Lesson 7: Markup and Markdown Problems

Student Outcomes

- Students understand the terms *original price, selling price, markup, markdown, markup rate,* and *markdown rate*.
- Students identify the original price as the whole and use their knowledge of percent and proportional relationships to solve multistep markup and markdown problems.
- Students understand equations for markup and markdown problems and use them to solve markup and markdown problems.

Lesson Notes

- In this lesson, students use algebraic equations to solve multi-step word problems involving markups and markdowns. This lesson extends the mathematical practices and terminology students were exposed to in Module 1, Lesson 14.
- New finance terms such as *retail price, consumer, cost price,* and *wholesale price* are introduced. Although students are not required to memorize these terms, they do provide a solid foundational knowledge for financial literacy. To make the lesson more meaningful to students, use examples from an actual newspaper circular.
- Students have had significant exposure to creating tables and graphs to determine proportional relationships in Module 3. Before the lesson, the teacher may need to review past student performance data to target students who might potentially struggle with discovering proportional relationships using percent problems in Exercise 4.

Definitions:

- A **markup** is the amount of increase in a price.
- A **markdown** is the amount of decrease in a price.
- The **original price** is the starting price. It is sometimes called the cost or wholesale price.
- The **selling price** is the original price plus the markup or minus the markdown.
- The **markup rate** is the percent increase in the price, and the markdown rate (discount rate) is the percent decrease in the price.
- Most markup problems can be solved by the equation: $Selling\ Price\ =\ (1+m)(Whole)$, where m is the markup rate, and the whole is the original price.
- Most markdown problems can be solved by the equation: $Selling\ Price\ =\ (1-m)(Whole)$, where m is the markdown rate, and the whole is the original price.

Classwork

Opening (3 minutes)

Pose the question to the class. Students, who have been placed in groups, discuss possible answers. Teacher asks a few students to share out.

- A brand of sneakers costs $\$29.00$ to manufacture in Omaha, Nebraska. The shoes are then shipped to shoe stores across the country. When you see them on the shelves, the price is $\$69.99$. How do you think the price you pay for the sneakers is determined? Use percent to describe the markup. Explain your reasoning.
  - *The store makes up a new price so they can make money. The store has to buy the sneakers and pay for any transportation costs to get the sneakers to the store.*
  - *The store marks up the price to earn a profit because they had to buy the shoes from the company.*
  - *Markup is the amount of increase in a price from the original price.*

Close the discussion by explaining how the price of an item sold in a store is determined. For example, in order for the manufacturer to make a profit, it has to pay for the cost to make the item. This is the first markup. Then, a store purchases the item at a *cost price* from the manufacturer. The store then increases the price of the item by a percent called the *markup rate* before it is sold to the store's customers. Stores do this in order to earn a *profit*.

Example 1 (5 minutes): A Video Game Markup

MP.6

Students construct an algebraic equation based on a word problem. They express the markup rate of 40% on a video game that costs $\$30.00$ as $1.40(30)$ to show that a markup means a percent increase. Students identify the quantity that corresponds with 100% (the whole).

Scaffolding:

- Use sentence strips to create a word wall for student reference throughout the lesson to avoid confusion over financial terms.

  Some words can be written on the same sentence strip to show they are synonyms, such as *discount price* and *sales price*, and *cost price* and *wholesale price*.

Example 1

Games Galore Super Store buys the latest video game at a wholesale price of $\$30.00$. The markup rate at Game's Galore Super Store is 40%. You use your allowance to purchase the game at the store. How much will you pay, not including tax?

a. **Write an equation to find the price of the game at Games Galore Super Store. Explain your equation.**

Let P represent the price of the video game.

$\textit{Quantity} = \textit{Percent} \times \textit{Whole}$

$P = (100\% + 40\%)(30)$

b. **Solve the equation from part (a).**

$P = (100\% + 40\%)(30)$

$P = (1.40)(30)$

$P = 42$ *I would pay $\$42$ if I bought it from Games Galore Super Store.*

c. **What was the total markup of the video game? Explain.**

The markup was $\$12$ because $\$42 - \$30 = \$12$.

d. **You and a friend are discussing markup rate. He says that an easier way to find the total markup is by multiplying the wholesale price of $\$30$ by 40%. Do you agree with him? Why or why not?**

Yes, I agree with him because $(0.40)(30) = 12$. The markup rate is a percent of the wholesale price. Therefore, it makes sense to multiply them together because $Quantity = Percent \times Whole$.

- Which quantity is the "whole" quantity in this problem?
  - *The wholesale price is the whole quantity.*
- How do 140% and 1.4 correspond in this situation?
  - *The markup price of the video game is 140% times the wholesale price. 140% and 1.4 are equivalent forms. In order to find the markup price, convert the percent to a decimal or fraction and multiply it by the whole.*
- What does a "markup" mean?
  - *A markup is the amount of increase in a price.*

Example 2 (7 minutes): Black Friday

Students discuss the greatest American shopping day of the year, Black Friday—the day after Thanksgiving. The teacher could share the history of Black Friday to engage students in the lesson by reading the article at http://www.marketplace.org/topics/life/commentary/history-black-friday. Students make the connection that markdown is a percent decrease.

MP.7 Students realize that the distributive property allows them to arrive at an answer in one step. They learn that in order to apply an additional discount, a new whole must be found first.

Scaffolding:

- Provide newspaper circulars from Black Friday sales, or print one from the Internet to access prior knowledge of discounts for all learners.
- Choose an item from the circular in lieu of the one provided in Example 1.

- Does it matter in what order we take the discount? Why or why not? Allow students time to conjecture in small groups or with elbow partners before problem solving. Monitor student conversations, providing clarification as needed.
  - *I think the order does matter because applying the first discount will lower the price. Then, you would multiply the second discount to the new lower price.*
  - *I do not think order matters because both discounts will be multiplied to the original price anyway and multiplication is commutative. For example, $2 \times 3 \times 4$ is the same as $3 \times 4 \times 2$.*

Example 2: Black Friday

A $\$300$ mountain bike is discounted by 30% and then discounted an additional 10% for shoppers who arrive before 5:00 a.m.

a. **Find the sales price of the bicycle.**

Find the price with 30% discount.

Let D represent the discount price of the bicycle with the 30% discount rate.

$$Quantity = Percent \times Whole$$
$$D = (100\% - 30\%)(300)$$
$$D = (0.70)(300)$$
$$D = 210$$

$\$210$ is the discount price of the bicycle with the 30% discount rate.

- Which quantity is the new whole?
  - *The discounted price of* 30% *off, which is* $\$210$.

Find the price with the additional $\mathbf{10\%}$ ***discount.***

Let $\boldsymbol{A}$ ***represent the discount price of the bicycle with the additional*** $\mathbf{10\%}$ ***discount.***

$$\begin{aligned} \boldsymbol{A} &= \mathbf{(100\% - 10\%)(210)} \\ &= \mathbf{(1 - 0.10)(210)} \\ &= \mathbf{(0.90)(210)} \\ &= \mathbf{189} \end{aligned}$$

$\mathbf{\$189}$ ***is the discount price of the bicycle with the additional*** $\mathbf{10\%}$ ***discount.***

b. In all, by how much has the bicycle been discounted in dollars? Explain.

$\mathbf{\$300 - \$189 = \$111}$. ***The bicycle has been discounted*** $\mathbf{\$111}$ ***because the original price was*** $\mathbf{\$300}$. ***With both discounts applied, the new price is*** $\mathbf{\$189}$.

c. After both discounts were taken, what was the total percent discount?

A final discount of $\mathbf{40\%}$ ***means that you would add*** $\mathbf{30\% + 10\%}$ ***and apply it to the same whole. This is not the case because the additional*** $\mathbf{10\%}$ ***discount is taken after the*** $\mathbf{30\%}$ ***discount has been applied, so you are only receiving that*** $\mathbf{10\%}$ ***discount on*** $\mathbf{70\%}$ ***of the original price. A*** $\mathbf{40\%}$ ***discount would make the final price*** $\mathbf{\$180}$ ***because*** $\mathbf{180 = (0.60)(300)}$.

However, the actual final discount as a percent is $\mathbf{37\%}$.

Let $\boldsymbol{P}$ ***be the percent the sales price is of the original price. Let*** $\boldsymbol{F}$ ***represent the actual final discount as a percent.***

$$\begin{aligned} \boldsymbol{Part} &= \boldsymbol{Percent \times Whole} \\ \mathbf{189} &= \boldsymbol{P \times Whole} \\ \mathbf{189} &= \boldsymbol{P} \times \mathbf{300} \\ \left(\frac{\mathbf{1}}{\mathbf{300}}\right)\mathbf{189} &= \boldsymbol{P} \times \mathbf{300}\left(\frac{\mathbf{1}}{\mathbf{300}}\right) \\ \mathbf{0.63} = \mathbf{63\%} &= \boldsymbol{P} \end{aligned}$$

$$\boldsymbol{F} = \mathbf{100\% - 63\% = 37\%}$$

- Teacher could also show students that a 30% discount means to multiply by 0.70, and an extra 10% means to multiply by 0.90. $(0.70)(0.90) = 0.63$, so it is the same as $100\% - 63\% = 37\%$ discount. This can help students perform the mathematics more efficiently.

d. Instead of purchasing the bike for $\$300$**, how much would you save if you bought it before 5:00 a.m.?**

You would save $\mathbf{\$111}$ ***if you bought the bike before 5:00 a.m. because*** $\mathbf{\$300 - \$189}$ ***is*** $\mathbf{\$111}$.

Exercises 1–3 (6 minutes)

Students complete the following exercises independently or in groups of two using $Quantity = Percent \times Whole$. Review the correct answers before moving to Example 3. The use of a calculator is recommended for the exercises.

Exercises 1–3

1. **Sasha went shopping and decided to purchase a set of bracelets for 25% off of the regular price. If Sasha buys the bracelets today, she will receive an additional 5%. Find the sales price of the set of bracelets with both discounts. How much money will Sasha save if she buys the bracelets today?**

   Let B be the sales price with both discounts in dollars.

   $B = (0.95)(0.75)(44) = 31.35$. ***The sales price of the set of bracelets with both discounts is $\$31.35$. Sasha will save $\$12.65$.***

2. **A golf store purchases a set of clubs at a wholesale price of $\$250$. Mr. Edmond learned that the clubs were marked up 200%. Is it possible to have a percent increase greater than 100%? What is the retail price of the clubs?**

   Yes, it is possible. Let C represent the retail price of the clubs in dollars.

$$\begin{aligned} C &= (100\% + 200\%)(250) \\ &= (1+2)(250) \\ &= (3)(250) \\ &= 750 \end{aligned}$$

   The retail price of the clubs is $\$750$.

3. **Is a percent increase of a set of golf clubs from $\$250$ to $\$750$ the same as a markup rate of 200%? Explain.**

   Yes, it is the same. In both cases, the percent increase and markup rate show by how much (in terms of percent) the new price is over the original price. The whole is $\$250$ and corresponds to 100%. $\frac{750}{250} = \frac{3}{1} \times 100\% = 300\%$. $\$750$ is 300% of $\$250$. $300\% - 100\% = 200\%$. From question 1, the markup is 200%. So, percent increase is the same as markup.

Example 3 (5 minutes): Working Backwards

Teacher refers to an item in the newspaper circular displayed to the class. Students find the markdown rate (discount rate) given an original price (regular price) and a sales price (discount price). Students find the total or final price, including sales tax.

Example 3: Working Backwards

A car that normally sells for $\$20,000$ is on sale for $\$16,000$. The sales tax is 7.5%.

- What is the "whole quantity" in this problem?
  - *The whole quantity is* $\$20,000$.

a. What percent of the original price of the car is the final price?

$$\begin{aligned}
Quantity &= Percent \times Whole \\
16,000 &= P(20,000) \\
16,000\left(\frac{1}{20,000}\right) &= P(20,000)\left(\frac{1}{20,000}\right) \\
0.8 &= P \\
0.8 &= \frac{80}{100} = 80\%
\end{aligned}$$

The final price is 80% of the original price.

b. Find the discount rate.

The discount rate is 20% because $100\% - 80\% = 20\%$.

c. By law, sales tax has to be applied to the discount price. However, would it be better for the consumer if the 7.5% sales tax were calculated before the 20% discount was applied? Why or why not?

Apply Sales Tax First

Apply the sales tax to the whole.

$(100\% + 7.5\%)(20,000)$

$(1 + 0.075)(20,000)$

$(1.075)(20,000)$

$\$21,500$ is the price of the car, including tax, before the discount.

Apply the discount to the new whole.

$(100\% - 20\%)(21,500)$

$(1 - 0.2)(21,500) = 17,200$

$\$17,200$ is the final price, including the discount.

Apply the Discount First

$(100\% + 7.5\%)(16,000)$

$(1 + 0.075)(16,000)$

$(1.075)(16,000)$

$\$17,200$ is the final price, including the tax.

Because both final prices are the same, it doesn't matter which is applied first. This is because multiplication is commutative. The discount rate and sales tax rate are both being applied to the whole, $\$20,000$.

d. Write an equation applying the commutative property to support your answer to part (c).

$$20,000(1.075)(0.8) = 20,000(0.8)(1.075)$$

Exercises 4–5 (9 minutes)

Students write a markup or markdown equation based on the context of the problem. They use algebraic equations of the form: $Quantity = (1 + m) \cdot Whole$ for markups, or $Quantity = (1 - m) \cdot Whole$ for markdowns. Students will use their equations to make a table and graph in order to interpret the unit rate (**7.RP.A.2**). Students may use a calculator for calculations, but their equations and steps should be shown for these exercises.

Scaffolding:

- Have visual learners refer to an anchor poster of proportional relationships to access prior knowledge. Poster should include items such as:
  - Word sentence
  - Equation
  - Graph of equation
  - Table of possible pairs of solutions
  - Meaning of $(1, r)$ and $(0,0)$ in context

Exercise 4

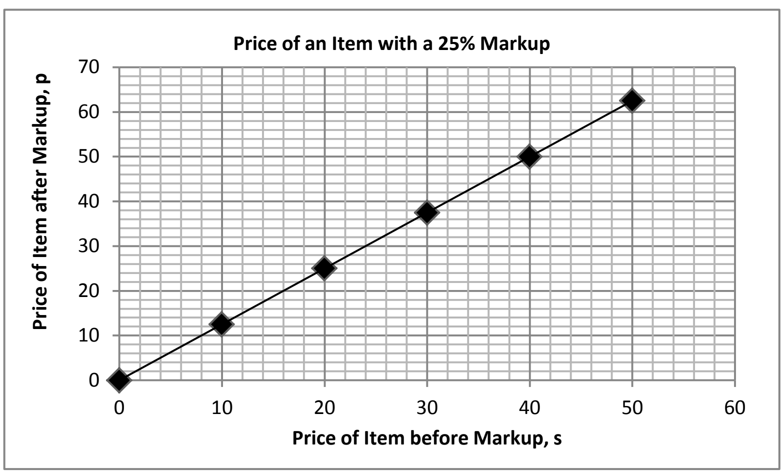
a. **Write an equation to determine the selling price, p, on an item that is originally priced s dollars after a markup of 25%.**

$\boldsymbol{p\ =\ 1.25s}$ ***or*** $\boldsymbol{p\ = (0.25 + 1)s}$

b. **Create a table (and label it) showing five possible pairs of solutions to the equation.**

Price of Item before Markup, s (in dollars)	Price of Item after Markup, p (in dollars)
$\mathbf{10}$	$\mathbf{12.50}$
$\mathbf{20}$	$\mathbf{25.00}$
$\mathbf{30}$	$\mathbf{37.50}$
$\mathbf{40}$	$\mathbf{50.00}$
$\mathbf{50}$	$\mathbf{62.50}$

c. **Create a graph (and label it) of the equation.**



d. **Interpret the points $(0, 0)$ and $(1, r)$.**

The point $\boldsymbol{(0, 0)}$ means that a $\boldsymbol{\$0}$ (free) item will cost $\boldsymbol{\$0}$ because the $\boldsymbol{25\%}$ markup is also $\boldsymbol{\$0}$. The point $\boldsymbol{(1, r)}$ is $\boldsymbol{(1, 1.25)}$. It means that a $\boldsymbol{\$1.00}$ item will cost $\boldsymbol{\$1.25}$ after it is marked up by $\boldsymbol{25\%}$; $\boldsymbol{r}$ is the unit rate.

Exercise 5

Use the following table to calculate the markup or markdown rate. Show your work. Is the relationship between the original price and selling price proportional or not? Explain.

Original Price, m (in dollars)	Selling Price, p (in dollars)
$\$1,750$	$\$1,400$
$\$1,500$	$\$1,200$
$\$1,250$	$\$1,000$
$\$1,000$	$\$800$
750	600

Because the selling price is less than the original price, use the equation: $\textit{Selling Price} = (1 - m) \times \textit{Whole}$.

$$1400 = (1 - m)(1750)$$
$$\frac{1400}{1750} = (1 - m)\frac{1750}{1750}$$
$$0.80 = 1 - m$$
$$0.20 = m$$

The markdown rate is 20%*. The relationship between the original price and selling price is proportional because the table shows the ratio* $\frac{p}{m} = \frac{0.80}{1}$ *for all possible pairs of solutions.*

Closing (3 minutes)

- How do you find the markup and markdown of an item?
  - *To find the markup of an item, you multiply the whole by* $(1 + m)$*, where* m *is the markup rate.*
  - *To find the markdown of an item, you multiply the whole by* $(1 - m)$*, where* m *is the markdown rate.*
- Discuss two ways to apply two discount rates to the price of an item when one discount follows the other.
  - *In order to apply two discounts, you must first multiply the original price (whole) by* 1 *minus the first discount rate to get the discount price (new whole). Then, you must multiply by* 1 *minus the second discount rate to the new whole to get the final price. For example, to find the final price of an item discounted by* 25%*, and then discounted by another* 10%*, you would first have to multiply by* 75% *to get a new whole. Then, you multiply the new whole by* 90% *to find the final price.*

Exit Ticket (7 minutes)

Name ____ Date ____________________

Lesson 7: Markup and Markdown Problems

Exit Ticket

1. A store that sells skis buys them from a manufacturer at a wholesale price of $\$57$. The store's markup rate is 50%.
   a. What price does the store charge its customers for the skis?

   b. What percent of the original price is the final price? Show your work.

   c. What is the percent increase from the original price to the final price?

Exit Ticket Sample Solutions

1. A store that sells skis buys them from a manufacturer at a wholesale price of $\$57$. The store's markup rate is 50%.
   a. What price does the store charge its customers for the skis?

      $57 \times (1 + 0.50) = 85.50$. *The store charges* $\$85.50$ *for the skis.*

   b. What percent of the original price is the final price? Show your work.

      $$\begin{aligned} \mathit{Quantity} &= \mathit{Percent} \times \mathit{Whole} \\ 85.50 &= P(57) \\ 85.50\left(\frac{1}{57}\right) &= P(57)\left(\frac{1}{57}\right) \\ 01.50 &= P \end{aligned}$$

      $1.50 = \frac{150}{100} = 150\%$. *The final price is* 150% *of the original price.*

   c. What is the percent increase from the original price to the final price?

      The percent increase is 50% *because* $150\% - 100\% = 50\%$.

Problem Set Sample Solutions

In the following problems, students solve markup problems by multiplying the whole by $(1+m)$, where m is the markup rate, and work backwards to find the whole by dividing the markup price by $(1+m)$. They also solve markdown problems by multiplying the whole by $(1-m)$, where m is the markdown rate, and work backwards to find the whole by dividing the markdown price by $(1-m)$. Students also solve percent problems learned so far in the module.

1. You have a coupon for an additional 25% off the price of any sale item at a store. The store has put a robotics kit on sale for 15% off the original price of $\$40$. What is the price of the robotics kit after both discounts?

   $(0.75)(0.85)(40) = 25.50$. *The price of the robotics kit after both discounts is* $\$25.50$.

2. A sign says that the price marked on all music equipment is 30% off the original price. You buy an electric guitar for the sale price of $\$315$.
   a. What is the original price?

      $\frac{315}{1-0.30} = \frac{315}{0.70} = 450$. *The original price is* $\$450$.

   b. How much money did you save off the original price of the guitar?

      $450 - 315 = 135$. *I saved* $\$135$ *off the original price of the guitar.*

   c. What percent of the original price is the sale price?

      $\frac{315}{450} = \frac{70}{100} = 70\%$

3. The cost of a New York Yankees baseball cap is $\$24.00$. The local sporting goods store sells it for $\$30.00$. Find the markup rate.

$30 = P(24)$

$P = \frac{30}{24} = 1.25 = (100\% + 25\%)$. *The markup rate is* 25%.

4. Write an equation to determine the selling price, p, on an item that is originally priced s dollars after a markdown of 15%.

$p = 0.85s$ *or* $p = (1 - 0.15)s$

a. Create a table (and label it) showing five possible pairs of solutions to the equation.

Price of Item before Markdown, s (in dollars)	Price of Item after Markdown, p (in dollars)
10	8.50
20	17.00
30	25.50
40	34.00
50	42.50

b. Create a graph (and label it) of the equation.

c. Interpret the points $(0, 0)$ and $(1, r)$.

The point $(0, 0)$ *means that a* $\$0$ *(free) item will cost* $\$0$ *because the* 15% *markdown is also* $\$0$. *The point* $(1, r)$ *is* $(1, 0.85)$, *which represents the unit rate. It means that a* $\$1.00$ *item will cost* $\$0.85$ *after it is marked down by* 15%.

5. At the amusement park, Laura paid $\$6.00$ for a small cotton candy. Her older brother works at the park, and he told her they mark up the cotton candy by 300%. Laura does not think that is mathematically possible. Is it possible, and if so, what is the price of the cotton candy before the markup?

Yes, it is possible. $\frac{6.00}{1+3} = \frac{6}{4} = 1.50$. *The price of the cotton candy before the markup is* $\$1.50$.

6. A store advertises that customers can take 25% off the original price and then take an extra 10% off. Is this 35% off? Explain.

No, because the 25% is taken first off the original price to get a new whole. Then, the extra 10% off is multiplied to the new whole. For example, $(1-0.25)(1-0.10)=0.675$ or $(0.75)(0.90)=0.675$. This is multiplied to the whole, which is the original price of the item. This is not the same as adding 25% and 10% to get 35%, then multiplying by $(1-0.35)$.

7. An item that costs $\$50$ is marked 20% off. Sales tax for the item is 8%. What is the final price, including tax?

   a. Solve the problem with the discount applied before the sales tax.

   $(1.08)(0.80)(50)=43.20$. *The final price is* $\$43.20$.

   b. Solve the problem with discount applied after the sales tax.

   $(0.80)(1.08)(50)=43.20$. *The final price is* $\$43.20$.

   c. Compare your answers in parts (a) and (b). Explain.

   My answers are the same. The final price is $\$43.20$. This is because multiplication is commutative.

8. The sale price for a bicycle is $\$315$ dollars. The original price was first discounted by 50% and then discounted an additional 10%. Find the original price of the bicycle.

$(315 \div 0.9) \div 0.5 = 700$. *The original price was* $\$700$.

9. A ski shop has a markup rate of 50%. Find the selling price of skis that cost the storeowner $\$300$.

Solution 1: Use the original price of $\$300$ as the whole. The markup rate is 50% of $\$300=\$150$.
The selling price is $\$300+\$150=\$450$.

Solution 2: Multiply $\$300$ by 1 plus the markup rate (i.e., the selling price is $(1.5)(\$300)=\$450$).

10. A tennis supply store pays a wholesaler $\$90$ for a tennis racquet and sells it for $\$144$. What is the markup rate?

Solution 1: Let the original price of $\$90$ be the whole. $Quantity = Percent \times Whole$.

$144 = Percent(90)$

$\frac{144}{90} = Percent = 0.6 = 160\%$. *This is a 60% increase. The markup rate is 60%.*

Solution 2:

$$\begin{aligned} Selling\ Price &= (1+m)(Whole) \\ 144 &= (1+m)90 \\ 1+m &= \frac{144}{90} = 1.6 \\ m &= 0.6 = 60\% \end{aligned}$$

11. A shoe store is selling a pair of shoes for $\$60$ that has been discounted by 25%. What was the original selling price?

Solution 1:

$\$60 \rightarrow 75\%$
$\$20 \rightarrow 25\%$
$\$80 \rightarrow 100\%$

The original price was $\$80$.

Solution 2: Let x be the original cost in dollars.

$$(1 - 0.25)x = 60$$
$$\frac{3}{4}x = 60$$
$$\left(\frac{4}{3}\right)\left(\frac{3}{4}x\right) = \frac{4}{3}(60) = 80$$

The original price was $\$80$.

12. A shoe store has a markup rate of 75% and is selling a pair of shoes for $\$133$. Find the price the store paid for the shoes.

Solution 1:

$\$133 \rightarrow 175\%$
$\$19 \rightarrow 25\%$
$\$76 \rightarrow 100\%$

The store paid $\$76$.

Solution 2: Divide the selling price by 1.75.

$$\frac{133}{1.75} = 76$$

The store paid $\$76$.

13. Write $5\frac{1}{4}\%$ as a simple fraction.

$$\frac{21}{400}$$

14. Write $\frac{3}{8}$ as a percent.

37.5%

15. If 20% of the 70 faculty members at John F. Kennedy Middle School are male, what is the number of male faculty members?

14

16. If a bag contains 400 coins, and $33\frac{1}{2}\%$ are nickels, how many nickels are there? What percent of the coins are not nickels?

There are 134 nickels. The percent of coins that are not nickels is $66\frac{1}{2}\%$.

17. The temperature outside is 60 degrees Fahrenheit. What would be the temperature if it is increased by 20%?

72 degrees Fahrenheit.