Lesson 5: Chance Experiments with Outcomes that are Not Equally Likely

Student Outcomes

- Students calculate probabilities for chance experiments that do not have equally likely outcomes.

Classwork

It is important that this lesson address the common student misconception that outcomes are always equally likely. Take the opportunity to remind them that this is not always the case. To make this point you can use a spinner that has sections with areas that are not equal as an example. Just because there are four sections on the spinner does not mean that each of the four possible outcomes will have a probability of $\frac{1}{4}$. A weighted die is another example you can use to show a situation where the six possible outcomes would not be equally likely to occur and so it is not reasonable to think that each of the six outcomes has a probability of $\frac{1}{6}$.

In previous lessons, you learned that when the outcomes in a sample space are equally likely, the probability of an event is the number of outcomes in the event divided by the number of outcomes in the sample space. However, when the outcomes in the sample space are *not* equally likely, we need to take a different approach.

Example 1 (5 minutes)

This straightforward example lets students experience probabilities given in a table, outcomes with unequal probabilities, adding probabilities, and finding the probability of a complement (i.e., the probability that an event does *not* happen).

Ask students:

MP.2

- What might be an alternative way of answering part (e)? Add the probabilities for $0, 1, 2, 4$, and 5 bananas.
  - $0.1 + 0.1 + 0.1 + 0.2 + 0.3 = 0.8$

Example 1

When Jenna goes to the farmer's market she usually buys bananas. The numbers of bananas she might buy and their probabilities are shown in the table below.

Number of Bananas	0	1	2	3	4	5
Probability	0.1	0.1	0.1	0.2	0.2	0.3

a. **What is the probability that Jenna buys exactly 3 bananas?**

You can see from the table that the probability that Jenna buys exactly 3 bananas is 0.2.

b. **What is the probability that Jenna doesn't buy any bananas?**

The probability that Jenna buys 0 *bananas is* 0.1.

c. **What is the probability that Jenna buys more than 3 bananas?**

The probability that Jenna buys 4 *or* 5 *bananas is* $0.2 + 0.3 = 0.5$.

d. **What is the probability that Jenna buys at least 3 bananas?**

The probability that Jenna buys , 4, or 5 *bananas is* $0.2 + 0.2 + 0.3 = 0.7$.

e. **What is the probability that Jenna doesn't buy exactly 3 bananas?**

Remember that the probability that an event does not happen is

$1-(\textit{the probability that the event does happen})$.

So, the probability that Jenna does not buy exactly 3 *bananas is*

$1-(\textit{the probability that she does buy exactly } 3 \textit{ bananas})$

$1 - 0.2 = 0.8$.

Notice that the probabilities in the table add to 1. $(0.1 + 0.1 + 0.1 + 0.2 + 0.2 + 0.3 = 1)$ **This is always true; when we add up the probabilities of all the possible outcomes, the result is always** 1. **So, taking** 1 **and subtracting the probability of the event gives us the probability of something NOT occurring.**

Exercises 1–2 (6 minutes)

Decide if you want to let students use calculators on this exercise. Doing the exercise using only pencil and paper would provide useful practice with adding decimals. However, you might decide that the time saved by allowing calculators on this exercise will give you more time to devote to working with fractions in the examples and exercises that follow. Let students work independently and confirm their answers with a neighbor.

Exercises 1–2

Jenna's husband, Rick, is concerned about his diet. On any given day, he eats 0, 1, 2, 3, **or** 4 **servings of fruit and vegetables. The probabilities are given in the table below.**

Number of Servings of Fruit and Vegetables	0	1	2	3	4
Probability	0.08	0.13	0.28	0.39	0.12

1. **On a given day, find the probability that Rick eats:**

a. **Two servings of fruit and vegetables.**

0.28

b. **More than two servings of fruit and vegetables.**

$0.39 + 0.12 = 0.51$

c. **At least two servings of fruit and vegetables.**

$0.28 + 0.39 + 0.12 = 0.79$

2. **Find the probability that Rick does not eat exactly two servings of fruit and vegetables.**

$1 - 0.28 = 0.72$

Example 2 (8 minutes)

Here, the concepts in Example 1 are repeated, this time using fractions. This provides an opportunity to remind students how to add and subtract fractions using common denominators, and how to reduce fractions.

Example 2

Luis works in an office, and the phone rings occasionally. The possible numbers of phone calls he receives in an afternoon and their probabilities are given in the table below.

Number of Phone Calls	0	1	2	3	4
Probability	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{2}{9}$	$\frac{1}{3}$	$\frac{1}{9}$

a. **Find the probability that Luis receives 3 or 4 phone calls.**

The probability that Luis receives 3 or 4 phone calls is $\frac{1}{3} + \frac{1}{9} = \frac{3}{9} + \frac{1}{9} = \frac{4}{9}$.

b. **Find the probability that Luis receives fewer than 2 phone calls.**

The probability that Luis receives fewer than 2 phone calls is $\frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$.

c. **Find the probability that Luis receives 2 or fewer phone calls.**

The probability that Luis receives 2 or fewer phone calls is $\frac{2}{9} + \frac{1}{6} + \frac{1}{6} = \frac{4}{18} + \frac{3}{18} + \frac{3}{18} = \frac{10}{18} = \frac{5}{9}$.

d. **Find the probability that Luis does not receive 4 phone calls.**

The probability that Luis does not receive 4 phone calls is $1 - \frac{1}{9} = \frac{9}{9} - \frac{1}{9} = \frac{8}{9}$.

If there is time available, ask students:

- How would you calculate the probability that Luis receives at least one call? What might be a quicker way of doing this?
  - *The straightforward answer is to add the probabilities for* $1, 2, 3,$ *and* 4*:*

    $$\frac{1}{6} + \frac{2}{9} + \frac{1}{3} + \frac{1}{9} = \frac{3}{18} + \frac{4}{18} + \frac{2}{18} = \frac{15}{18} = \frac{5}{6}$$

    The quicker way is to say that the probability he gets at least one call is the probability that he doesn't get zero calls, and so the required probability is $1 - \frac{1}{6} = \frac{5}{6}$.

Exercises 3–6 (8 minutes)

The main challenge this question presents is adding and subtracting fractions, so it is important that students do not use calculators on this exercise or on the other questions that involve fractions. Students can either work independently or with a partner. Teachers should use discretion on how important it is to complete this exercise. This exercise provides students an opportunity to review their work with fractions.

Exercises 3–6

When Jenna goes to the farmer's market, she also usually buys some broccoli. The possible number of heads of broccoli that she buys and the probabilities are given in the table below.

Number of Heads of Broccoli	0	1	2	3	4
Probability	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{5}{12}$	$\frac{1}{4}$	$\frac{1}{12}$

Find the probability that Jenna:

3. **buys exactly 3 heads of broccoli.** $\frac{1}{4}$

4. **does not buy exactly 3 heads of broccoli.** $1-\frac{1}{4}=\frac{3}{4}$

5. **buys more than 1 head of broccoli.** $\frac{5}{12}+\frac{1}{4}+\frac{1}{12}=\frac{3}{4}$

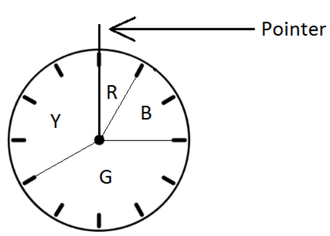
6. **buys at least 3 heads of broccoli.** $\frac{1}{4}+\frac{1}{12}=\frac{1}{3}$

Exercises 7–10 (8 minutes)

Here, students calculate the probabilities of a given scenario. Because this is the first time this is covered in this class, some students might need assistance.

Exercises 7–10

The diagram below shows a spinner designed like the face of a clock. The sectors of the spinner are colored red (R), blue (B), green (G), and yellow (Y).



Spin the pointer, and award the player a prize according to the color on which the pointer stops.

7. Writing your answers as fractions in lowest terms, find the probability that the pointer stops on

a. red: $\frac{1}{12}$

b. blue: $\frac{2}{12} = \frac{1}{6}$

c. green: $\frac{5}{12}$

d. yellow: $\frac{4}{12} = \frac{1}{3}$

8. Complete the table of probabilities below.

Color	Red	Blue	Green	Yellow
Probability	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{5}{12}$	$\frac{1}{3}$

MP.4

9. Find the probability that the pointer stops in either the blue region or the green region.

$\frac{1}{6} + \frac{5}{12} = \frac{7}{12}$

10. Find the probability that the pointer does not stop in the green region.

$1 - \frac{5}{12} = \frac{7}{12}$

Closing

Discuss with students the Lesson Summary.

Lesson Summary

In a probability experiment where the outcomes are not known to be equally likely, the formula for the probability of an event does not necessarily apply:

$$P(event) = \frac{Number\ of\ outcomes\ in\ the\ event}{Number\ of\ outcomes\ in\ the\ sample\ space}.$$

For example:

- **To find the probability that the score is greater than 3, add the probabilities of all the scores that are greater than 3.**
- **To find the probability of not getting a score of 3, calculate $1 - (the\ probability\ of\ getting\ a\ 3)$.**

Exit Ticket (5 minutes)

Name ____ Date____________________

Lesson 5: Chance Experiments with Outcomes that are Not Equally Likely

Exit Ticket

Carol is sitting on the bus on the way home from school and is thinking about the fact that she has three homework assignments to do tonight. The table below shows her estimated probabilities of completing $0, 1, 2$, or all 3 of the assignments.

Number of Homework Assignments Completed	0	1	2	3
Probability	$\frac{1}{6}$	$\frac{2}{9}$	$\frac{5}{18}$	$\frac{1}{3}$

1. Writing your answers as fractions in lowest terms, find the probability that Carol completes
   a. exactly one assignment.

   b. more than one assignment.

   c. at least one assignment.

2. Find the probability that the number of homework assignments Carol completes is not exactly 2.

3. Carol has a bag of colored chips. She has 3 red chips, 10 blue chips, and 7 green chips in the bag. Estimate the probability (as a fraction or decimal) of Carol reaching into her bag and pulling out a green chip.

Exit Ticket Sample Solutions

Carol is sitting on the bus on the way home from school and is thinking about the fact that she has three homework assignments to do tonight. The table below shows her estimated probabilities of completing 0, 1, 2, or all 3 of the assignments.

Number of Homework Assignments Completed	0	1	2	3
Probability	$\frac{1}{6}$	$\frac{2}{9}$	$\frac{5}{18}$	$\frac{1}{3}$

1. Writing your answers as fractions in lowest terms, find the probability that Carol completes

   a. exactly one assignment.

   $\frac{2}{9}$

   b. more than one assignment.

   $\frac{5}{18}+\frac{1}{3}=\frac{5}{18}+\frac{6}{18}=\frac{11}{18}$

   c. at least one assignment.

   $\frac{2}{9}+\frac{5}{18}+\frac{1}{3}=\frac{4}{18}+\frac{5}{18}+\frac{6}{18}=\frac{15}{18}=\frac{5}{6}$

2. Find the probability that the number of homework assignments Carol completes is not exactly 2.

   $1-\frac{5}{18}=\frac{13}{18}$

3. Carol has a bag of colored chips. She has 3 red chips, 10 blue chips, and 7 green chips in the bag. Estimate the probability (as a fraction or decimal) of Carol reaching into her bag and pulling out a green chip.

   An estimate of the probability would be 7 out of 20, or $\frac{7}{20}$, or 0.35.

Problem Set Sample Solutions

1. The "Gator Girls" are a soccer team. The possible number of goals the Gator Girls will score in a game and their probabilities are shown in the table below.

Number of Goals	0	1	2	3	4
Probability	0.22	0.31	0.33	0.11	0.03

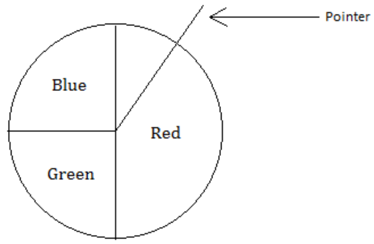
   Find the probability that the Gator Girls

   a. score more than two goals: $0.11 + 0.03 = 0.14$

   b. score at least two goals: $0.33 + 0.11 + 0.03 = 0.47$

   c. do not score exactly 3 goals: $1-0.11 = 0.89$

2. The diagram below shows a spinner. The pointer is spun, and the player is awarded a prize according to the color on which the pointer stops.



a. What is the probability that the pointer stops in the red region?

$\frac{1}{2}$

b. Complete the table below showing the probabilities of the three possible results.

Color	Red	Green	Blue
Probability	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$

c. Find the probability that the pointer stops on green or blue.

$\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$

d. Find the probability that the pointer does not stop on green.

$1-\frac{1}{4}=\frac{3}{4}$

3. Wayne asked every student in his class how many siblings (brothers and sisters) they had. Survey results are shown in the table below. (Wayne included himself in the results.)

Number of Siblings	0	1	2	3	4
Number of Students	4	5	14	6	3

(Note: The table tells us that 4 students had no siblings, 5 students had one sibling, 14 students had two siblings, and so on.)

a. How many students are in Wayne's class?

$4 + 5 + 14 + 6 + 3 = 32$

b. What is the probability that a randomly selected student does not have any siblings? Write your answer as a fraction in lowest terms.

$\frac{4}{32}=\frac{1}{8}$

c. The table below shows the possible number of siblings and the probabilities of each number. Complete the table by writing the probabilities as fractions in lowest terms.

Number of Siblings	0	1	2	3	4
Probability	$\frac{1}{8}$	$\frac{5}{32}$	$\frac{7}{16}$	$\frac{3}{16}$	$\frac{3}{32}$

d. Writing your answers as fractions in lowest terms, find the probability that the student:

i. has fewer than two siblings.

$$\frac{1}{8}+\frac{5}{32}=\frac{9}{32}$$

ii. has two or fewer siblings.

$$\frac{1}{8}+\frac{5}{32}+\frac{7}{16}=\frac{23}{32}$$

iii. does not have exactly one sibling.

$$1-\frac{5}{32}=\frac{27}{32}$$