Lesson 10: Writing and Interpreting Inequality Statements Involving Rational Numbers

Student Outcomes

- Students write and explain inequality statements involving rational numbers.
- Students justify inequality statements involving rational numbers.

Lesson Notes

Lessons 6, 7, 8, and 9 prepare students for this lesson. Students have developed an understanding of how to represent, order, and compare rational numbers, and will now write and interpret inequality statements involving rational numbers. In Grade 5, Module 1, students write inequality statements involving decimals, and that experience serves as a foundation for this lesson as well.

Classwork

Opening Exercise (3 minutes)

As students enter the room, the following question is posted on the board (and is in the student materials).

"The amount of money I have in my pocket is less than $5 but greater than $4."

Direct students to discuss with their group or elbow partner a possible value for the amount of money in your pocket, and then complete the following:

Scaffolding:

Use play money (including bills and coins) to represent $4 and $5. Students then create representations involving $4 and some change, so as not to exceed $5.

MP.1

Opening Exercises

"The amount of money I have in my pocket is less than $5 but greater than $4."

1. **One possible value for the amount of money in my pocket is:** $4.75

2. **Write an inequality statement comparing the possible value of the money in your pocket to $4.**

   $4.00 < 4.75$

3. **Write an inequality statement comparing the possible value of the money in your pocket to $5.**

   $4.75 < 5.00$

Discussion (5 minutes)

Allow time for students to share with the class their answers from part (a) of the opening activity.

Discuss answers that *would and would not* fall between 4 and 5. Elicit student responses. Discuss potential answers that *would or would not make sense* in the context of the situation (e.g., 4.308 cannot be represented with physical money).

- What are some possible values for the amount of money in my pocket?

Have several students write their inequality statements on the board from parts (b) and (c). Select a student's answer (or create another example of a correct answer) to use as a model. Write the answers to parts (b) and (c) side by side as shown below.

  - $4.00 < 4.75$ *and* $4.75 < 5.00$
- Are there any integer solutions? Why or why not?
- Are there any numbers between 4 and 5 that are not possible values for the amount of money in my pocket? Why or why not?
- Could *any number* between 4 and 5 be a solution?
- Is there a way to write one inequality statement that describes both of these relationships?

Exercises 1–4 (4 minutes)

Students use a number line model to represent the order of the numbers used in their Opening Exercise (or in the example just discussed as a whole group). Students then graph three points: 4, 5, and a value that falls in between 4 and 5. Students should use the model and number line ordering to write one inequality statement relating the three numbers.

Exercises 1–4

1. **Graph your answer from the Opening Exercise, part (a) on the number line below.**

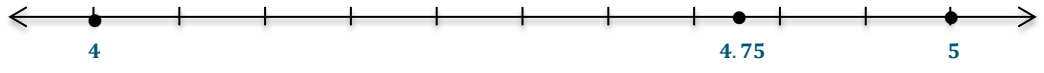
2. **Also graph the points associated with 4 and 5 on the number line.**

3. **Explain in words how the location of the three numbers on the number line supports the inequality statements you wrote in parts (b) and (c).**

   The numbers go from least to greatest when I look at the number line from left to right. So 4 is less than 4.75 and 4.75 is less than 5.

4. **Write one inequality statement that shows the relationship among all three numbers.**

   $4 < 4.75 < 5$



If students struggle with Exercise 4, spend adequate time as a whole group doing several examples of ordering students' sets of numbers using one statement of inequality.

The following two examples should be conducted as whole-group instruction.

Example 1 (4 minutes): Writing Inequality Statements Involving Rational Numbers

Students should recall using inequality symbols to compare decimal numbers from Grade 5, Module 1. As needed, refer to the number line representation of non-integer rational numbers and their opposites from Lesson 6 of this module, and revisit the orientation of the less than symbol (<) and greater than symbol (>). Note that students may erroneously write $8 < 10\frac{1}{2} > 9$, which does not accurately describe the order of all three numbers.

Example 1: Writing Inequality Statements Involving Rational Numbers

Write one inequality statement to show the relationship between the following shoe sizes: $10\frac{1}{2}$, 8, and 9.

a. From Least to Greatest:

$8 < 9 < 10\frac{1}{2}$

b. From Greatest to Least:

$10\frac{1}{2} > 9 > 8$

Example 2 (4 minutes): Interpreting Data and Writing Inequality Statements

Example 2: Interpreting Data and Writing Inequality Statements

Mary is comparing the rainfall totals for May, June and July. The data is reflected in the table below. Fill in the blanks below to create inequality statements that compare the *Changes in Total Rainfall* for each month (the right-most column of the table).

Month	This Year's Total Rainfall (in inches)	Last Year's Total Rainfall (in inches)	Change in Total Rainfall from last year to this year (in inches)
May	2.3	3.7	-1.4
June	3.8	3.5	0.3
July	3.7	3.2	0.5

Order the Changes in Total Rainfall: $-1.4 < 0.3 < 0.5$ or $0.5 > 0.3 > -1.4$

From Least to Greatest **From Greatest to Least**

In this case, does the greatest number indicate the greatest change in rainfall? Explain.

No, in this situation, the greatest change is for the month of May since the average total rainfall went down from last year by 1.4 inches, but the greatest number in the inequality statement is 0.5.

Exercises 5–8 (8 minutes)

Students work independently to answer the following questions. Allow time for students to present their answers and share their thought process to the class. Use the following as an optional task: Have students transfer their word problems for Exercise 8 onto paper using colorful markers or colored pencils and display them in the classroom.

Exercises 5–8

5. **Mark's favorite football team lost yards on two plays back-to-back. They lost 3 yards on the first play. They lost 1 yard on the second play. Write an inequality statement using integers to compare the forward progress made on each play.**

   $-3 < -1$

6. **Sierra had to pay the school for two textbooks that she lost. One textbook costs $55 and the other cost $75. Her mother wrote two separate checks for each expense. Write two integers that represent the change to her mother's checking account balance. Then write an inequality statement that shows the relationship between these two numbers.**

   -55 *and* -75; $-55 > -75$

7. **Jason ordered the numbers, -70, -18, and -18.5, from least to greatest by writing the following statement: $-18 < -18.5 < -70$. Is this a true statement? Explain.**

   No, it is not a true statement because $18 < 18.5 < 70$, so the opposites of these numbers are in the opposite order. The order should be: $-70 < -18.5 < -18$.

8. **Write a real-world situation that is represented by the following inequality: $-19 < 40$. Explain the position of the numbers on a number line.**

   The coldest temperature in January was -19 degrees and the warmest temperature was 40 degrees. Since the point associated with 40 is above zero on a vertical number line and -19 is below zero, we know that 40 is greater than -19. This means that 40 degree is warmer than -19 degrees.

Exercises 9–10 (5 minutes): Fluency Builder – Writing Inequalities

Photocopy the attached 2-page fluency-building exercises, so that each student receives a copy. Time the students; allowing *one* minute to complete Side A. Before students begin, inform them that they may not skip over questions, and that they must move in order. *After all students complete the activity, discuss the* answers. Before administering Side B, elicit strategies from those students who were able to accurately complete many problems on Side A. Administer Side B in the same fashion, and review the answers.

Exercise 11 (4 minutes): A Closer Look at the Fluency Builder

MP.7 Students are asked to look closely at two related examples from the Fluency Builder (Exercises 9 and 10), and explain the relationship between the numbers' order, the inequality symbols, and the graphs of the numbers on the number line.

11. Look at the following two examples from the fluency builder.

a. Fill in the numbers in the correct order.

☐ < ☐ < ☐   -¼ , -1, 0
☐ > ☐ > ☐   -¼ , -1, 0

$-1 < -\frac{1}{4} < 0$ *and* $0 > -\frac{1}{4} > -1$

b. Explain how the position of the numbers on the number line supports the inequality statements you created.

-1 is the farthest left on the number line, so it is the least; 0 is farthest right, so it is the greatest value; and $-\frac{1}{4}$ is in between.

c. Create a new pair of *greater than* and *less than* inequality statements using three other rational numbers.

$8 > 0 > -1$ *and* $-1 < 0 < 8$

Closing (3 minutes)

- What can you do before writing an inequality statement involving three numbers that makes it easier to write the inequality statement? For example, explain the process for writing one inequality statement comparing -3, 8, and -10.
  - *First, I would order the numbers, either from least to greatest or greatest to least.*
- If you know the order of a set of numbers, how can you represent the order using inequality symbols?
  - *For example, if the numbers are 1, 2, and 3, you can either write: $1 < 2 < 3$ or $3 > 2 > 1$*
- If two negative numbers are ordered using the $<$ symbol, what must be true about their positions on a horizontal number line? On a vertical number line?
  - *The first number must be associated with a point to the left of the second number on a horizontal number line. The first number must be associated with below the second number on a vertical number line.*

Exit Ticket (5 minutes)

Name _____ Date_____

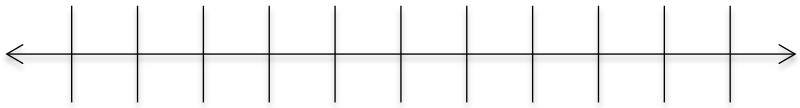
Lesson 10: Writing and Interpreting Inequality Statements Involving Rational Numbers

Exit Ticket

1. Kendra collected data for her Science project. She surveyed people asking them how many hours they sleep during a typical night. The chart below shows how each person's response compares to 8 hours (which is the answer she expected most people to say).

Name	Number of Hours (usually slept each night)	Compared to 8 hours
Frankie	8.5	0.5
Mr. Fields	7	-1.0
Karla	9.5	1.5
Louis	8	0
Tiffany	$7\frac{3}{4}$	$-\frac{1}{4}$

a. Plot and label each of the numbers in the right-most column of the table above on the number line below.



b. List the numbers from least to greatest.

c. Using your answer from part (b) and inequality symbols, write one statement that shows the relationship among all of the numbers.

Exit Ticket Sample Solutions

1. Kendra collected data for her science project. She surveyed people asking them how many hours they sleep during a typical night. The chart below shows how each person's response compares to 8 hours (which is the answer she expected most people to say).

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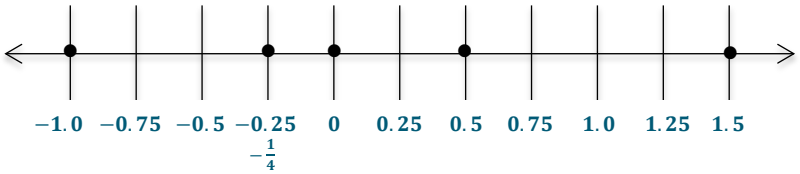

b. List the numbers from least to greatest.

$-1.0, -\frac{1}{4}, 0, 0.5, 1.5$

c. Using your answer from part (b) and inequality symbols, write one statement that shows the relationship among all the numbers.

$$-1.0 < -\frac{1}{4} < 0 < 0.5 < 1.5 \quad \textit{or} \quad 1.5 > 0.5 > 0 > -\frac{1}{4} > -1.0$$

Problem Set Sample Solutions

For each of the relationships described below, write an inequality that relates the rational numbers.

1. Seven feet below sea level is farther below sea level than $4\frac{1}{2}$ feet below sea level. $-7 < -4\frac{1}{2}$

2. Sixteen degrees Celsius is warmer than zero degrees Celsius. $16 > 0$

3. Three and one-half yards of fabric is less than five and one-half yards of fabric. $3\frac{1}{2} < 5\frac{1}{2}$

4. A loss of $\$500$ in the stock market is worse than a gain of $\$200$ in the stock market. $-500 < 200$

5. A test score of 64 is worse than a test score of 65, and a test score of 65 is worse than a test score of $67\frac{1}{2}$.

   $64 < 65 < 67\frac{1}{2}$

6. In December the total snowfall was 13.2 inches, which is more than the total snowfall in October and November, which was 3.7 inches and 6.15 inches, respectively.

   $13.2 > 6.15 > 3.7$

For each of the following use the information given by the inequality to describe the relative position of the numbers on a horizontal number line.

7. $-0.2 < -0.1$

   -0.2 is to the left of -0.1 or -0.1 is to the right of -0.2.

8. $8\frac{1}{4} > -8\frac{1}{4}$

   $8\frac{1}{4}$ is to the right of $-8\frac{1}{4}$ or $-8\frac{1}{4}$ is to the left of $8\frac{1}{4}$.

9. $-2 < 0 < 5$

   -2 is to the left of zero and zero is to the left of 5 or 5 is to the right of zero and zero is to the right of -2.

10. $-99 > -100$

    -99 is to the right of -100 or -100 is to the left of -99.

11. $-7.6 < -7\frac{1}{2} < -7$

    -7.6 is to the left of $-7\frac{1}{2}$ and $-7\frac{1}{2}$ is to the left of -7 or -7 is to the right of $-7\frac{1}{2}$ and $-7\frac{1}{2}$ is to the right of -7.6.

Fill in the blanks with numbers that correctly complete each of the inequalities statements.

12. Three integers between -4 and 0. $-3 < -2 < -1$

13. Three rational numbers between 16 and 15. $15.7 > 15.6 > 15.3$ *Other answers are possible.*

14. Three rational numbers between -1 and -2. $-1.9 < -1.55 < -1.02$ *Other answers are possible.*

15. Three integers between 2 and -2. $1 > 0 > -1$

Exercise 9: Fluency Builder Rational Number Inequality Statements - Side A

Work in numerical order to answer #1-33. Arrange each set of numbers in order according to the inequality symbols.

1. □ < □ < □ 1 , −1 , 0	12. □ > □ > □ 7 , −6 , 6	23. □ > □ > □ 25 , ¾ , − ¾
2. □ > □ > □ 1 , −1 , 0	13. □ > □ > □ 17 , 4 , 16	24. □ < □ < □ 25 , ¾ , − ¾
3. □ < □ < □ 3 ½ , −3 ½ , 0	14. □ < □ < □ 17 , 4 , 16	25. □ > □ > □ 2.2 , 2.3 , 2.4
4. □ > □ > □ 3 ½ , −3 ½ , 0	15. □ < □ < □ 0 , 12 , −11	26. □ > □ > □ 1.2 , 1.3 , 1.4
5. □ > □ > □ 1 , − ½ , ½	16. □ > □ > □ 0 , 12 , −11	27. □ > □ > □ 0.2 , 0.3 , 0.4
6. □ < □ < □ 1 , − ½ , ½	17. □ > □ > □ 1 , ¼ , ½	28. □ > □ > □ −0.5 , − 1 , − 0.6
7. □ < □ < □ −3 , −4 , −5	18. □ < □ < □ 1 , ¼ , ½	29. □ < □ < □ −0.5 , −1 , −0.6
8. □ < □ < □ −13 , −14 , −15	19. □ < □ < □ − ½ , ½ , 0	30. □ < □ < □ −8 , −9 , 8
9. □ > □ > □ −13 , −14 , −15	20. □ > □ > □ −½ , ½ , 0	31. □ < □ < □ −18 , −19 , −2
10. □ < □ < □ − ¼ , −1 , 0	21. □ < □ < □ 50 , −10 , 0	32. □ > □ > □ −2 , − 3 , 1
11. □ > □ > □ −¼ , −1 , 0	22. □ < □ < □ −50 , 10 , 0	33. □ < □ < □ −2 , − 3 , 1

Exercise 10: Fluency Builder Rational Number Inequality Statements - Side B

Work in numerical order to answer #1-33. Arrange each set of numbers in order according to the inequality symbols.

1. $\square < \square < \square$ $1/7, -1/7, 0$	12. $\square > \square > \square$ $1\frac{1}{4}, 1, 1\frac{1}{2}$	23. $\square > \square > \square$ $1, 1\frac{3}{4}, -1\frac{3}{4}$
2. $\square > \square > \square$ $1/7, -1/7, 0$	13. $\square > \square > \square$ $11\frac{1}{4}, 11, 11\frac{1}{2}$	24. $\square < \square < \square$ $1, 1\frac{3}{4}, -1\frac{3}{4}$
3. $\square < \square < \square$ $3/7, 2/7, -1/7$	14. $\square < \square < \square$ $11\frac{1}{4}, 11, 11\frac{1}{2}$	25. $\square > \square > \square$ $-82, -93, -104$
4. $\square > \square > \square$ $3/7, 2/7, -1/7$	15. $\square < \square < \square$ $0, 0.2, -0.1$	26. $\square < \square < \square$ $-82, -93, -104$
5. $\square > \square > \square$ $-4/5, 1/5, -1/5$	16. $\square > \square > \square$ $0, 0.2, -0.1$	27. $\square > \square > \square$ $0.5, 1, 0.6$
6. $\square < \square < \square$ $-4/5, 1/5, -1/5$	17. $\square > \square > \square$ $1, 0.7, 1/10$	28. $\square > \square > \square$ $-0.5, -1, -0.6$
7. $\square < \square < \square$ $-8/9, 5/9, 1/9$	18. $\square < \square < \square$ $1, 0.7, 1/10$	29. $\square < \square < \square$ $-0.5, -1, -0.6$
8. $\square > \square > \square$ $-8/9, 5/9, 1/9$	19. $\square < \square < \square$ $0, -12, -12\frac{1}{2}$	30. $\square < \square < \square$ $1, 8, 9$
9. $\square > \square > \square$ $-30, -10, -50$	20. $\square > \square > \square$ $0, -12, \quad -12\frac{1}{2}$	31. $\square < \square < \square$ $-1, -8, -9$
10. $\square < \square < \square$ $-30, -10, -50$	21. $\square < \square < \square$ $5, -1, 0$	32. $\square > \square > \square$ $-2, -3, -5$
11. $\square > \square > \square$ $-40, -20, -60$	22. $\square < \square < \square$ $-5, 1, 0$	33. $\square > \square > \square$ $2, 3, 5$