Lesson 11: Absolute Value—Magnitude and Distance

Student Outcomes

- Students understand the absolute value of a number as its distance from zero on the number line.
- Students use absolute value to find the magnitude of a positive or negative quantity in a real-world situation.

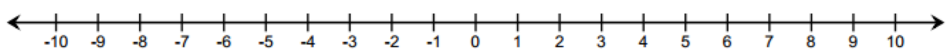
Classwork

Opening Exercise (4 minutes)

MP.8

For this warm-up exercise, students work individually to record two different rational numbers that are the same distance from zero. Students find as many examples as possible, and reach a conclusion about what must be true for every pair of numbers that lie that same distance from zero.

Opening Exercise



After two minutes:

- What are some examples you found (pairs of numbers that are the same distance from zero)?
  - *Answers may vary from the following examples:* $-\frac{1}{2}$ *and* $\frac{1}{2}$, 8.01 *and* -8.01, -7 *and* 7, *etc.*
- What is the relationship between each pair of numbers?
  - *They are opposites.*
- How do each pair of numbers relate to zero?
  - *Both numbers in each pair are the same distance from zero.*

Discussion (3 minutes)

We just saw that every number and it opposite are the same distance from zero on the number line. The **absolute value** of a number is the distance between the number and zero on a number line.

In other words, a number and its opposite have the same *absolute value*.

- What is the absolute value of 5? 5.
- What is the absolute value of -5? 5.
- Both 5 and -5 are five units from zero.

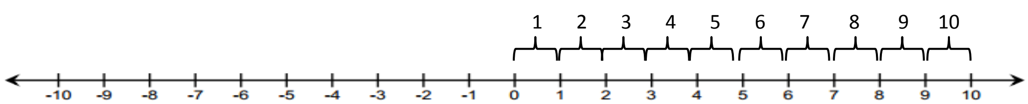
> *Scaffolding:*
> Provide students with a number line so they can physically count the number of units between a number and zero.

- What is the absolute value of -1?
  - 1
- What is the absolute value of 0?
  - 0

Example 1 (3 minutes): The Absolute Value of a Number

Example 1: The Absolute Value of a Number

The absolute value of ten is written as: $|10|$. On the number line, count the number of units from 10 to 0. How many units is 10 from 0? $\underline{|10| = 10}$



What other number has an absolute value of 10? Why?

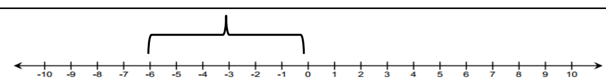
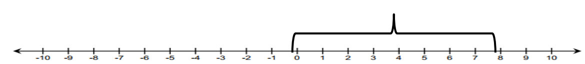
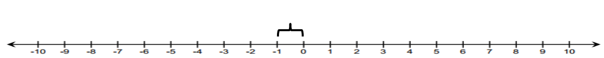
$|-10| = 10$ because -10 is 10 units from zero.

The <u>absolute value</u> of a number is the distance between the number and zero on the number line.

Exercises 1–3 (4 minutes)

Exercises 1–3

Complete the following chart.

	Number	Absolute Value	Number Line Diagram	Different Number with the same Absolute Value
1.	-6	$|-6| = 6$		6
2.	8	$|8| = 8$		-8
3.	-1	$|-1| = 1$		1

Example 2 (3 minutes): Using Absolute Value to Find Magnitude

The **magnitude** of a quantity is found by taking the absolute value of its numerical part.

Example 2: Using Absolute Value to Find Magnitude

Mrs. Owens received a call from her bank because she had a checkbook balance of -45 dollars. What was the magnitude of the amount overdrawn?

$|-45| = 45$ *Mrs. Owens overdrew her checking account by \$45.*

The magnitude of a quantity is found by taking the absolute value of its numerical part

Exercises 4–8 (6 minutes)

Exercises 4–8

For each scenario below, use absolute value to determine the magnitude of each quantity.

MP.6

4. **Maria was sick with the flu and her weight change as a result of it is represented by -4 pounds. How much weight did Maria lose?**

   $|-4| = 4$ *Maria lost 4 pounds.*

5. **Jeffrey owes his friend \$5. How much is Jeffrey's debt?**

   $|-5| = 5$ *Jeffrey has a \$5 debt.*

6. **The elevation of Niagara Falls, which is located between Lake Erie and Lake Ontario, is 326 feet. How far is this above sea level?**

   $|326| = 326$ *It is 326 feet above sea level.*

7. **How far below zero is -16 degrees Celsius?**

   $|-16| = 16$ *$-16°C$ is 16 degrees below zero.*

8. **Frank received a monthly statement for his college savings account. It listed a deposit of \$100 as $+100.00$. It listed a withdrawal of \$25 as -25.00. The statement showed an overall ending balance of \$835.50. How much money did Frank add to his account that month? How much did he take out? What is the total amount Frank has saved for college?**

   $|100| = 100$ *Frank added \$100 to his account.*

   $|-25| = 25$ *Frank took \$25 out of his account.*

   $|835.50| = 835.50$ *The total amount of Frank's savings for college is \$835.50.*

Exercises 9–19 (13 minutes)

Students work independently for 8–10 minutes. Allow 3–5 minutes to go over the answers as a whole group.

Exercises 9–19

9. **Meg is playing a card game with her friend Iona. The cards have positive and negative numbers printed on them. Meg exclaims: "The absolute value of the number on my card equals 8!" What is the number on Meg's card?**

   $|-8| = 8$ or $|8| = 8$ Meg either has an 8 or a -8 on her card.

10. **List a positive and negative number whose absolute value is greater than 3. Explain how to justify your answer using the number line.**

    Answers may vary. $|-4| = 4$ and $|7| = 7$; $4 > 3$ and $7 > 3$. On a number line the distance from zero to -4 is 4 units. So the absolute value of -4 is 4. The number 4 is to the right of 3 on the number line, so 4 is greater than 3. The distance from zero to 7 on a number line is 7 units, so the absolute value of 7 is 7. Since 7 is to the right of 3 on the number line, 7 is greater than 3.

11. **Which of the following situations can be represented by the absolute value of 10? Check all that apply.**

    ______ **The temperature is 10 degrees below zero. Express this as an integer.**

    __X__ **Determine the size of Harold's debt if he owes $\$10$.**

    __X__ **Determine how far -10 is from zero on a number line.**

    __X__ **10 degrees is how many degrees above zero?**

12. **Julia used absolute value to find the distance between 0 and 6 on a number line. She then wrote a similar statement to represent the distance between 0 and -6. Below is her work. Is it correct? Explain.**

    $$|6| = 6 \text{ and } |-6| = -6$$

    No. The distance is 6 units whether you go from 0 to 6 or 0 to -6. So the absolute value of -6 should also be 6, but Julia said it was -6.

13. **Use absolute value to represent the amount, in dollars, of a $\$238.25$ profit.**

    $|238.25| = 238.25$

14. **Judy lost 15 pounds. Use absolute value to represent the number of pounds Judy lost.**

    $|-15| = 15$

15. **In math class, Carl and Angela are debating about integers and absolute value. Carl said two integers can have the same absolute value and Angela said one integer can have two absolute values. Who is right? Defend your answer.**

    Carl is right. An integer and its opposite are the same distance from zero. So, they have the same absolute values because absolute value is the distance between the number and zero.

16. **Jamie told his math teacher: "Give me any absolute value, and I can tell you two numbers that have that absolute value." Is Jamie correct? For any given absolute value, will there always be two numbers that have that absolute value?**

    No, Jamie is not correct because zero is its own opposite. Only one number has an absolute value of 0, and that would be 0.

17. **Use a number line to show why a number and its opposite have the same absolute value.**

    A number and its opposite are the same distance from zero, but on opposite sides. An example is 5 and -5. These numbers are both 5 units from zero. Their distance is the same, so they have the same absolute value, 5.

18. **A bank teller assisted two customers with transactions. One customer made a $\$25.00$ withdrawal from a savings account. The other customer made a $\$15$ deposit. Use absolute value to show the size of each transaction. Which transaction involved more money?**

    $|-25| = 25$ ***and*** $|15| = 15$ ***The $\$25$ withdrawal involved more money.***

19. **Which is farther from zero: $-7\frac{3}{4}$ or $7\frac{1}{2}$? Use absolute value to defend your answer.**

    The number that is farther from 0 is $-7\frac{3}{4}$. This is because $|-7\frac{3}{4}| = 7\frac{3}{4}$ ***and*** $|7\frac{1}{2}| = 7\frac{1}{2}$. ***Absolute value is a number's distance from zero. I compared the absolute value of each number to determine which was farther from zero. The absolute value of $-7\frac{3}{4}$ is $7\frac{3}{4}$. The absolute value of $7\frac{1}{2}$ is $7\frac{1}{2}$. We know that $7\frac{3}{4}$ is greater than $7\frac{1}{2}$.***

Closing (3 minutes)

- I am thinking of two numbers. Both numbers have the same absolute value. What must be true about the two numbers?
  - *The numbers are opposites.*
- Can the absolute value of a number ever be a negative number? Why or why not?
  - *No. Absolute value is the distance a number is from zero. If you count the number of units from zero to the number, the number of units is its absolute value. You could be on the right or left side of zero, but the number of units you count represents the distance or absolute value, and that will always be a positive number.*
- How can we use absolute value to determine magnitude? For instance, how far below zero is -8 degrees?
  - *Absolute value represents magnitude. This means that -8 degrees is 8 units below zero.*

Exit Ticket (6 minutes)

Name ____ Date____________________

Lesson 11: Absolute Value—Magnitude and Distance

Exit Ticket

1. Jessie and his family drove up to a picnic area on a mountain. In the morning, they followed a trail that led to the mountain summit, which was 2,000 feet above the picnic area. They then returned to the picnic area for lunch. After lunch, they hiked on a trail that led to the mountain overlook, which was 3,500 feet below the picnic area.

   a. Locate and label the elevation of the mountain summit and mountain overlook on a vertical number line. The picnic area represents zero. Write a rational number to represent each location.

   picnic area: ___0___

   mountain summit: _________

   mountain overlook: _________

   b. Use absolute value to represent the distance on the number line of each location from the picnic area.

   Distance from the picnic area to the mountain summit: ___________

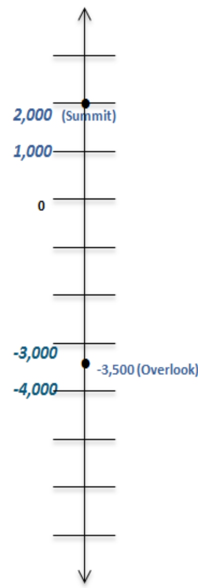
   Distance from the picnic area to the mountain overlook: ___________

   c. What is the distance between the elevations of the summit and overlook? Use absolute value and your number line from part (a) to explain your answer.

Exit Ticket Sample Solutions

1. Jessie and his family drove up to a picnic area on a mountain. In the morning, they followed a trail that led to the mountain summit, which was $2,000$ feet above the picnic area. They then returned to the picnic area for lunch. After lunch, they hiked on a trail that led to the mountain overlook, which was $3,500$ feet below the picnic area.

   a. Locate and label the elevation of the mountain summit and mountain overlook on a vertical number line. The picnic area represents zero. Write a rational number to represent each location.

   Picnic Area: 0

   Mountain Summit: *$2,000$*

   Mountain Overlook: *$-3,500$*

   b. Use absolute value to represent the distance on the number line of each location from the picnic area.

   Distance from the picnic area to the mountain summit: *$|2,000| = 2,000$*

   Distance from the picnic area to the mountain overlook: *$|-3,500| = 3,500$*

   c. What is the distance between the elevations of the summit and overlook? Use absolute value and your number line from Part a to explain your answer

   Summit to picnic area and picnic area to overlook: $2,000 + 3,500 = 5,500$ $5,500$ feet

   There are $2,000$ units from zero to $2,000$ on the number line.

   There are $3,500$ units from zero to $-3,500$ on the number line.

   Altogether that equals $5,500$ units, which represents the distance on the number line between the two elevations. So the difference in elevations is $5,500$ feet.

Problem Set Sample Solutions

For each of the following two quantities, which has the greater magnitude? (Use absolute value to defend your answers.)

1. 33 dollars and -52 dollars

   $|-52| = 52$ $|33| = 33$ *$52 > 33$, so -52 dollars has the greater magnitude.*

2. -14 feet and 23 feet

   $|-14| = 14$ $|23| = 23$ *$14 < 23$, so 23 feet has the greater magnitude.*

3. -24.6 pounds and -24.58 pounds

   $|-24.6| = 24.6$ $|-24.58| = 24.58$ *$24.6 > 24.58$, so -24.6 pounds has the greater magnitude.*

4. $-11\frac{1}{4}$ degrees and 11 degrees

$|-11\frac{1}{4}| = 11\frac{1}{4}$ $|-11| = 11$ $11\frac{1}{4} > 11$, so $-11\frac{1}{4}$ degrees has the greater magnitude.

For questions 5–7, answer true or false. If false, explain why.

5. The absolute value of a negative number will always be a positive number.

   True.

6. The absolute value of any number will always be a positive number.

   False. Zero is the exception since the absolute value of zero is zero, and zero is not positive.

7. Positive numbers will always have a higher absolute value than negative numbers.

   False. A number and it opposite have the same absolute value.

8. Write a word problem whose solution is: $|20| = 20$.

   Answers will vary.

9. Write a word problem whose solution is: $|-70| = 70$.

   Answers will vary.

10. Look at the bank account transactions listed below and determine which has the greatest impact on the account balance. Explain.

    i. A withdrawal of $\$60$.

    ii. A deposit of $\$55$.

    iii. A withdrawal of $\$58.50$.

    $|-60| = 60$ $|55| = 55$ $|-58.50| = 58.50$

    $60 > 58.50 > 55$, so a withdrawal of $\$60$ has the greatest impact on the account balance.