



Lesson 16: Symmetry in the Coordinate Plane

Student Outcomes

- Students understand that two numbers are said to differ only by signs if they are opposite of each other.
- Students recognize that when two ordered pairs differ only by sign of one or both of the coordinates, then the locations of the points are related by reflections across one or both axes.

Classwork

Opening Exercise (3 minutes)

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Give an example of two opposite numbers and describe where the numbers lie on the number line. How are opposite numbers similar and how are they different?

Example 1 (14 minutes): Extending Opposite Numbers to the Coordinate Plane

Students locate and label points whose ordered pairs differ only by the sign of one or both coordinates. Together, students and their teacher examine the relationships of the points on the coordinate plane, and express these relationships in a graphic organizer.

- Locate and label the points $(3,4)$ and $(-3,4)$.
- Record observations in the first column of the graphic organizer.

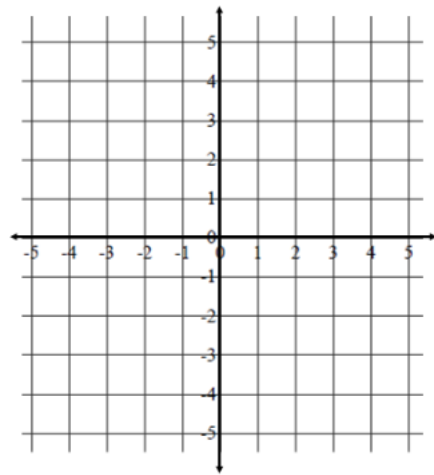
The first column of the graphic organizer is teacher-led so that students can pay particular attention to the absolute values of coordinates and the general locations of the corresponding points with regard to each axis. Following this lead, columns 2 and 3 are more student-led.

- Locate and label the point $(3,-4)$.
- Record observations in the second column of the graphic organizer.
- Locate and label the point $(-3,-4)$.
- Record observations in the third column of the graphic organizer.

MP.8

Extending Opposite Numbers to the Coordinates of Points on the Coordinate Plane

Locate and label your points on the coordinate plane to the right. For each given pair of points in the table below, record your observations and conjectures in the appropriate cell. Pay attention to the absolute values of the coordinates and where the points lie in reference to each axis.



	(3, 4) and (−3, 4)	(3, 4) and (3, −4)	(3, 4) and (−3, −4)
Similarities of Coordinates	<i>Same y-coordinates. The x-coordinates have the same absolute value.</i>	<i>Same x-coordinates. The y-coordinates have the same absolute value.</i>	<i>The x-coordinates have the same absolute value. The y-coordinates have the same absolute value.</i>
Differences of Coordinates	<i>The x-coordinates are opposite numbers.</i>	<i>The y-coordinates are opposite numbers.</i>	<i>Both the x- and y-coordinates are opposite numbers.</i>
Similarities in Location	<i>Both points are 4 units above the x-axis; and 3 units away from the y-axis.</i>	<i>Both points are 3 units to the right of the y-axis; and 4 units away from the x-axis.</i>	<i>Both points are 3 units from the y-axis; and 4 units from the x-axis.</i>
Differences in Location	<i>One point is 3 units to the right of the y-axis; the other is 3 units to the left of the y-axis.</i>	<i>One point is 4 units above the x-axis; the other is 4 units below.</i>	<i>One point is 3 units right of the x-axis; the other is 3 units left. One point is 4 units above the y-axis; the other is 4 units below.</i>
Relationship between Coordinates and Location on the Plane	<i>The x-coordinates are opposite numbers so the points lie on opposite sides of the y-axis. Because opposites have the same absolute value, both points lie the same distance from the y-axis. The points lie the same distance above the x-axis, so the points are symmetric about the y-axis. A reflection across the y-axis takes one point to the other.</i>	<i>The y-coordinates are opposite numbers so the points lie on opposite sides of the x-axis. Because opposites have the same absolute value, both points lie the same distance from the x-axis. The points lie the same distance right of the y-axis, so the points are symmetric about the x-axis. A reflection across the x-axis takes one point to the other.</i>	<i>The points have opposite numbers for x- and y-coordinates, so the points must lie on opposite sides of each axis. Because the numbers are opposites, and opposites have the same absolute values each point must be the same distance from each axis. A reflection across one axis followed by the other will take one point to the other.</i>

Exercise (5 minutes)

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In each column, write the coordinates of the points that are related to the given point by the criteria listed in the first column of the table. Point $S(5, 3)$ has been reflected over the x - and y -axes for you as a guide and its images are shown on the coordinate plane. Use the coordinate grid to help you locate each point and its corresponding coordinates.

Given Point:	$S(5, 3)$	$(-2, 4)$	$(3, -2)$	$(-1, -5)$
Reflected across the x -axis.	$M(5, -3)$	$(-2, -4)$	$(3, 2)$	$(-1, 5)$
Reflected across the y -axis.	$L(-5, 3)$	$(2, 4)$	$(-3, -2)$	$(1, -5)$
Reflected first across the x -axis then across the y -axis.	$A(-5, -3)$	$(2, -4)$	$(-3, 2)$	$(1, 5)$
Reflected first across the y -axis then across the x -axis.	$A(-5, -3)$	$(2, -4)$	$(-3, 2)$	$(1, 5)$

a. When the coordinates of two points are (x, y) and $(-x, y)$, what line of symmetry do the points share? Explain.
They share the y -axis, because the y -coordinates are the same and the x -coordinates are opposites, which means the points will be the same distance from the y -axis, but on opposite sides.

b. When the coordinates of two points are (x, y) and $(x, -y)$, what line of symmetry do the points share? Explain.
They share the x -axis, because the x -coordinates are the same and the y -coordinates are opposites, which means the points will be the same distance from the x -axis but on opposite sides.

Example 2 (8 minutes): Navigating the Coordinate Plane using Reflections

Have students use a pencil eraser or their finger to navigate the coordinate plane given verbal prompts. Then circulate the room during the example to assess students' understanding and provide assistance as needed.

- Begin at $(7, 2)$. Move 3 units down, then reflect over the y -axis. Where are you?
 - $(-7, -1)$
- Begin at $(4, -5)$. Reflect over the x -axis, then move 7 units down, then to the right 2 units. Where are you?
 - $(6, -2)$



- Begin at $(-3, 0)$. Reflect over the x -axis then move 6 units to the right. Move up two units, then reflect over the x -axis again. Where are you?
 - $(3, -2)$
- Begin at $(-2, 8)$. Decrease the y -coordinate by 6. Reflect over the y -axis, then move down 3 units. Where are you?
 - $(2, -1)$
- Begin at $(5, -1)$. Reflect over the x -axis, then reflect over the y -axis. Where are you?
 - $(-5, 1)$

Example 3 (7 minutes): Describing How to Navigate the Coordinate Plane

Given a starting point and an ending point, students describe a sequence of directions using at least one reflection about an axis to navigate from the starting point to the ending point. Once students have found a sequence, have them find another sequence while their classmates finish the task.

- Begin at $(9, -3)$ and end at $(-4, -3)$. Use exactly one reflection.
 - *Possible Answer: Reflect over the y -axis then move 5 units to the right.*
- Begin at $(0, 0)$ and end at $(5, -1)$. Use exactly one reflection.
 - *Possible Answer: Move 5 units right, 1 unit up, then reflect over the x -axis.*
- Begin at $(0, 0)$ and end at $(-1, -6)$. Use exactly two reflections.
 - *Possible Answer: Move right 1 unit, reflect over the y -axis, up 6 units, then reflect over the x -axis.*

Closing (4 minutes)

- When the coordinates of two points differ only by one sign, such as $(-8, 2)$ and $(8, 2)$, what do the similarities and differences in the coordinates tell us about their relative locations on the plane?
- What is the relationship between $(5, 1)$ and $(5, -1)$? Given one point, how can you locate the other?

Exit Ticket (4 minutes)

Exit Ticket Sample Solutions

- How are the ordered pairs $(4, 9)$ and $(4, -9)$ similar and how are they different? Are the two points related by a reflection over an axis in the coordinate plane? If so, indicate which axis is the line of symmetry between the points. If they are not related by a reflection over an axis in the coordinate plane, explain how you know?

The x-coordinates are the same, but the y-coordinates are opposites, meaning they are the same distance from zero on the x-axis, and the same distance but opposite sides of zero on the y-axis. Reflecting about the x-axis interchanges these two points.

- Given the point $(-5, 2)$, write the coordinates of a point that is related by a reflection over the x- or y-axis. Specify which axis is the line of symmetry.

Using the x-axis as a line of symmetry, $(-5, -2)$; using the y-axis as a line of symmetry, $(5, 2)$.

Problem Set Sample Solutions

- Locate a point in Quadrant IV of the coordinate plane. Label the point A and write its ordered pair next to it.

Answers will vary; Quadrant IV $(5, -3)$;

- Reflect point A over an axis so that its image is in Quadrant III. Label the image B and write its ordered pair next to it. Which axis did you reflect over? What is the only difference in the ordered pairs of points A and B?

$B(-5, -3)$; Reflected over the y-axis.

The ordered pairs differ only by the sign of their x-coordinates: $A(5, -3)$ and $B(-5, -3)$.

- Reflect point B over an axis so that its image is in Quadrant II. Label the image C and write its ordered pair next to it. Which axis did you reflect over? What is the only difference in the ordered pairs of points B and C? How does the ordered pair of point C relate to the ordered pair of point A?

$C(-5, 3)$; Reflected over the x-axis.

The ordered pairs differ only by the sign of their y-coordinates: $B(-5, -3)$ and $C(-5, 3)$.

The ordered pair for point C differs from the ordered pair for point A by the signs of both coordinates: $A(5, -3)$ and $C(-5, 3)$.

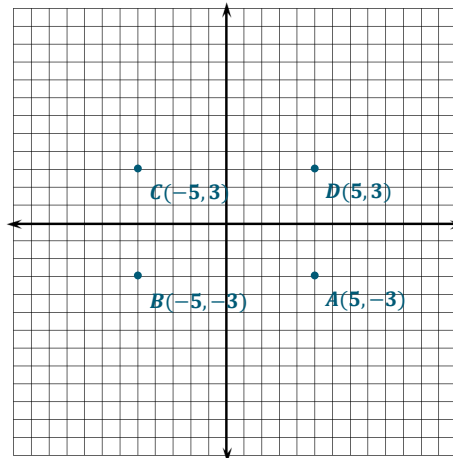
- Reflect point C over an axis so that its image is in Quadrant I. Label the image D and write its ordered pair next to it. Which axis did you reflect over? How does the ordered pair for point D compare to the ordered pair for point C? How does the ordered pair for point D compare to points A and B?

$D(5, 3)$; Reflected over the y-axis again.

Point D differs from point C by only the sign of its x-coordinate: $D(5, 3)$ and $C(-5, 3)$.

Point D differs from point B by the signs of both coordinates: $D(5, 3)$ and $B(-5, -3)$.

Point D differs from point A by only the sign of the y-coordinate: $D(5, 3)$ and $A(5, -3)$.





2. Bobbie listened to her teacher's directions and navigated from the point $(-1, 0)$ to $(5, -3)$. She knows that she has the correct answer but, she forgot part of the teacher's directions. Her teacher's directions included the following:

"Move 7 units down, reflect about the ?-axis, move up 4 units, then move right 4 units."

Help Bobbie determine the missing axis in the directions, and explain your answer.

The missing line is a reflection over the y -axis. The first line would move the location to $(-1, -7)$. A reflection over the y -axis would move the location to $(1, -7)$ in Quadrant IV, which is 4 units left and 4 units down from the end point $(5, -3)$.