic equation that they have the skills to solve:

$$\begin{aligned} 2x &= 64 && \textit{vert. } \angle s \\ \left(\frac{1}{2}\right)2x &= \left(\frac{1}{2}\right)64 \\ x &= 32 \end{aligned}$$

Measurement of each angle in the vertical pair: $3(32°) = 96°$

Extension:

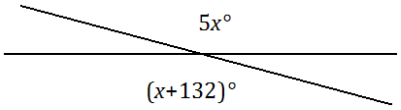
$$\begin{aligned} 3x &= x + 64 && \textit{vert. } \angle s \\ 3x - x &= x - x + 64 \\ 2x &= 64 \\ \left(\frac{1}{2}\right)2x &= \left(\frac{1}{2}\right)64 \\ x &= 32 \end{aligned}$$

Measurement of each angle in the vertical pair: $3(32°) = 96°$

Exercise 1 (4 minutes)

Exercise 1

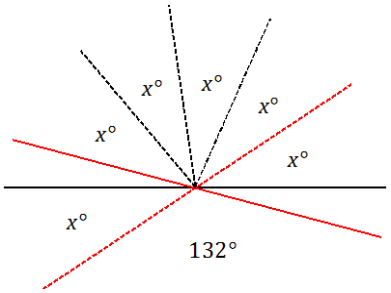
Set up and solve an equation to find the value of x. List the relevant angle relationship in the diagram. Find the measurement of one of the vertical angles.



Students use information in the figure and a protractor to solve for x.

i) Measure a 66° angle as shown; the remaining portion of the original angle must be x ($\angle s$ add).

ii) Partition the other angle in the vertical pair into equal angles of $x°$.



Students should perform a check (as in Example 1) before solving an equation that matches the modified figure.

Extension:

$$4x = 132 \qquad \textit{vert. } \angle s$$
$$\left(\frac{1}{4}\right)4x = \left(\frac{1}{4}\right)132$$
$$x = 33$$

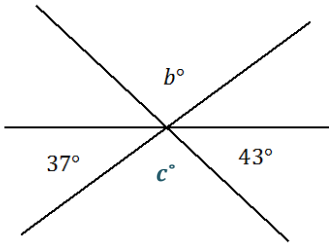
Measurement of each vertical angle: $5(33) = 165°$

Note: Students can check their answers for any question by measuring each unknown angle with a protractor, as all diagrams are drawn to scale.

Example 2 (4 minutes)

Example 2

Three lines meet at a point. List the relevant angle relationships in the diagram. Set up and solve an equation to find the value of b.



Let $b = c$.

$$c + 37 + 43 = 180 \qquad \textit{$\angle s$ on a line}$$
$$c + 80 = 360$$
$$c + 80 - 80 = 360 - 80$$
$$c = 100$$

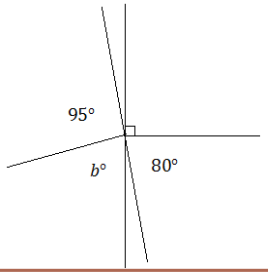
Since $b = c$, $c = 100$

Exercise 2 (4 minutes)

Students set up and solve an equation for the unknown angle based on the relevant angle relationships in the diagram. List the appropriate angle fact abbreviation in the initial equation.

Exercise 2

Two lines meet at the common vertex of two rays. List the relevant angle relationships in the diagram. Set up and solve an equation to find the value of b.



$$b + 95 = 90 + 80 \qquad \textit{vert. } \angle s$$
$$b + 95 - 95 = 170 - 95$$
$$b = 75$$

Example 3 (6 minutes)

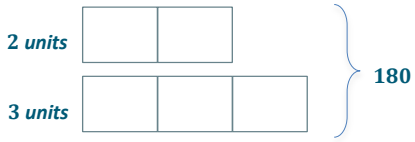
Students set up and solve an equation for the unknown angle based on the relevant angle relationship in the question. In this case, suggest that students use the words "angle" and "supplement" as placeholders in their equations. Students can use a tape diagram to solve for the unknown angles.

Example 3

The measurement of an angle is $\frac{2}{3}$ the measurement of its supplement. Find the measurement of the angle.

$answer = \frac{2}{3}(180 - angle)$

Using a tape diagram:



5 *units* $= 180$
1 *unit* $= 36$
2 *units* $= 72$
3 *units* $= 108$

The measurements of the two supplementary angles that satisfy these criteria are $72°$ and $108°$.

The tape diagram model is an ideal strategy for this question. If students are not familiar with the tape diagram model, use a 'guess and check' table with them. Here is an example of such a table with two entries for guesses that did not result in a correct answer.

Guess	$\frac{2}{3}$(Guess)	Sum (should be $180°$)
60	$\frac{2}{3}(60) = 40$	$60 + 40 = 100$; not the answer
90	$\frac{2}{3}(90) = 60$	$90 + 60 = 150$; not the answer

Exercise 3 (5 minutes)

Students set up and solve an equation for the unknown angle based on the relevant angle relationship in the question. In this case, suggest that students use the words "angle" and "complement" as placeholders in their equations. Students can use a tape diagram to solve for the unknown angles.

Exercise 3

The measurement of an angle is $\frac{1}{4}$ the measurement of its complement. Find the measurement of the angle.

$answer = \frac{1}{4}(90 - angle)$

Using a tape diagram:

5 *units* $= 90$
1 *unit* $= 18$
4 *units* $= 72$

The measurements of the two complementary angles that satisfy these criteria are $18°$ and $72°$.

Example 4 (4 minutes)

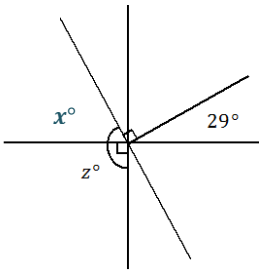
Example 4

Three lines meet at the common vertex of a ray. List the relevant angle relationships in the diagram. Set up and solve an equation to find the value of z.

Let $x°$ be the measurement of the indicated angle.

$x + 90 + 29 = 180$ *∠s on a line*
$x + 119 = 180$
$x + 119 - 119 = 180 - 119$
$x = 61$

$z = x + 90$ *∠s add*
$z = 61 + 90$
$z = 151$



Exercise 4 (4 minutes)

Exercise 4

Two lines meet at the common vertex of two rays. Set up and solve an equation to find the value of x. Find the measurement of $\angle GAF$ and of $\angle BAC$.

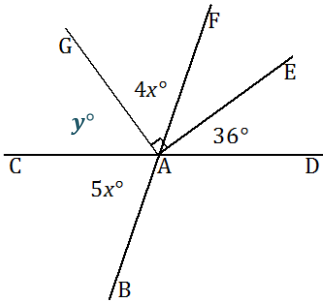
Let $y°$ be the measurement of the indicated angle.

$y = 180 - (90 + 36)$ *∠s on a line*
$y = 54$

$4x + y + 5x = 180$ *∠s on a line*
$4x + 54 + 5x = 180$
$9x + 54 = 180$
$9x + 54 - 54 = 180 - 54$
$9x = 126$
$\left(\frac{1}{9}\right)9x = \left(\frac{1}{9}\right)126$
$x = 14$

The measurement of $\angle GAF$: $4(14°) = 56°$.

The measurement of $\angle BAC$: $5(14°) = 70°$.



Closing (1 minute)

- In every unknown angle problem, it is important to identify the angle relationship(s) correctly in order to set up an equation that will yield the unknown value.
- Check your answer by substituting and/or measuring to be sure it is correct.

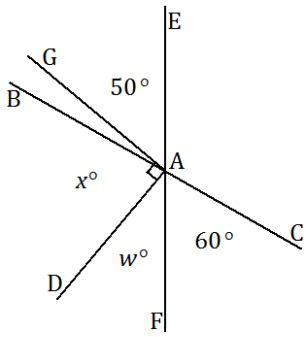
Exit Ticket (4 minutes)

Name _____ Date _____

Lesson 4: Solving for Unknown Angles Using Equations

Exit Ticket

BC and EF meet at A. AG and AD form a right angle. Set up and solve an equation to find the values of x and w.



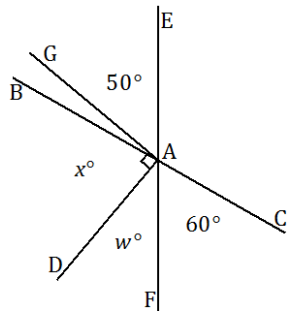
Exit Ticket Sample Solutions

Lines BC and EF meet at A. Rays AG and AD form a right angle. Set up and solve an equation to find the values of x and w.

$\angle BAE = 60$ *vert. ∠s*
$\angle BAG = 10$ *∠s add*

$x + \angle BAG = 90$ *complementary ∠s*
$x + 10 = 90$
$x + 10 - 10 = 90 - 10$
$x = 80$

$x + w + 60 = 180$ *∠s on a line*
$80 + w + 60 = 180$
$140 + w = 180$
$140 + w - 140 = 180 - 140$
$w = 40$



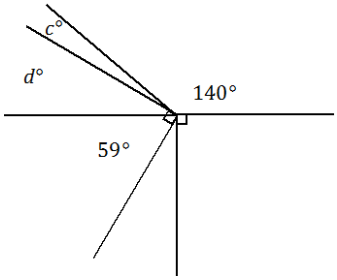
Problem Set Sample Solutions

Set up and solve an equation for the unknown angle based on the relevant angle relationships in the diagram. Add labels to diagrams as needed to facilitate their solutions. List the appropriate angle fact abbreviation for any step that depends on an angle relationship.

1. **Four rays have a common vertex on a line. Set up and solve an equation to find the value of c.**

$59 + d = 90$ *complementary ∠s*
$59 - 59 + d = 90 - 59$
$d = 31$

$d + c + 140 = 180$ *∠s on a line*
$31 + c + 140 = 180$
$c + 171 = 180$
$c + 171 - 171 = 180 - 171$
$c = 9$

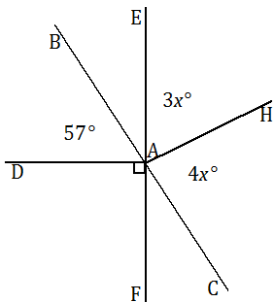


2. **Lines $\overleftrightarrow{BC}$ and $\overleftrightarrow{EF}$ meet at A. Set up and solve an equation to find the value of x. Find the measurements of $\angle EAH$ and $\angle HAC$.**

$\angle BAE + 57 = 90$ *complementary ∠s*
$\angle BAE + 57 - 57 = 90 - 57$
$\angle BAE = 33$

$\angle BAE + 3x + 4x = 180$ *∠s on a line*
$33 + 3x + 4x = 180$
$33 + 7x = 180$
$33 - 33 + 7x = 180 - 33$
$7x = 147$
$\left(\frac{1}{7}\right)7x = \left(\frac{1}{7}\right)147$
$x = 21$

The measurement of $\angle EAH = 3(21°) = 63°$.
The measurement of $\angle HAC = 4(21°) = 84°$.

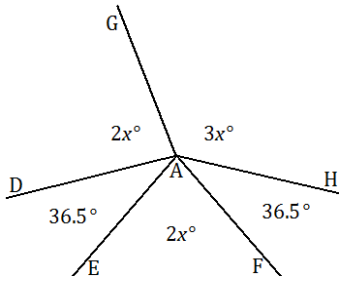


Scaffolding:

Students struggling to organize their solution may benefit from prompts such as the following:

- Write an equation to model this situation. Explain how your equation describes the situation. Solve and interpret the solution. Is it reasonable?

3. Five rays meet at a point. Set up and solve an equation to find the values of x. Find the measurements of $\angle DAG$ and $\angle GAH$.

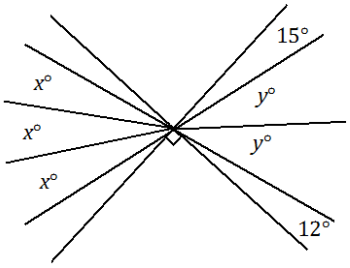


$2x + 36.5 + 36.5 + 2x + 3x = 360$ *∠s at a point*
$7x + 73 = 360$
$7x + 73 - 73 = 360 - 73$
$7x = 287$
$\left(\frac{1}{7}\right)7x = \left(\frac{1}{7}\right)287$
$x = 41$

The measurement of $\angle EAF = 2(41°) = 82°$.

The measurement of $\angle GAH = 3(41°) = 123°$.

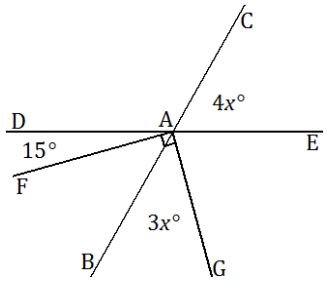
4. Two perpendicular lines meet at the common vertex of seven rays. Set up and solve an equation to find the values of x and y.



$2y + 12 + 15 + 90 = 180$ *∠s on a line*
$2y + 117 = 180$
$2y + 117 - 117 = 180 - 117$
$2y = 63$
$\left(\frac{1}{2}\right)2y = \left(\frac{1}{2}\right)63$
$y = 31.5$

$3x = 2y$ *vert. ∠s*
$3x = 2(31.5)$
$3x = 63$
$\left(\frac{1}{3}\right)3x = \left(\frac{1}{3}\right)63$
$x = 21$

5. Two lines meet at the common vertex of two perpendicular rays. Set up and solve an equation to find the value of x. Find the measurements of $\angle CAE$ and $\angle BAG$.



The measurement of $\angle DAB = 4x°$ *vert. ∠s*
The measurement of $\angle DAG = 90° + 15° = 105°$ *∠s add*
$7x = 105$ *∠s add*
$\left(\frac{1}{7}\right)7x = \left(\frac{1}{7}\right)105$
$x = 15$

The measurement of $\angle CAE = 4(15°) = 60°$.

The measurement of $\angle BAG = 3(15°) = 45°$.

6. Three angles are at a point. The second angle is $20°$ more than the first, and the third angle is $20°$ more than the second. Find the measurements of all three angles.

$x + (x + 20) + (x + 20 + 20) = 360$
$3x + 60 = 360$
$3x + 60 - 60 = 360 - 60$
$3x = 300$
$\left(\frac{1}{3}\right)3x = \left(\frac{1}{3}\right)300$
$x = 100$

Angle 1: $100°$
Angle 2: $100° + 20° = 120°$
Angle 3: $100° + 20° + 20° = 140°$

7. Five angles are at a point. The measurement of each angle is one of five consecutive, positive whole numbers.

a. Determine the measurements of all five angles.

$$\begin{aligned} x + (x+1) + (x+2) + (x+3) + (x+4) &= 360 \\ 5x + 10 &= 360 \\ 5x + 10 - 10 &= 360 - 10 \\ 5x &= 350 \\ \left(\frac{1}{5}\right)5x &= \left(\frac{1}{5}\right)350 \\ x &= 70 \end{aligned}$$

Angle measures are: $70°$, $71°$, $72°$, $73°$, and $74°$.

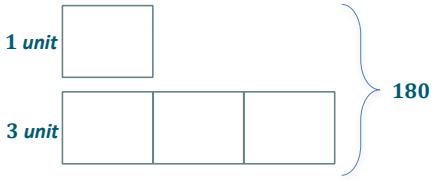
MP.2 & MP.7

b. Compare the expressions you used for the three angles and their combined expression. Explain how they are equivalent and how they reveal different information about this situation.

By the commutative and associative laws, $x + (x+1) + (x+2) + (x+3) + (x+4)$ is equal to $(x + x + x + x + x) + (1 + 2 + 3 + 4)$, which is equal to $5x + 10$. The first expression, $x + (x+1) + (x+2) + (x+3) + (x+4)$, shows the sum of five unknown numbers where the second is $1°$ more than the first, the third is $1°$ more than the second, and so on. The expression $5x + 10$ shows the sum of five times an unknown number with 10.

8. Let $x°$ be the measurement of an angle. The ratio of the measurement of the complement of the angle to the measurement of the supplement of the angle is $\frac{1}{3}$. Use a tape diagram to find the measurement of this angle.

$(90 - x) : (180 - x) = 1 : 3$



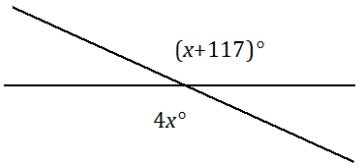
4 *units* $= 180$

1 *unit* $= 45$

3 *units* $= 135$

The measurement of the angle that satisfies these criteria is $45°$.

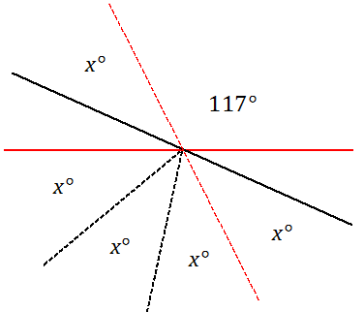
9. Two lines meet at a point. Set up and solve an equation to find the value of x. Find the measurement of one of the vertical angles.



A solution can include a modified diagram (as shown) and the supporting algebra work:

$$\begin{aligned} 3x &= 117 \qquad \text{vert. } \angle s \\ \left(\frac{1}{3}\right)3x &= \left(\frac{1}{3}\right)117 \\ x &= 39 \end{aligned}$$

Each vertical angle: $4(39°) = 156°$.



Solutions may also include the full equation and solution:

$$\begin{aligned} 4x &= x + 117 \\ 4x - x &= x - x + 117 \\ 3x &= 117 \\ \left(\frac{1}{3}\right)3x &= \left(\frac{1}{3}\right)117 \\ x &= 39 \end{aligned}$$

10. The difference between three times the measurement of the complement of an angle and the measurement of the supplement of that angle is $20°$. What is the measurement of the angle?

$$\begin{aligned} 3(90 - x) - (180 - x) &= 20 \\ 270 - 3x - 180 + x &= 20 \\ 90 - 2x &= 20 \\ 90 - 90 - 2x &= 20 - 90 \\ -2x &= -70 \\ \left(-\frac{1}{2}\right)(-2x) &= \left(-\frac{1}{2}\right)(-70) \\ x &= 35 \end{aligned}$$

The measurement of the angle is $35°$.