Lesson 3: The Area of Acute Triangles Using Height and Base

Student Outcomes

- Students show the area formula for a triangular region by decomposing a triangle into right triangles. For a given triangle, the height of the triangle is the length of the altitude. The length of the base is either called the length base or, more commonly, the base.
- Students understand that the height of the triangle is the perpendicular segment from a vertex of a triangle to the line containing the opposite side. The opposite side is called the base. Students understand that any side of a triangle can be considered a base and that the choice of base determines the height.

Lesson Notes

For this lesson, students will need the triangle template to this lesson and a ruler.

Throughout the lesson, students will determine if the area formula for right triangles is the same as the formula used to calculate the area of acute triangles.

Fluency Exercise (5 minutes)

Multiplication of Decimals Sprint

Classwork

Discussion (5 minutes)

- What is different between the two triangles below?

  - *One triangle is a right triangle because it has one right angle; the other does not have a right angle, so it is not a right triangle.*
- How do we find the area of the right triangle?
  - $A = \frac{1}{2} \times base \times height$
- How do we know which side of the right triangle is the base and which is the height?
  - *If you choose one of the two shorter sides to be the base, then the side that is perpendicular to this side will be the height.*

- How do we calculate the area of the other triangle?
  - *We do not know how to calculate the area of the other triangle because we do not know its height.*

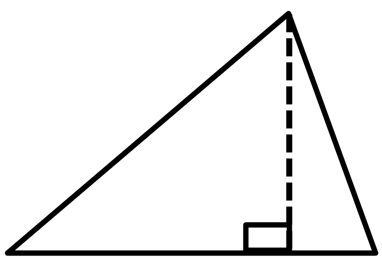
Mathematical Modeling Exercise (10 minutes)

Students will need the triangle template found at the end of the lesson and a ruler to complete this example. To save class time, cut out the triangles ahead of time.

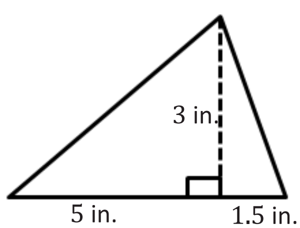
- The height of a triangle does not always have to be a side of the triangle. The height of a triangle is also called the altitude, which is a line segment from a vertex of the triangle and perpendicular to the opposite side.

NOTE: English learners may benefit from a poster showing each part of a right triangle and acute triangle (and eventually an obtuse triangle) labeled, so they can see the height and altitude and develop a better understanding of the new vocabulary words.

Model how to draw the altitude of the given triangle.



MP.3

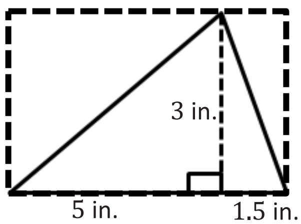
- Fold the paper to show where the altitude would be located, and then draw the altitude or the height of the triangle.
- Notice that by drawing the altitude we have created two right triangles. Using the knowledge we gained yesterday, can we calculate the area of the entire triangle?
  - *We can calculate the area of the entire triangle by calculating the area of the two right triangles.*
- Measure and label each base and height. Round your measurements to the nearest half inch.
  - 

> *Scaffolding:*
> Outline or shade each right triangle with a different color to help students see the two different triangles.

- Calculate the area of each right triangle.
  - $A = \frac{1}{2}bh = \frac{1}{2}(5\ in.)(3\ in.) = 7.5\ in^2$
  - $A = \frac{1}{2}bh = \frac{1}{2}(1.5\ in.)(3\ in.) = 2.25\ in^2$
- Now that we know the area of each right triangle, how can we calculate the area of the entire triangle?
  - *To calculate the area of the entire triangle, we can add the two areas together.*
- Calculate the area of the entire triangle.
  - $A = 7.5\ in^2 + 2.25\ in^2 = 9.75\ in^2$

- Talk to your neighbor and try to determine a more efficient way to calculate the area of the entire triangle.
  - *Allow students some time to discuss their thoughts.*
  - *Answers will vary. Allow a few students to share their thoughts.*

Test a few of the students' predictions on how to find the area of the entire triangle faster. The last prediction you should try is the correct one shown below.

- In the previous lesson, we said that the area of right triangles can be calculated using the formula $A = \frac{1}{2} \times base \times height$. Some of you believe we can still use this same formula for the given triangle.
- Draw a rectangle around the given triangle.
  - 

MP.3

- Does the triangle represent half of the area of the rectangle? Why or why not?
  - *The triangle does represent half of the area of the rectangle. If the altitude of the triangle splits the rectangle into two separate rectangles, then the slanted sides of the triangle split these rectangles into two equal parts.*
- What is the length of the base?
  - *The length of the base is* 6.5 *inches because we have to add the two parts together.*
- What is the length of the altitude (the height)?
  - *The height is* 3 *inches because that is the length of the line segment that is perpendicular to the base.*
- Calculate the area of the triangle using the formula we discovered yesterday, $A = \frac{1}{2} \times base \times height$.
  - $A = \frac{1}{2}bh = \frac{1}{2}(6.5\ in.)(3\ in.) = 9.75\ in^2$
- Is this the same area we got when we split the triangle into two right triangles?
  - *Yes.*
- It is important to determine if this is true for more than just this one example.

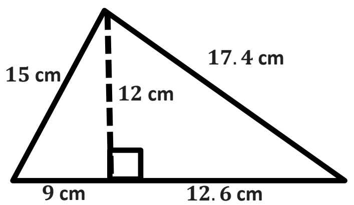
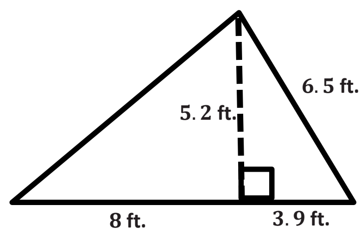
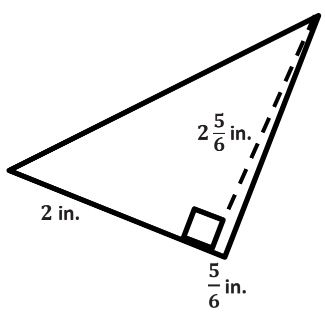
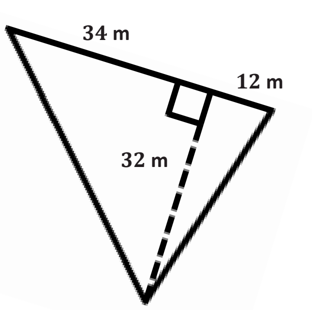
Exercises (15 minutes)

Have students work with partners on the exercises below. The purpose of the first exercise is to determine if the area formula, $A = \frac{1}{2}bh$, is always correct. One partner calculates the area of the given triangle by calculating the area of two right triangles, and the other partner calculates the area just as one triangle. Partners should switch who finds each area so that every student has a chance to practice both methods. Students may use a calculator as long as they record their work on their paper as well.

MP.2

Exercises

1. Work with a partner on the exercises below. Determine if the area formula $A = \frac{1}{2}bh$ is always correct. You may use a calculator, but be sure to record your work on your paper as well.

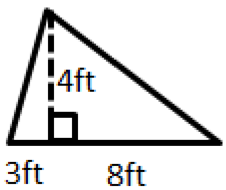
	Area of Two Right Triangles	Area of Entire Triangle
	$A = \frac{1}{2}(9\ cm)(12\ cm)$ $A = 54\ cm^2$ $A = \frac{1}{2}(12.6\ cm)(12\ cm)$ $A = 75.6\ cm^2$ $A = 54 + 75.6 = 129.6\ cm^2$	$base = 9\ cm + 12.6\ cm = 21.6\ cm$ $A = \frac{1}{2}(21.6\ cm)(12\ cm)$ $A = 129.6\ cm^2$
	$A = \frac{1}{2}(3.9\ ft.)(5.2\ ft.)$ $A = 10.14\ ft^2$ $A = \frac{1}{2}(8\ ft.)(5.2\ ft.)$ $A = 20.8\ ft^2$ $A = 10.14 + 20.8 = 30.94\ ft^2$	$base = 8\ ft. + 3.9\ ft. = 11.9\ ft.$ $A = \frac{1}{2}(11.9\ ft.)(5.2\ ft.)$ $A = 30.94\ ft^2$
	$A = \frac{1}{2}(2\ in.)\left(2\frac{5}{6}\ in.\right)$ $A = \frac{1}{2}\left(\frac{2}{1}\ in.\right)\left(\frac{17}{6}\ in.\right)$ $A = \frac{34}{12} = 2\frac{5}{6}\ in^2$ $A = \frac{1}{2}\left(\frac{5}{6}\ in.\right)\left(2\frac{5}{6}\ in.\right)$ $A = \frac{1}{2}\left(\frac{5}{6}\ in.\right)\left(\frac{17}{6}\ in.\right)$ $A = \frac{85}{72} = 1\frac{13}{72}\ in^2$ $A = 2\frac{5}{6} + 1\frac{13}{72} = 2\frac{60}{72} + 1\frac{13}{72}$ $= 4\frac{1}{72}\ in^2$	$base = 2\ in. + \frac{5}{6}\ in. = 2\frac{5}{6}\ in.$ $A = \frac{1}{2}\left(2\frac{5}{6}\ in.\right)\left(2\frac{5}{6}\ in.\right)$ $A = \frac{1}{2}\left(\frac{17}{6}\ in.\right)\left(\frac{17}{6}\ in.\right)$ $A = \frac{289}{72} = 4\frac{1}{72}\ in^2$
	$A = \frac{1}{2}(34\ m)(32\ m)$ $A = 544\ m^2$ $A = \frac{1}{2}(12\ m)(32\ m)$ $A = 192\ m^2$ $A = 544 + 192 = 736\ m^2$	$base = 12\ m + 34\ m = 46\ m$ $A = \frac{1}{2}(46\ m)(32\ m)$ $A = 736\ m^2$

MP.2

2. Can we use the formula $A = \frac{1}{2} \times base \times height$ to calculate the area of triangles that are not right triangles? Explain your thinking.

Yes, the formula $A = \frac{1}{2} \times base \times height$ can be used for more than just right triangles. We just need to be able to determine the height, even if it isn't the length of one of the sides.

3. Examine the given triangle and expression.



$\frac{1}{2}(11\text{ ft.})(4\text{ ft.})$

Explain what each part of the expression represents according to the triangle.

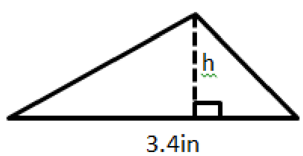
11 ft. represents the base of the triangle because 8 ft. $+ 3$ ft. $= 11$ ft.

4 ft. represents the altitude of the triangle because this length is perpendicular to the base.

4. Joe found the area of a triangle by writing $A = \frac{1}{2}(11\text{ in.})(4\text{ in.})$, while Kaitlyn found the area by writing $A = \frac{1}{2}(3\text{ in.})(4\text{ in.}) + \frac{1}{2}(8\text{ in.})(4\text{ in.})$. Explain how each student approached the problem.

Joe combined the two bases of the triangle first, and then calculated the area, whereas Kaitlyn calculated the area of two smaller triangles, and then added these areas together.

5. The triangle below has an area of 4.76 sq. in. If the base is 3.4 in., let h be the height in inches.



a. Explain how the equation $4.76\text{ in}^2 = \frac{1}{2}(3.4\text{ in.})(h)$ represents the situation.

The equation shows the area, 4.76 in², is one half the base, 3.4 in., times the height in inches, h.

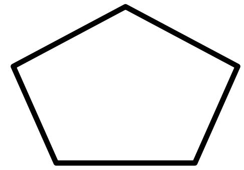
b. Solve the equation.

$$4.76\ in^2 = \frac{1}{2}(3.4\ in.)(h)$$
$$4.76\ in^2 = (1.7\ in.)(h)$$
$$4.76\ in^2 \div 1.7\ in. = (1.7\ in.)(h) \div 1.7\ in.$$
$$2.8\ in. = h$$

Closing (5 minutes)

- When a triangle is not a right triangle, how can you determine its base and height?
    - *The height of a triangle is the length of the altitude. The altitude is the line segment from a vertex of a triangle to the line containing the opposite side (or the base) that is perpendicular to the base.*

- How can you use your knowledge of area to calculate the area of more complex shapes?
  - *Split the shape into smaller shapes for which we know how to calculate the area.*



Exit Ticket (5 minutes)

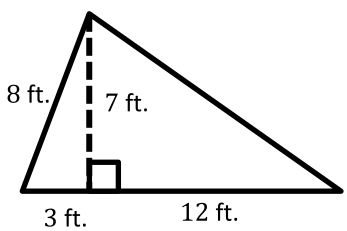
Name ____ Date ____________________

Lesson 3: The Area of Acute Triangles Using Height and Base

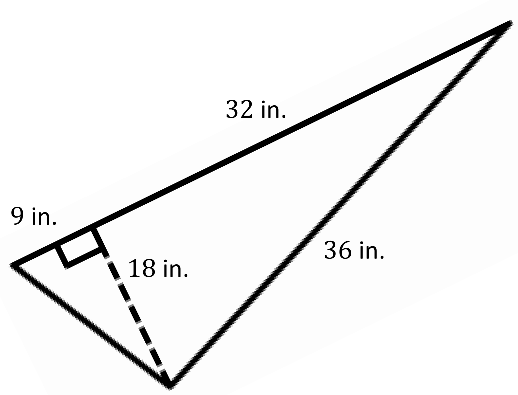
Exit Ticket

Calculate the area of each triangle using two different methods. Figures are not drawn to scale.

1.



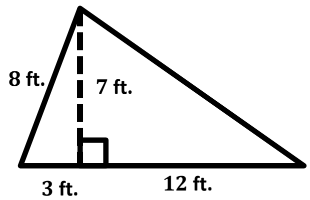
2.



Exit Ticket Sample Solutions

Calculate the area of each triangle. Figures are not drawn to scale.

1.

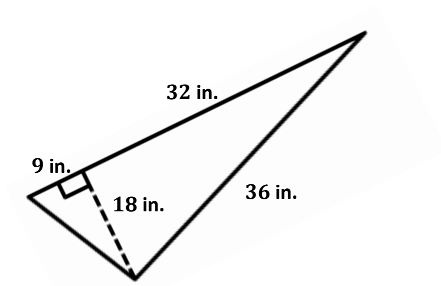


$A = \frac{1}{2}(3\ ft.)(7\ ft.) = 10.5\ ft^2$ $A = \frac{1}{2}(12\ ft.)(7\ ft.) = 42\ ft^2$ $A = 10.5\ ft^2 + 42\ ft^2 = 52.5\ ft^2$

or

$A = \frac{1}{2}(15\ ft.)(7\ ft.) = 52.5\ ft^2$

2.



$A = \frac{1}{2}(9\ in.)(18\ in.) = 81\ in^2$ $A = \frac{1}{2}(32\ in.)(18\ in.) = 288\ in^2$ $A = 81\ in^2 + 288\ in^2 = 369\ in^2$

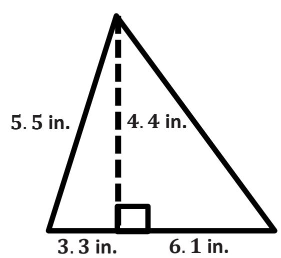
or

$A = \frac{1}{2}(41\ in.)(18\ in.) = 369\ in^2$

Problem Set Sample Solutions

Calculate the area of each shape below. Figures are not drawn to scale.

1.

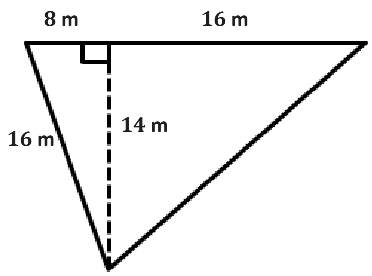


$A = \frac{1}{2}(3.3\ in.)(4.4\ in.) = 7.26\ in^2$ $A = \frac{1}{2}(6.1\ in.)(4.4\ in.) = 13.42\ in^s$ $A = 7.26\ in^2 + 13.42\ in^2$
$= 20.68\ in^2$

or

$A = \frac{1}{2}(9.4\ in.)(4.4\ in.) = 20.68\ in^2$

2.

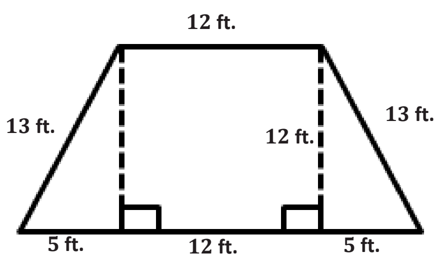


$A = \frac{1}{2}(8\ m)(14\ m) = 56\ m^2$; $A = \frac{1}{2}(16\ m)(14\ m) = 112\ m^2 \rightarrow A = 56\ m^2 + 112\ m^2 = 168\ m^2$

or

$A = \frac{1}{2}(24\ m)(14\ m) = 168\ m^2$

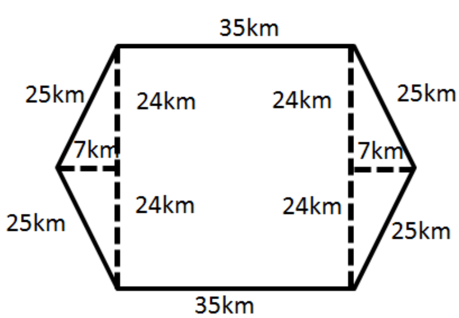
3.



$A = \frac{1}{2}(5\ ft.)(12\ ft.) = 30\ ft^2$; $A = (12\ ft.)(12\ ft.) = 144\ ft^2$; $A = \frac{1}{2}(5\ ft.)(12\ ft.) = 30\ ft^2$

$A = 30\ ft^2 + 144\ ft^2 + 30\ ft^2 = 204\ ft^2$

4.



$A = \frac{1}{2}(48\ km)(7\ km) = 168\ km^2$; $A = 35\ km(48\ km) = 1680\ km^2$; $A = \frac{1}{2}(48\ km)(7\ km) = 168\ km^2$

$A = 168\ km^2 + 1680\ km^2 + 168\ km^2 = 2016\ km^2$

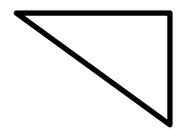
5. Immanuel is building a fence to make an enclosed play area for his dog. The enclosed area will be in the shape of a triangle with a base of 48 in. and an altitude of 32 in. How much space does the dog have to play?

$A = \frac{1}{2}bh = \frac{1}{2}(48\ in.)(32\ in.) = 768\ in^2$

The dog will have 768 in² to play.

6. Chauncey is building a storage bench for his son's playroom. The storage bench will fit into the corner and then go along the wall to form a triangle. Chauncey wants to buy a cover for the bench.

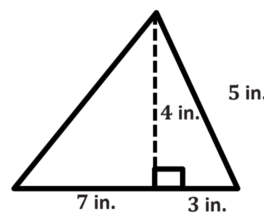
If the storage bench is $2\frac{1}{2}$ ft. along one wall and $4\frac{1}{4}$ ft. along the other wall, how big will the cover have to be to cover the entire bench?



$A = \frac{1}{2}\left(2\frac{1}{2}\ ft.\right)\left(4\frac{1}{4}\ ft.\right) = \frac{1}{2}\left(\frac{5}{2}\ ft.\right)\left(\frac{17}{4} ft.\right) = \frac{85}{16}\ ft^2 = 5\frac{5}{16}\ ft^2$

Chauncey would have to buy a cover that has an area of $5\frac{5}{16}$ ft² to cover the entire bench.

7. Examine the triangle to the right.



a. Write an expression to show how you would calculate the area.

$\frac{1}{2}(7\ in.)(4\ in.) + \frac{1}{2}(3\ in.)(4\ in.)$ *or* $\frac{1}{2}(10\ in.)(4\ in.)$

b. Identify each part of your expression as it relates to the triangle.

If students wrote the first expression: 7 in. and 3 in. represent the two parts of the base, and 4 in. is the height or altitude of the triangle.

If students wrote the second expression: 10 in. represents the base because 7 in. $+\ 3$ in. $=10$ in., and 4 in. represents the height or the altitude of the triangle.

8. A triangular room has an area of $32\frac{1}{2}$ sq. m. If the height is $7\frac{1}{2}$ m, write an equation to determine the length of the base, b, in meters. Then solve the equation.

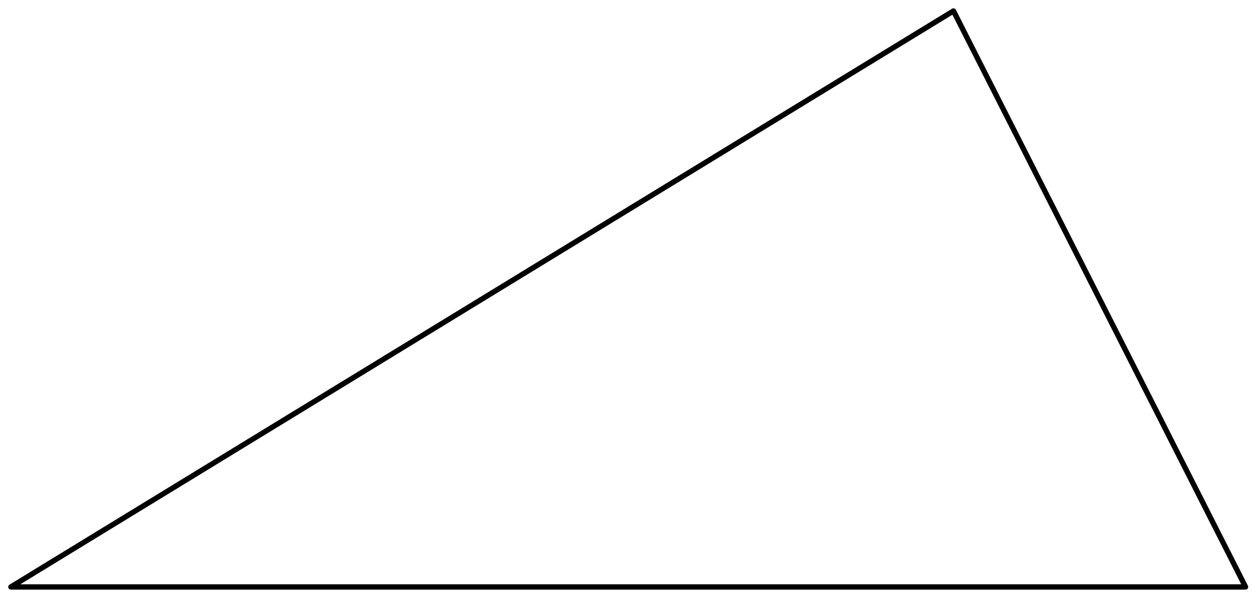
$$32\frac{1}{2}\ m^2 = \frac{1}{2}(b)\left(7\frac{1}{2}\ m\right)$$

$$32\frac{1}{2}\ m^2 = \left(\frac{15}{4}\ m\right)(b)$$

$$32\frac{1}{2}m^2 \div \frac{15}{4}m = \left(\frac{15}{4}m\right)(b) \div \frac{15}{4}m$$

$$\frac{26}{3}m = b$$

$$8\frac{2}{3}m = b$$



Multiplication of Decimals – Round 1

Directions: *Determine the products of the decimals.*

Number Correct: _______

1.	4.5×3		19.	3.56×4.12	
2.	7.2×8		20.	9.32×1.74	
3.	9.4×6		21.	10.43×7.61	
4.	10.2×7		22.	2.77×8.39	
5.	8.3×4		23.	1.89×7.52	
6.	5.8×2		24.	7.5×10.91	
7.	7.1×9		25.	7.28×6.3	
8.	5.9×10		26.	1.92×8.34	
9.	3.4×3		27.	9.81×5.11	
10.	3.2×4.1		28.	18.23×12.56	
11.	6.3×2.8		29.	92.38×45.78	
12.	9.7×3.6		30.	13.41×22.96	
13.	8.7×10.2		31.	143.8×32.81	
14.	4.4×8.9		32.	82.14×329.4	
15.	3.9×7.4		33.	34.19×84.7	
16.	6.5×5.5		34.	23.65×38.83	
17.	1.8×8.1		35.	72.5×56.21	
18.	9.6×2.3		36.	341.9×24.56	

Multiplication of Decimals – Round 1 [KEY]

Directions: *Determine the products of the decimals.*

1.	4.5×3	**13.5**	19.	3.56×4.12	**14.6672**
2.	7.2×8	**57.6**	20.	9.32×1.74	**16.2168**
3.	9.4×6	**56.4**	21.	10.43×7.61	**79.3723**
4.	10.2×7	**71.4**	22.	2.77×8.39	**23.2403**
5.	8.3×4	**33.2**	23.	1.89×7.52	**14.2128**
6.	5.8×2	**11.6**	24.	7.5×10.91	**81.825**
7.	7.1×9	**63.9**	25.	7.28×6.3	**45.864**
8.	5.9×10	**59**	26.	1.92×8.34	**16.0128**
9.	3.4×3	**10.2**	27.	9.81×5.11	**50.1291**
10.	3.2×4.1	**13.12**	28.	18.23×12.56	**228.9688**
11.	6.3×2.8	**17.64**	29.	92.38×45.78	**4,229.1564**
12.	9.7×3.6	**34.92**	30.	13.41×22.96	**307.8936**
13.	8.7×10.2	**88.74**	31.	143.8×32.81	**4,718.078**
14.	4.4×8.9	**39.16**	32.	82.14×329.4	**27,056.916**
15.	3.9×7.4	**28.86**	33.	34.19×84.7	**2,895.893**
16.	6.5×5.5	**35.75**	34.	23.65×38.83	**9,18.3295**
17.	1.8×8.1	**14.58**	35.	72.5×56.21	**4,075.225**
18.	9.6×2.3	**22.08**	36.	341.9×24.56	**8,397.064**

Multiplication of Decimals – Round 2

Number Correct: ______

Improvement: ______

Directions: *Determine the products of the decimals.*

1.	3.7×8		19.	4.67×5.21	
2.	9.2×10		20.	6.81×1.94	
3.	2.1×3		21.	7.82×10.45	
4.	4.8×9		22.	3.87×3.97	
5.	3.3×5		23.	9.43×4.21	
6.	7.4×4		24.	1.48×9.52	
7.	8.1×9		25.	9.41×2.74	
8.	1.9×2		26.	5.6×4.22	
9.	5.6×7		27.	8.65×3.1	
10.	3.6×8.2		28.	14.56×98.36	
11.	4.1×9.8		29.	33.9×10.23	
12.	5.2×8.7		30.	451.8×32.04	
13.	1.4×7.2		31.	108.4×32.71	
14.	3.4×10.2		32.	40.36×190.3	
15.	2.8×6.4		33.	75.8×32.45	
16.	3.9×9.3		34.	56.71×321.8	
17.	8.2×6.5		35.	80.72×42.7	
18.	4.5×9.2		36.	291.08×41.23	

Multiplication of Decimals – Round 2 [KEY]

Directions: *Determine the products of the decimals.*

1.	3.7×8	**29.6**	19.	4.67×5.21	**24.3307**
2.	9.2×10	**92**	20.	6.81×1.94	**13.2114**
3.	2.1×3	**6.3**	21.	7.82×10.45	**81.719**
4.	4.8×9	**43.2**	22.	3.87×3.97	**15.3639**
5.	3.3×5	**16.5**	23.	9.43×4.21	**39.7003**
6.	7.4×4	**29.6**	24.	1.48×9.52	**14.0896**
7.	8.1×9	**72.9**	25.	9.41×2.74	**25.7834**
8.	1.9×2	**3.8**	26.	5.6×4.22	**23.632**
9.	5.6×7	**39.2**	27.	8.65×3.1	**26.815**
10.	3.6×8.2	**29.52**	28.	14.56×98.36	**1,432.1216**
11.	4.1×9.8	**40.18**	29.	33.9×10.23	**346.797**
12.	5.2×8.7	**45.24**	30.	451.8×32.04	**14,475.672**
13.	1.4×7.2	**10.08**	31.	108.4×32.71	**3,545.764**
14.	3.4×10.2	**34.68**	32.	40.36×190.3	**7,680.508**
15.	2.8×6.4	**17.92**	33.	75.8×32.45	**2,459.71**
16.	3.9×9.3	**36.27**	34.	56.71×321.8	**18,249.278**
17.	8.2×6.5	**53.3**	35.	80.72×42.7	**3,446.744**
18.	4.5×9.2	**41.4**	36.	291.08×41.23	**12,001.2284**