



## Lesson 3: Solving for Unknown Angles Using Equations

### Student Outcomes

- Students solve for unknown angles in word problems and in diagrams involving all learned angle facts.

### Classwork

#### Opening Exercise (5 minutes)

**Opening Exercise**

Two lines meet at the common vertex of two rays; the measurement of  $\angle COF = 134^\circ$ . Set up and solve an equation to find the value of  $x$  and  $y$ . Are your answers reasonable? How do you know?

$$14 + x + 90 = 134 \quad \angle s \text{ add}$$

$$x + 104 = 134$$

$$x + 104 - 104 = 134 - 104$$

$$x = 30$$
  

$$y + 134 = 180 \quad \angle s \text{ on a line}$$

$$y + 134 - 134 = 180 - 134$$

$$y = 46$$

*The answers are reasonable because the angle marked  $y^\circ$  appears to be approximately half the measurement of a right angle, and the angle marked  $x^\circ$  appears to be approximately double in measurement of  $\angle AOC$ .*

In the following examples and exercises, students set up and solve an equation for the unknown angle based on the relevant angle relationships in the diagram. Encourage students to note the appropriate angle fact abbreviation for any step that depends on an angle relationship.

#### Example 1 (4 minutes)

**Example 1**

Set up and solve an equation to find the value of  $x$ .

$$x + 90 + 123 = 360 \quad \angle s \text{ at a point}$$

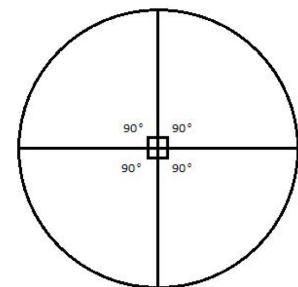
$$x + 213 = 360$$

$$x + 213 - 213 = 360 - 213$$

$$x = 147$$

#### Scaffolding:

Remind students that a full rotation or turn through a circle is  $360^\circ$ .



A circular protractor may help to demonstrate this.

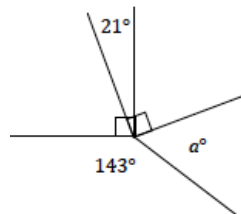
Exercise 1 (4 minutes)

Exercise 1

Five rays meet at a common vertex. In a complete sentence, describe the relevant angle relationships in the diagram. Set up and solve an equation to find the value of  $a$ .

The sum of angles at a point is  $360^\circ$ .

$$\begin{aligned} 90 + 90 - 21 + a + 143 &= 360 && \angle s \text{ at a point} \\ 302 + a &= 360 \\ 302 - 302 + a &= 360 - 302 \\ a &= 58 \end{aligned}$$



Example 2 (4 minutes)

Example 2

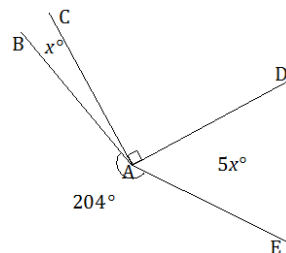
Four rays meet at a common vertex. In a complete sentence, describe the relevant angle relationships in the diagram. Set up and solve an equation to find the value of  $x$ . Find the measurements of angles  $\angle BAC$  and  $\angle DAE$ .

The sum of the degree measurements of  $\angle BAC$ ,  $\angle CAD$ ,  $\angle DAE$ , and the arc that measures  $204^\circ$  is  $360^\circ$ .

$$\begin{aligned} x + 90 + 5x + 204 &= 360 && \angle s \text{ at a point} \\ 6x + 294 &= 360 \\ 6x + 294 - 294 &= 360 - 294 \\ 6x &= 66 \\ \left(\frac{1}{6}\right) 6x &= \left(\frac{1}{6}\right) 66 \\ x &= 11 \end{aligned}$$

The measurement of  $\angle BAC = 11^\circ$

The measurement of  $\angle DAE = 5(11^\circ) = 55^\circ$



Exercise 2 (4 minutes)

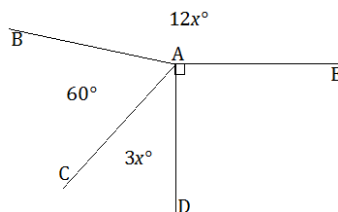
Exercise 2

Four rays meet at a common vertex. In a complete sentence, describe the relevant angle relationships in the diagram. Set up and solve an equation to find the value of  $x$ . Find the measurement of  $\angle CAD$ .

$\angle BAC$ ,  $\angle CAD$ ,  $\angle DAE$ , and  $\angle EAD$  are angles at a point and sum to  $360^\circ$ .

$$\begin{aligned} 3x + 60 + 12x + 90 &= 360 && \angle s \text{ at a point} \\ 15x + 150 &= 360 \\ 15x + 150 - 150 &= 360 - 150 \\ 15x &= 210 \\ \left(\frac{1}{15}\right) 15x &= \left(\frac{1}{15}\right) 210 \\ x &= 14 \end{aligned}$$

The measurement of  $\angle CAD = 3(14^\circ) = 42^\circ$



**Example 3 (4 minutes)**

**Example 3**

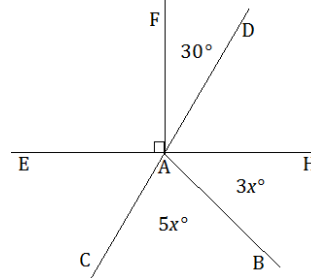
Two lines meet at the common vertex of two rays. In a complete sentence, describe the relevant angle relationships in the diagram. Set up and solve an equation to find the value of  $x$ . Find the measurements of angles  $\angle BAC$  and  $\angle BAH$ .

$\angle DAE$  is formed by adjacent angles  $\angle EAF$  and  $\angle FAD$ ; the measurement of  $\angle DAE$  is equal to the sum of the measurements of the adjacent angles. This is also true for measurement of  $\angle CAH$ , formed by adjacent angles  $\angle CAB$  and  $\angle BAH$ .  $\angle CAH$  is vertically opposite from and equal in measurement to  $\angle DAE$ .

$$\begin{aligned} 90^\circ + 30^\circ &= 120^\circ && \angle DAE, \angle s \text{ add} \\ 5x^\circ + 3x^\circ &= 8x^\circ && \angle CAH, \angle s \text{ add} \\ 8x &= 120 && \text{vert. } \angle s \\ 8x &= 120 && \\ \left(\frac{1}{8}\right) 8x &= \left(\frac{1}{8}\right) 120 && \\ x &= 15 && \end{aligned}$$

The measurement of  $\angle BAC = 5(15^\circ) = 75^\circ$

The measurement of  $\angle BAH = 3(15^\circ) = 45^\circ$



**Exercise 3 (4 minutes)**

**Exercise 3**

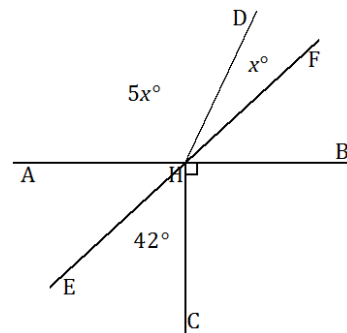
Two lines meet at the common vertex of two rays. In a complete sentence, describe the relevant angle relationships in the diagram. Set up and solve an equation to find the value of  $x$ . Find the measurements of angles  $\angle DHF$  and  $\angle AHD$ .

which The measurement of  $\angle AHF$  formed by adjacent angles  $\angle AHD$  and  $\angle DHF$  is equal to the sum of the measurements of the adjacent angles. This is also true for measurement of  $\angle EHB$ , which is formed by adjacent angles  $\angle EHC$  and  $\angle CHB$ .  $\angle AHF$  is vertically opposite from and equal to  $\angle EHB$ .

$$\begin{aligned} 5x^\circ + x^\circ &= 6x^\circ && \angle AHF, \angle s \text{ add} \\ 42^\circ + 90^\circ &= 132^\circ && \angle EHB, \angle s \text{ add} \\ 6x &= 132 && \text{vert. } \angle s \\ 6x &= 132 && \\ \left(\frac{1}{6}\right) 6x &= \left(\frac{1}{6}\right) 132 && \\ x &= 22 && \end{aligned}$$

The measurement of  $\angle DHF = 22^\circ$

The measurement of  $\angle AHF = 5(22^\circ) = 110^\circ$

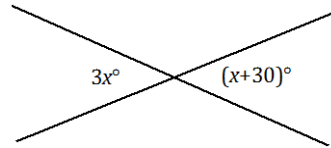


The following examples are designed to highlight Mathematical Practice 7 by helping students to see the connection between an angle diagram and the equation used to model it. Solving equations with variables on both sides is a topic Grade 8 teachers may choose to show that solution method if they so choose.

**Example 4 (6 minutes)**

**Example 4**

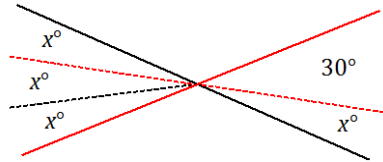
Two lines meet at a point. Set up and solve an equation to find the value of  $x$ . Find the measurement of one of the vertical angles.



Students use information in the figure and a protractor to solve for  $x$ .

- i) Students will measure a  $30^\circ$  angle as shown; the remaining portion of the angle must be  $x^\circ$  ( $\angle s$  add).
- ii) Students can use their protractor to find the measurement of  $x$  and use this measurement to partition the other angle in the vertical pair.

As a check, students should substitute the measured  $x$  value into each expression and evaluate; each angle of the vertical pair should equal the other. Students can also use their protractor to measure each angle of the vertical angle pair.



MP.7

With a modified figure, students can write an algebraic equation that they have the skills to solve:

$$2x = 30 \quad \text{vert. } \angle s$$

$$\left(\frac{1}{2}\right)2x = \left(\frac{1}{2}\right)30$$

$$x = 15$$

*Measurement of each angle in the vertical pair:  $3(15^\circ) = 45^\circ$*

Extension: The algebra steps above are particularly helpful as a stepping-stone in demonstrating how to solve the equation that takes care of the problem in one shot:

$$3x = x + 30 \quad \text{vert. } \angle s$$

$$3x - x = x - x + 30$$

$$2x = 30$$

$$\left(\frac{1}{2}\right)2x = \left(\frac{1}{2}\right)30$$

$$x = 15$$

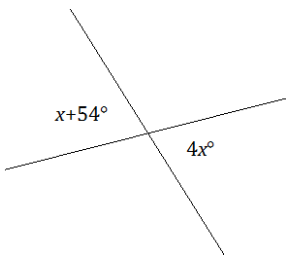
*Measurement of each angle in the vertical pair:  $3(15^\circ) = 45^\circ$*

Students understand the first line of this solution because of their knowledge of vertical angles. In fact, the only line they are not familiar with is the second line of the solution, which is a skill that they learn in Grade 8. Showing students this solution is simply a preview.

**Exercise 4 (4 minutes)**

**Exercise 4**

Set up and solve an equation to find the value of  $x$ . Find the measurement of one of the vertical angles.



Students use information in the figure and a protractor to solve for  $x$ .

- i) Students will measure a  $54^\circ$  angle as shown; the remaining portion of the angle must be  $x$  ( $\angle$ s add).
- ii) Students can use their protractor to find the measurement of  $x$  and use this measurement to partition the other angle in the vertical pair.

Students should perform a check as in Example 4 before solving an equation that matches the modified figure.

$$54 = 3x \quad \text{vert. } \angle\text{s}$$

$$\left(\frac{1}{3}\right) 3x = \left(\frac{1}{3}\right) 54$$

$$x = 18$$

*Measurement of each vertical angle:  $4(18^\circ) = 72^\circ$*

**Extension:**

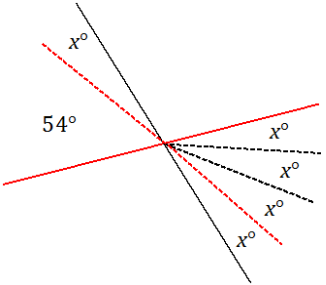
$$x + 54 = 4x \quad \text{vert. } \angle\text{s}$$

$$x - x + 54 = 4x - x$$

$$54 = 3x$$

$$\left(\frac{1}{3}\right) 54 = \left(\frac{1}{3}\right) 3x$$

$$x = 18$$



**Closing (1 minute)**

- In every unknown angle problem, it is important to identify the angle relationship(s) correctly in order to set up an equation that will yield the unknown value.
- Check your answer by substituting and/or measuring to be sure it is correct.

**Exit Ticket (5 minutes)**

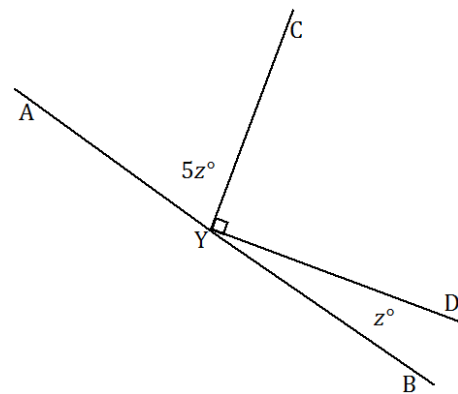
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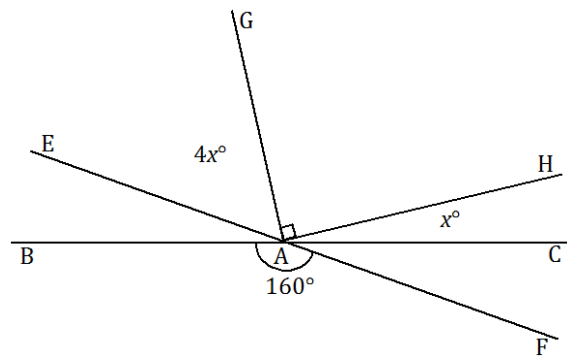
## Lesson 3: Solving for Unknown Angles Using Equations

### Exit Ticket

- Two rays have a common vertex on a line. Set up and solve an equation to find the value of  $z$ . Find the measurements of angles  $\angle AYC$  and  $\angle DYB$ .



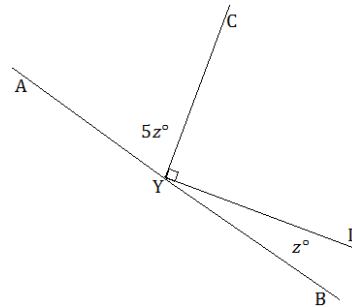
- Two lines meet at the common vertex of two rays. Set up and solve an equation to find the value of  $x$ . Find the measurements of angles  $\angle CAH$  and  $\angle EAG$ .



Exit Ticket Sample Solutions

1. Two rays have a common vertex on a line. Set up and solve an equation to find the value of  $z$ . Find the measurements of angles  $\angle AYC$  and  $\angle DYB$ .

$$\begin{aligned} 5z + 90 + z &= 180 && \angle s \text{ on a line} \\ 6z + 90 &= 180 \\ 6z + 90 - 90 &= 180 - 90 \\ 6z &= 90 \\ \left(\frac{1}{6}\right)6z &= \left(\frac{1}{6}\right)90 \\ z &= 15 \end{aligned}$$



The measurement of  $\angle AYC = 5(15^\circ) = 75^\circ$

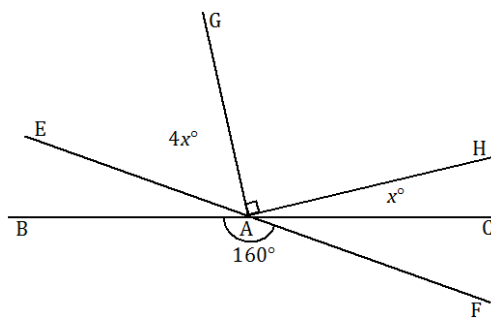
The measurement of  $\angle DYB = 15^\circ$

Scaffolded solutions:

- Use equation above.
- The angle marked  $z^\circ$ , the right angle, and the angle with measurement  $5z^\circ$  are angles on a line. Their measurements sum to  $180^\circ$ .
- The answers seem reasonable because the measurement of  $\angle AYC$  is  $75^\circ$  once  $15^\circ$  is substituted, which is slightly smaller than a right angle, and the measurement of  $\angle DYB$  is  $15^\circ$ , which is an acute angle.

2. Two lines meet at the common vertex of two rays. Set up and solve an equation to find the value of  $x$ . Find the measurements of angles  $\angle CAH$  and  $\angle EAG$ .

$$\begin{aligned} 4x + 90 + x &= 160 && \text{vert. } \angle s \\ 5x + 90 &= 160 \\ 5x + 90 - 90 &= 160 - 90 \\ 5x &= 70 \\ \left(\frac{1}{5}\right)5x &= \left(\frac{1}{5}\right)70 \\ x &= 14 \end{aligned}$$



The measurement of  $\angle CAH = 14^\circ$

The measurement of  $\angle EAG = 4(14^\circ) = 56^\circ$

Scaffolding:

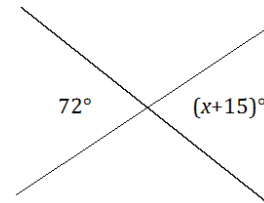
Students struggling to organize their solution may benefit from prompts such as the following: Write an equation to model this situation. Explain how your equation describes the situation. Solve and interpret the solution. Is it reasonable?

**Problem Set Sample Solutions**

Set up and solve an equation for the unknown angle based on the relevant angle relationships in the diagram. Add labels to diagrams as needed to facilitate their solutions. List the appropriate angle fact abbreviation for any step that depends on an angle relationship.

1. Two lines meet at a point. Set up and solve an equation to find the value of  $x$ .

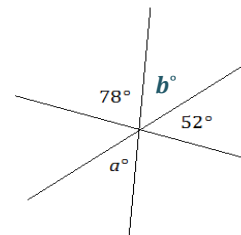
$$\begin{aligned} x + 15 &= 72 && \text{vert. } \angle\text{s} \\ x &= 57 \end{aligned}$$



2. Three lines meet at a point. Set up and solve an equation to find the value of  $a$ . Is your answer reasonable? Explain how you know.

Let  $b = a$ .

$$\begin{aligned} 78 + b + 52 &= 180 && \text{vert. } \angle\text{s} \\ b + 130 &= 180 && \angle\text{s on a line} \\ b + 130 - 130 &= 180 - 130 \\ b &= 50 \end{aligned}$$



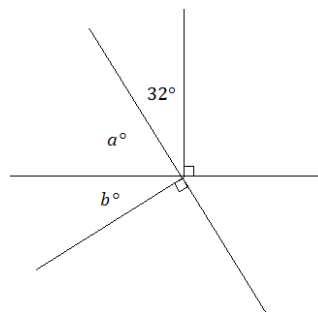
Since  $b = a$ ,  $a = 50$ .

The answers seem reasonable since they are similar in magnitude to the  $52^\circ$  angle.

3. Two lines meet at the common vertex of two rays. Set up and solve an equation to find the values of  $a$  and  $b$ .

$$\begin{aligned} a + 32 + 90 &= 180 && \angle\text{s on a line} \\ a + 122 &= 180 \\ a + 122 - 122 & && \\ = 180 - 122 & && \\ a &= 58 \end{aligned}$$

$$\begin{aligned} a + b + 90 &= 180 && \angle\text{s on a line} \\ 58 + b + 90 &= 180 \\ b + 148 &= 180 \\ b + 148 - 148 &= 180 - 148 \\ b &= 32 \end{aligned}$$



**Scaffolding:**  
Students struggling to organize their solution may benefit from prompts such as the following:

- Write an equation to model this situation. Explain how your equation describes the situation. Solve and interpret the solution. Is it reasonable?

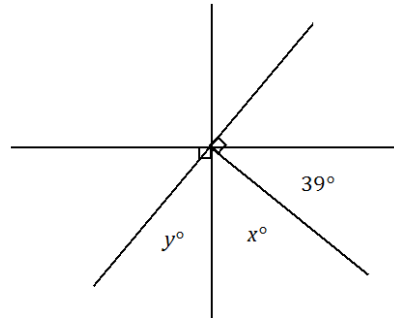
**Scaffolded solutions:**

- Use equation above.
- The angle marked  $a^\circ$ , the angle with measurement  $32^\circ$ , and the right angle are angles on a line. Their measurements sum to  $180^\circ$ .
- The answers seem reasonable because once the values of  $a$  and  $b$  are substituted, it appears that the two angles ( $a^\circ$  and  $b^\circ$ ) form a right angle. We know those two angles should form a right angle because the angle adjacent to it is a right angle.

4. Two lines meet at the common vertex of two rays. Set up and solve an equation to find the values of  $x$  and  $y$ .

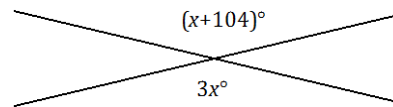
$$\begin{aligned} x + 39 + 90 &= 180 && \angle S \text{ on a line} \\ x + 129 &= 180 \\ x + 129 - 129 &= 180 - 129 \\ x &= 51 \end{aligned}$$

$$\begin{aligned} y + x + 90 &= 180 && \angle S \text{ on a line} \\ y + 51 + 90 &= 180 \\ y + 141 &= 180 \\ y + 141 - 141 &= 180 - 141 \\ y &= 39 \end{aligned}$$



5. Two lines meet a point. Find the measurement of a vertical angle. Is your answer reasonable? Explain how you know.

$$\begin{aligned} 2x &= 104 && \text{vert. } \angle s \\ \left(\frac{1}{2}\right)2x &= \left(\frac{1}{2}\right)104 \\ x &= 52 \end{aligned}$$

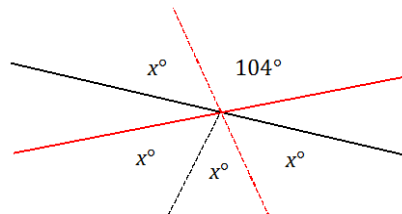


A solution can include a modified diagram, as shown and the supporting algebra work:

The answer seems reasonable because a rounded value of 50 would make the numeric value of each expression 150 and 154, which are reasonable close for a check.

Solutions may also include the full equation and solution:

$$\begin{aligned} 3x &= x + 104 && \text{vert. } \angle s \\ 3x - x &= x - x + 104 \\ 2x &= 104 \\ \left(\frac{1}{2}\right)2x &= \left(\frac{1}{2}\right)104 \\ x &= 52 \end{aligned}$$



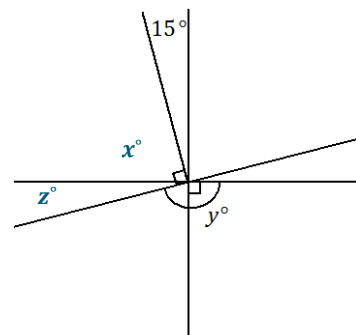
6. Three lines meet at the vertex of a ray. Set up and solve an equation to find the value of  $y$ .

Let  $x^\circ$  and  $y^\circ$  be the measurements of the indicated angles.

$$\begin{aligned} x + 15 &= 90 && \text{vert. } \angle s \\ x + 15 - 15 &= 90 - 15 \\ x &= 75 \end{aligned}$$

$$\begin{aligned} x + z &= 90 && \text{complementary } \angle s \\ 75 + z &= 90 \\ 75 - 75 + z &= 90 - 75 \\ z &= 15 \end{aligned}$$

$$\begin{aligned} z + y &= 180 && \angle S \text{ on a line} \\ 15 + y &= 180 \\ 15 - 15 + y &= 180 - 15 \\ y &= 165 \end{aligned}$$



MP.2  
&  
MP.7

7. Three angles are at a point. The second angle is  $20^\circ$  more than the first, and the third angle is  $20^\circ$  more than the second angle.

- a. Find the measurement of all three angles.

$$\begin{aligned} x + (x + 20) + (x + 20 + 20) &= 360 && \angle\text{s at a point} \\ 3x + 60 &= 360 \\ 3x + 60 - 60 &= 360 - 60 \\ 3x &= 300 \\ \left(\frac{1}{3}\right) 3x &= \left(\frac{1}{3}\right) 300 \\ x &= 100 \end{aligned}$$

Angle 1:  $100^\circ$

Angle 2:  $100^\circ + 20^\circ = 120^\circ$

Angle 3:  $100^\circ + 20^\circ + 20^\circ = 140^\circ$

- b. Compare the expressions you used for the three angles and their combined expression. Explain how they are equal and how they reveal different information about this situation.

*By the commutative and associative laws,  $x + (x + 20) + (x + 20 + 20)$  is equal to  $(x + x + x) + (20 + 20 + 20)$ , which is equal to  $3x + 60$ . The first expression,  $x + (x + 20) + (x + 20 + 20)$ , shows the sum of three unknown numbers, where the second is 20 more than the first, and the third is 20 more than the second. The expression  $3x + 60$  shows the sum of three times an unknown number with 60.*

8. Four adjacent angles are on a line. The measurements of the four angles are four consecutive even numbers. Determine the measurements of all four angles.

$$\begin{aligned} x + (x + 2) + (x + 4) + (x + 6) &= 180 && \angle\text{s on a line} \\ 4x + 12 &= 180 \\ 4x &= 168 \\ \left(\frac{1}{4}\right) 4x &= \left(\frac{1}{4}\right) 168 \\ x &= 42 \end{aligned}$$

The four angle measures are:  $42^\circ$ ,  $44^\circ$ ,  $46^\circ$ , and  $48^\circ$ .

*Scaffolding:*

Teachers may need to review the term “consecutive” for students to successfully complete Problem Set 7.

9. Three angles are at a point. The ratio of the measurement of the second angle to the measurement of the first angle is 4:3. The ratio of the measurement of the third angle to the measurement of the second angle is 5:4.

*Let the smallest measure of the three angles be  $3x^\circ$ . Then the measure of the second angle is  $4x^\circ$ , and the measure of the third angle is  $5x^\circ$ .*

$$\begin{aligned} 3x + 4x + 5x &= 360 && \angle\text{s at a point} \\ 12x &= 360 \\ \left(\frac{1}{12}\right) 12x &= \left(\frac{1}{12}\right) 360 \\ x &= 30 \end{aligned}$$

Angle 1 =  $3(30^\circ) = 90^\circ$

Angle 2 =  $4(30^\circ) = 120^\circ$

Angle 3 =  $5(30^\circ) = 150^\circ$

10. Solve for  $x$  and  $y$  in the following diagram.

$$2x + 18 + 90 = 180 \quad \angle s \text{ on a line}$$

$$2x + 108 = 180$$

$$2x + 108 - 108 = 180 - 108$$

$$2x = 72$$

$$\left(\frac{1}{2}\right) 2x = \left(\frac{1}{2}\right) 72$$

$$x = 36$$

$$2x = 3y$$

vert.  $\angle s$

$$2(36) = 3y$$

$$72 = 3y$$

$$\left(\frac{1}{3}\right) 72 = \left(\frac{1}{3}\right) 3y$$

$$y = 24$$

