Lesson 23: Solving Equations Using Algebra

Student Outcomes

- Students use algebra to solve equations (of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r, are specific rational numbers); using techniques of making zero (adding the additive inverse) and making one (multiplying by the multiplicative inverse) to solve for the variable.
- Students identify and compare the sequence of operations used to find the solution to an equation algebraically, with the sequence of operations used to solve the equation with tape diagrams. They recognize the steps as being the same.
- Students solve equations for the value of the variable using inverse operations; by making zero (adding the additive inverse) and making one (multiplying by the multiplicative inverse).

Classwork

MP. 1-3

As in Lesson 22, students continue solving equations using properties of equality and inverse operations to relate their steps to the steps taken when solving problems arithmetically. In this lesson, students decontextualize word problems to create equations that model given situations. Students justify their solutions by comparing their algebraic steps to the steps taken when using a tape diagram. Have the students work in cooperative groups and share out their solutions on chart paper. Use the share out as a way to have students view the differences in problem solving approaches.

Exercises 1–3

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1. Youth Group Trip

The youth group is going on a trip to an amusement park in another part of the state. The trip costs each group member of the group $\$150$, which includes $\$85$ for the hotel and two one-day combination entrance and meal plan passes.

a. Write an equation representing the cost of the trip. Let P be the cost of the park pass.

$85 + 2P = 150$

Scaffolding:

Provide a review card showing examples of fraction multiplication and division for students who do not have adequate prerequisite skills.

b. Solve the equation algebraically to find the cost of the park pass. Then write the reason that justifies each step, using if-then statements.

If: $85 + 2P = 150$,

Then: $85 - 85 + 2P = 150 - 85$ *Subtraction Property of Equality for the Additive Inverse of* 85

If: $0 + 2P = 65$

Then: $2P = 65$ *Additive Identity*

If: $2P = 65$

Then: $\left(\frac{1}{2}\right) 2P = \left(\frac{1}{2}\right) 65$ *Multiplication Property of Equality using the Multiplicative Inverse of* 2

If: $1P = 32.5$

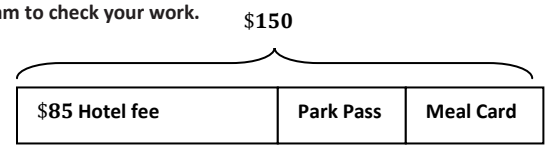
Then: $P = 32.5$ *Multiplicative Identity*

The park pass costs $32.50.

c. Model the problem using a tape diagram to check your work.

$150 - 85 = 65$

$65 \div 2 = 32.50$



Suppose you want to buy your favorite ice cream bar while at the amusement park and it costs $\$2.89$. If you purchase the ice cream bar and 3 bottles of water, and pay with a $\$10$ bill and receive no change, then how much did each bottle of water cost?

d. Write an equation to model this situation.

$2.89 + 3W = 10$

e. Solve the equation to determine the cost of one water bottle. Let W be the cost of the water bottle. Then, write the reason that justifies each step, using if-then statements.

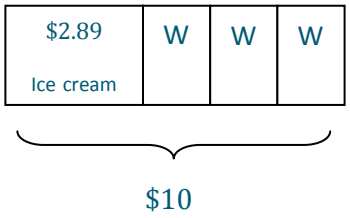
		Justification:
If:	$2.89 + 3W = 10$	
Then:	$2.89 - 2.89 + 3W = 10 - 2.89$	*Subtraction Property of Equality for the Additive Inverse of* 2.89
If:	$0 + 3W = 7.11$	
Then:	$3W = 7.11$	*Additive Identity*
If:	$3W = 7.11$	
Then:	$\frac{1}{3}(3w) = \frac{1}{3}(7.11)$	*Multiplication Property of Equality using the Multiplicative Inverse of* 3
If:	$1W = 2.37$	
Then:	$W = 2.37$	*Multiplicative Identity*

The cost of a water bottle is $\$2.37$.

f. **Model the problem using a tape diagram to check your work.**

$10 - 2.89 = 7.11$

$\frac{7.11}{3} = 2.37$



2. **Weekly Allowance**

Charlotte receives a weekly allowance from her parents. She spent half of this week's allowance at the movies, but earned an additional $4 for performing extra chores. If she didn't spend any additional money and finished the week with $12, what is Charlotte's weekly allowance? Write an equation that can be used to find the original amount of Charlotte's weekly allowance. Let A be the value of Charlotte's original weekly allowance.

$\frac{1}{2}A + 4 = 12$

a. **Solve the equation to find the original amount of allowance. Then, write the reason that justifies each step, using if-then statements.**

If: $\frac{1}{2}A + 4 = 12$

Then: $\frac{1}{2}A + 4 - 4 = 12 - 4$ *Subtraction Property of Equality for Additive Inverse of 4*

If: $\frac{1}{2}A + 0 = 8$

Then: $\frac{1}{2}A = 8$ *Additive Identity*

If : $\frac{1}{2}A = 8$

Then: $(2)\frac{1}{2}A = (2)8$ *Multiplication Property of Equality using the Multiplicative Inverse of* $\frac{1}{2}$

If: $1A = 16$

Then: $A = 16$ *Multiplicative Identity*

The original allowance was $16.

b. **Explain your answer in the context of this problem.**

Charlotte's weekly allowance is $16.

c. **Charlotte's goal is to save $100 for her beach trip at the end of the summer. Use the amount of weekly allowance you found in part (c) to write an equation to determine the number of weeks that Charlotte must work to meet her goal. Let w represent the number of weeks.**

$16w = 100$

$\left(\frac{1}{16}\right)16w = \left(\frac{1}{16}\right)100$

$1w = 6.25$

$w = 6.25$

d. In looking at your answer to part (d), and based on the story above, do you think it will take Charlotte that many weeks to meet her goal? Why or Why not?

Charlotte needs more than 6 weeks' allowance, so she will need to save 7 weeks' allowance, (and not spend any of it). There are 10–12 weeks in the summer; so, yes, she can do it.

3. Travel Baseball Team

Allen is very excited about joining a travel baseball team for the fall season. He wants to determine how much money he should save to pay for the expenses related to this new team. Players are required to pay for uniforms, travel expenses, and meals.

a. If Allen buys 4 uniform shirts at one time, he gets a $\$10.00$ discount so that the total cost of 4 shirts would be $\$44$. Write an algebraic equation that represents the regular price of one shirt. Solve the equation. Write the reason that justifies each step, using if-then statements.

If: $4s - 10 = 44$

Then: $4s - 10 + 10 = 44 + 10$ *Addition Property of Equality using the Additive Inverse of* -10

If: $4s + 0 = 54$

Then: $4s = 54$ *Additive Identity*

If: $4s = 54$

Then: $\left(\frac{1}{4}\right)4s = \left(\frac{1}{4}\right)54,$ *Multiplication Property of Equality using Multiplicative Inverse of 4*

If: $1s = 13.50$

Then: $s = 13.50$ *Multiplicative Identity*

b. What is the cost of one shirt without the discount?

The cost of one shirt is $\$13.50$

c. What is the cost of one shirt with the discount?

$4s = 44$

$\left(\frac{1}{4}\right)4s = \left(\frac{1}{4}\right)44$

$1s = 11$

$s = 11$

d. How much more do you pay per shirt if you buy them one at a time (rather than in bulk)?

$13.50 - 11.00 = 2.50$

One shirt costs $\$11$ *if you buy them in bulk. So, Allen would pay* $\$2.50$ *more per shirt if he bought them one at a time.*

Allen's team was also required to buy two pairs of uniform pants and two baseball caps, which total $\$68$. A pair of pants costs $\$12$ more than a baseball cap.

e. Write an equation that models this situation. Let c represent the cost of a baseball cap.

$2(\text{cap} + 1 \text{ pair of pants}) = 68$

$2(c + c + 12) = 68$ *or* $2(2c + 12) = 68$ *or* $4c + 24 = 68$

f. **Solve the equation algebraically to find the cost of a baseball cap., Write the reason that justifies each step, using if-then statements.**

If: $2\,(2\,c\,+\,12) = 68$

Then: $\left(\frac{1}{2}\right)(2\,)(2\,c\,+\,12) = \left(\frac{1}{2}\right)68$ *Multiplication Property of Equality using the Multiplicative Inverse of* 2

If: $1(2\,c\,+\,12) = 34$

Then: $2\,c\,+\,12\,= 34$ *Multiplicative Identity*

If: $2\,c\,+\,12\,= 34$

Then: $2\,c\,+\,12\,-12\,= 34\,-\,12,$ *Subtraction Property of Equality for the Additive Inverse of 12*

If: $2\,c + 0\,=\,22$

Then: $2\,c = 22$ *Additive Identity*

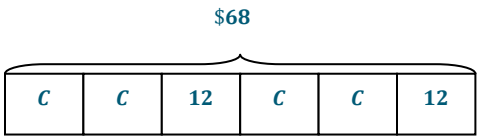
If: $2\,c = 22$

Then: $\left(\frac{1}{2}\right)2\,c = \left(\frac{1}{2}\right)22,$ *Multiplication Property of Equality using the Multiplicative Inverse of 2*

If: $1\,c\,\,= 11$

Then: $c\,\,= 11$ *Multiplicative Identity*

g. **Model the problem using a tape diagram in order to check your work.**



$68 - 24\,=\,44$

$\frac{44}{4} = 11$

h. **What is the cost of one cap?**

The cost of one cap is $\$11.$

i. **What is the cost of one pair of pants?**

$11\,+\,12\,=\,23$ *The cost of one pair of pants is* $\$23.$

Closing (5 minutes)

- How do we translate a word problem into an equation? For instance, in Exercise 1 about the youth group trip, what key words and statements helped you determine the operations and values used in the equation?
- How do we make sense of a word problem and model it with an equation?

Lesson Summary

Equations are useful to model and solve real-world problems. The steps taken to solve an algebraic equation are the same steps used in an arithmetic solution.

Exit Ticket (5 minutes)

Name _____ Date_____

Lesson 23: Solving Equations Using Algebra

Exit Ticket

Andrew's math teacher entered the 7^{th} grade students in a math competition. There was an enrollment fee of $\$30$ and also an $\$11$ charge for each packet of 10 tests. The total cost was $\$151$. How many tests were purchased?
Set up an equation to model this situation, solve it using if-then statements, and justify the reasons for each step in your solution.

Exit Ticket Sample Solutions

Andrew's math teacher entered the 7th grade students in a math competition. There was an enrollment fee of $\$30$ and also an $\$11$ charge for each packet of 10 tests. The total cost was $\$151$. How many tests were purchased? Set up an equation to model this situation, solve it and justify your answer.

Let p = the number of test packets.

Enrollment fee + cost of test = 151

If: $30 + 11p = 151$

Then: $30 - 30 + 11p = 151 - 30$ *Subtraction Property of Equality for the Additive Inverse of 30*

If: $0 + 11p = 121$

Then: $11p = 121$ *Additive Identity*

If: $11p = 121$

Then: $\frac{1}{11}(11p) = \frac{1}{11}(121)$ *Multiplication Property of Equality using the Multiplicative Inverse of 11*

If: $1p = 11$

Then: $p = 11$ *Multiplicative Identity*

Andrew's math teacher bought 11 packets of tests. There were 10 tests in each packet, and $10 \times 11 = 110$.

So, there were 110 tests purchased.

Problem Set Sample Solutions

For Exercises 1–4, solve each equation algebraically and justify your steps.

1. $\frac{2}{3}x - 4 = 20$

If: $\frac{2}{3}x - 4 = 20$

Then: $\frac{2}{3}x - 4 + 4 = 20 + 4$ *Addition Property of Equality using the Additive Inverse of -4*

If: $\frac{2}{3}x + 0 = 24$

Then: $\frac{2}{3}x = 24$ *Additive Identity*

If: $\frac{2}{3}x = 24$

Then: $\left(\frac{3}{2}\right)\frac{2}{3}x = \left(\frac{3}{2}\right)24$ *Multiplication Property of Equality using the Multiplicative Inverse of $\frac{2}{3}$*

If: $1x = 36$

Then: $x = 36$ *Multiplicative Identity*

2. $4 = \frac{-1+x}{2}$

If: $4 = \frac{-1+x}{2}$

Then: $2\,(4) = 2\,\left(\frac{-1+x}{2}\right)$ *Multiplication Property of Equality using the Multiplicative Inverse of* $\frac{1}{2}$

If: $8 = 1\,(-1+x)$

Then: $8 = -1+x$ *Multiplicative Identity*

If: $8 = -1+x$

Then: $8-(-1) = -1-(-1)+x$ *Subtraction Property of Equality for the Additive Inverse of* -1

If: $9 = 0+x$

Then: $9 = x$ *Additive Identity*

3. $12(x+9) = -108$

If: $12(x+9) = -108$

Then: $\left(\frac{1}{12}\right)12(x+9) = \left(\frac{1}{12}\right)(-108)$ *Multiplication Property of Equality using the Multiplicative Inverse of* 12

If: $1\,(x+9) = -9$

Then: $x+9 = -9$ *Multiplicative Identity*

If: $x+9 = -9$

Then: $x+9-9 = -9-9$ *Subtraction Property of Equality for the Additive Inverse of* 9

If: $x+0 = -18$

Then: $x = -18$ *Additive Identity*

4. $5x + 14 = -7$

If: $5x + 14 = -7$

Then: $5x + 14 - 14 = -7 - 14$ *Subtraction Property of Equality for the Additive Inverse of* 14

If: $5x + 0 = -21$

Then: $5x = -21$ *Additive Identity*

If: $5x = -21$

Then: $\left(\frac{1}{5}\right)5x = \left(\frac{1}{5}\right)(-21)$ *Multiplication Property of Equality using the Multiplicative Inverse of* 5

If: $1x = -4.2$

Then: $x = -4.2$ *Multiplicative Identity*

For Exercises 5–7, write an equation to represent each word problem. Solve the equation showing the steps and then state the value of the variable in the context of the situation.

5. **A plumber has a very long piece of pipe that is used to run city water parallel to a major roadway. The pipe is cut into two sections. One section of pipe is 12 ft. shorter than the other. If $\frac{3}{4}$ of the length of the shorter pipe is 120 ft., how long is the longer piece of the pipe?**

Let x = the longer piece of pipe

If: $\frac{3}{4}(x-12)=120$

Then: $\frac{4}{3}\left(\frac{3}{4}\right)(x-12)=\left(\frac{4}{3}\right)120$ *Multiplication Property of Equality using the Multiplicative Inverse of 3*

If: $1(x-12)=160$

Then: $x-12=160$ *Multiplicative Identity*

If: $x-12=160$

Then: $x-12+12=160+12$ *Addition Property of Equality for the Additive Inverse of -12*

If: $x+0=172$

Then: $x=172$ *Additive Identity*

The longer piece of pipe is 172 ft.

6. **Bob's monthly phone bill is made up of a $\$10$ fee plus $\$0.05$ per minute. Bob's phone bill for July was $\$22$. Write an equation to model the situation, using m to represent the number of minutes. Solve the equation to determine the number of phone minutes Bob used in July.**

Let m = the number of phone minutes Bob used

If: $10+0.05m=22$

Then: $10-10+0.05m=22-10$ *Subtraction Property of Equality for the Additive Inverse of 30*

If: $0+0.05m=12$

Then: $0.05m=12$ *Additive Identity*

If: $0.05m=12$

Then: $\left(\frac{1}{0.05}\right)0.05m=\left(\frac{1}{0.05}\right)12$ *Multiplication Property of Equality using the Multiplicative Inverse of 0.05*

If: $1m=240$

Then: $m=240$ *Multiplicative Identity*

Bob used 240 phone minutes in July.

7. Kym switched cell phone plans. She signed up for a new plan that will save her $\$3.50$ per month compared to her old cell phone plan. The cost of the new phone plan for an entire year is $\$294$. How much did Kym pay per month under her old phone plan?

Let n = the amount Kym paid per month for her old cell phone plan

If: $294 = 12(n - 3.50)$

Then: $\left(\frac{1}{12}\right)(294) = \left(\frac{1}{12}\right)12(n - 3.50)$ *Multiplication Property of Equality using the Multiplicative Inverse of* 12

If: $24.5 = 1\,(n - 3.50)$

Then: $24.5 = n - 3.50$ *Multiplicative Identity*

If: $24.5 = n - 3.50$

Then: $24.5 + 3.50 = n - 3.50 + 3.50$ *Addition Property of Equality for the Additive Inverse of* -3.50

If: $28 = n + 0$

Then: $28 = n$ *Additive Identity*

Kym paid $\$28$ per month for her old cell phone plan.