Lesson 6: Rational Numbers on the Number Line

Student Outcomes

- Students use number lines that extend in both directions and use 0 and 1 to locate integers and rational numbers on the number line. Students know that the sign of a nonzero rational number is positive or negative, depending on whether the number is greater than zero (positive) or less than zero (negative), and use an appropriate scale when graphing rational numbers on the number line.
- Students know that the opposites of rational numbers are similar to the opposites of integers. Students know that two rational numbers have opposite signs if they are on different sides of zero, and that they have the same sign if they are on the same side of zero on the number line.

Classwork

Opening Exercises (5 minutes)

Students work independently for 5 minutes to review fractions and decimals.

Opening Exercises

1. **Write the decimal equivalent of each fraction.**
   a. $\frac{1}{2}$
   0.5
   b. $\frac{4}{5}$
   0.8
   c. $6\frac{7}{10}$
   6.70

2. **Write the fraction equivalent of each decimal.**
   a. 0.42
   $\frac{42}{100} = \frac{21}{50}$
   b. 3.75
   $3\frac{75}{100} = 3\frac{3}{4}$
   c. 36.90
   $36\frac{90}{100} = 36\frac{9}{10}$

Scaffolding:
- Use polling software to elicit immediate feedback from the class to engage all learners.
- Display each problem one at a time, and use student whiteboards for kinesthetic learners.

Scaffolding:
- Use edges of square tiles on the floor as a number line to illustrate how to connect segments of equal length for visual and kinesthetic learners.
- Provide green and red pencils to help with modeling the example for visual learners.

Example 1 (10 minutes): Graphing Rational Numbers

The purpose of this example is to show students how to graph non-integer rational numbers on a real number line. Students will complete the example by following along with the teacher.

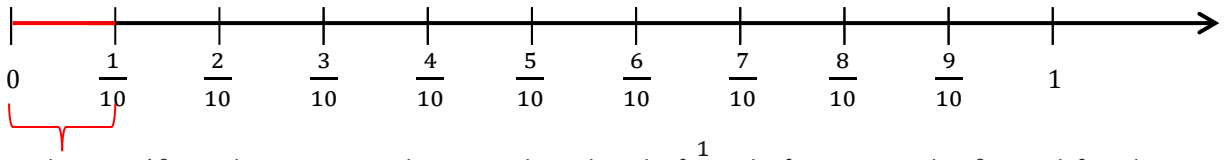
Locate and graph the number $\frac{3}{10}$ and its opposite on a number line.

- Before modeling the example, the teacher should review graphing a fraction on the number line to the whole class by first reviewing fraction definitions with respect to the number line.

> **Example 1: Graphing Rational Numbers**
>
> **If b is a nonzero whole number, then the unit fraction $\frac{1}{b}$ is located on the number line by dividing the segment between 0 and 1 into b segments of equal length. One of the b segments has 0 as its left endpoint; the right endpoint of this segment corresponds to the unit fraction $\frac{1}{b}$.**

- In this example, the denominator is 10. To locate the rational number $\frac{3}{10}$ on the number line, divide the interval from zero to one into ten equal segments.
- Since the number is a **rational number**, a number can be represented as a fraction, determine how the number line should be scaled.[1]
  - First, divide the number line into two halves to represent positive and negative numbers.
  - Next, divide the right half of the number line segment between 0 and 1 into ten segments of equal length; each segment has a length of $\frac{1}{10}$. *Students will divide their number lines into ten equal segments as shown. Check for accuracy.*

MP.4



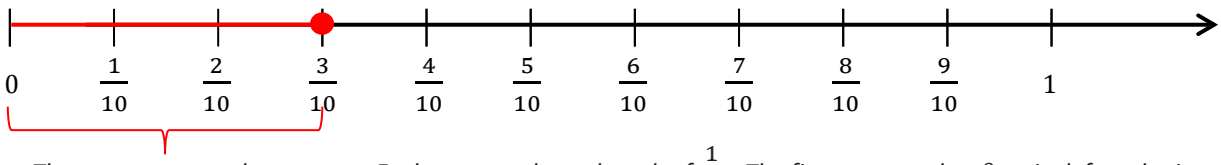
There are 10 equal segments. Each segment has a length of $\frac{1}{10}$. The first segment has 0 as its left endpoint, and the right endpoint corresponds to $\frac{1}{10}$.

[1] Activity:

- Have four students each stand in a square floor tile forming a straight line facing the class. Give each student a number to tie around his neck: $0, \frac{1}{10}, \frac{2}{10}$, or $\frac{3}{10}$. (Use index cards or construction paper.)
- Ask a fifth student to assist by giving one end of a ball of string to the person at 0. This person will hold one end of the string and pass the rest to the person to the left. (So the class sees it moving to the right.)
- As the string gets passed down the line, each person will announce his or her number, "$\frac{1}{10}, \frac{2}{10}, \frac{3}{10}$" stopping at $\frac{3}{10}$.
- The assistant will cut the string at $\frac{3}{10}$ and give that end of the string to the person holding $\frac{3}{10}$, making one segment of length $\frac{3}{10}$.
- Have students turn over their numbers to reveal their opposites and rearrange themselves to represent the opposite of $\frac{3}{10}$ using the same process. This time students will pass the string to the right of the person standing at 0.

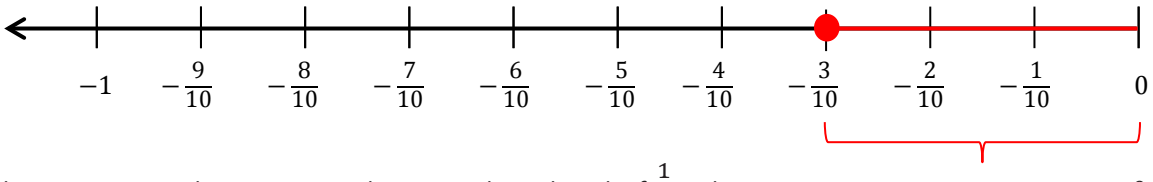
The fraction $\frac{a}{b}$ is located on the number line by joining a segments of length $\frac{1}{b}$, so that: (1) the left endpoint of the first segment is 0, and (2) the right endpoint of each segment is the left endpoint of the next segment. The right endpoint of the last segment corresponds to the fraction $\frac{a}{b}$.

- To locate the number $\frac{a}{b}$ on a number line, the students should divide the interval between zero and 1 into b equal parts. Starting at 0, move along the number line a number of times.



There are ten equal segments. Each segment has a length of $\frac{1}{10}$. The first segment has 0 as its left endpoint, and the right endpoint of the third segment corresponds to $\frac{3}{10}$. The point is located at $\frac{3}{10}$.

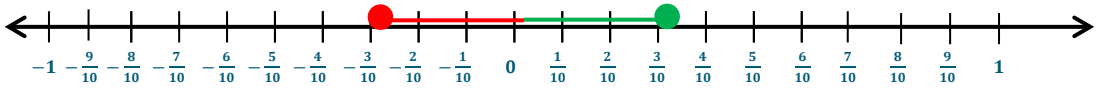
- The opposite of $\frac{3}{10}$ is located the same distance from zero as $\left(\frac{3}{10}\right)$ but in the opposite direction, or to the left.
- To locate $-\frac{3}{10}$ on the number line, divide the interval between zero and -1 into ten equal segments. Starting at zero, move to the left along the number line 3 times.



There are ten equal segments. Each segment has a length of $\frac{1}{10}$. Three consecutive segments, starting at 0 and moving to the left would have a total length of $\frac{3}{10}$. The point is located at $-\frac{3}{10}$.

- Counting three consecutive segments of length of $\frac{1}{10}$ from 0 moving to the left and taking the endpoint of the last segment corresponds to the number $-\frac{3}{10}$.

Locate and graph the number $\frac{3}{10}$ and its opposite on a number line.

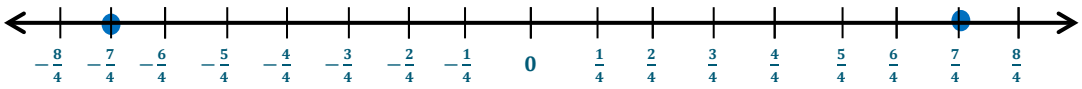


Exercise 1 (5 minutes)

Students work independently to practice graphing a non-integer rational number and its opposite on the number line. Allow 2–3 minutes for review as a whole group.

Exercise 1

Use what you know about the points $-\frac{7}{4}$ and its opposite to graph both points on the number line below. The fraction, $-\frac{7}{4}$ is located between which two consecutive integers? Explain your reasoning.



On the number line, each segment will have an equal length of $\frac{1}{4}$. In the fraction $-\frac{7}{4}$, the numerator is -7 and the denominator is 4. The fraction is located between -1 and -2.

Explanation:

$\frac{7}{4}$ is the opposite of $-\frac{7}{4}$. It is the same distance from zero but on the opposite side. Since $-\frac{7}{4}$ is to the left of zero, $\frac{7}{4}$ is to the right of zero. The fraction is located between -2, or $-\frac{8}{4}$, and -1, or $-\frac{4}{4}$.

Example 2 (7 minutes): Rational Numbers and the Real World

Display the following vertical number line model on the board. Students are to follow along in their student materials to answer the questions. Pose additional questions to the class throughout the example.

Example 2: Rational Numbers and the Real World

The water level of a lake rose 1.25 feet after it rained. Answer the questions below using the diagram below.

a. Write a rational number to represent the situation.

1.25

b. What two integers is 1.25 between on a number line?

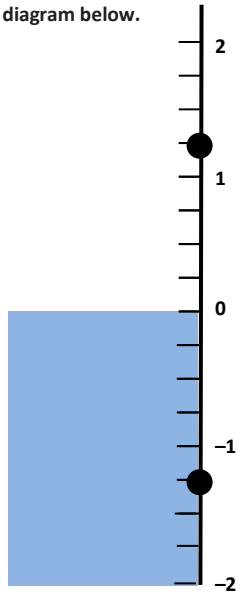
1 and 2

c. Write the length of each segment on the number line as a decimal and a fraction.

0.25 *and* $\frac{1}{4}$

d. What will be the water level after it rained? Graph the point on the number line.

1.25 *feet above the original lake level*



e. **After two weeks of rain, the water level of the lake is the opposite of the water level before it rained. What will be the new water level? Graph the point on the number line. Explain how you got your answer.**

The water level would be 1.25 feet below the original lake level. If the water level was 1.25, the opposite of 1.25 is -1.25.

f. **State a rational number that is not an integer whose value is less than 1.25, and describe its location between two consecutive integers on the number line.**

A rational number whose value is less than 1.25 is 0.75. It would be located between 0 and 1 on a number line.

Possible Discussion Questions

- What units are we using to measure the water level?
  - *Feet*
- What was the water level after the rain? How do you know?
  - *If zero represents the original water level on the number line, the water level after rain is 1.25 feet. From 0 to 1, there are four equal segments. This tells me that the scale is $\frac{1}{4}$. The top of the water is represented on the number line at one mark above 1, which represents $\frac{5}{4}$ feet or 1.25 feet.*
- What strategy could we use to determine the water level after it rained?
  - *I started at 0 and counted by $\frac{1}{4}$ for each move. I counted $\frac{1}{4}$ five times to get $\frac{5}{4}$, which is the same as $1\frac{1}{4}$, which is the same as 1.25. I know the number is positive because I moved up. Since the measurements are in feet, the answer is 1.25 feet.*
- For the fraction $\frac{5}{4}$, what is the value of *the numerator* and *denominator*?
  - *The numerator is 5 and the denominator is 4.*
- What do the negative numbers represent on the number line?
  - *They represent the number of feet below the original lake level.*

Exercise 2 (10 minutes)

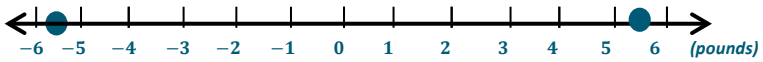
Students are seated in groups of three or four. Distribute one sheet of grid paper and a ruler to each student. Each group will complete the following tasks:

1. Write a real-world story problem using a rational number and its opposite.
2. Create a horizontal or vertical number line diagram to represent your situation:
   a. Determine an appropriate scale and label the number line.
   b. Write the units of measurement (if needed).
   c. Graph the rational number and its opposite that represent the situation.
3. Describe what points 0 and the opposite number represent on the number line.
4. Give a rational number to the left and right of the rational number you initially chose.

Scaffolding:

- Project the directions for the activity as a way for groups to make sure they are completing all task requirements.
- Have students write their story problems and draw their number lines on large wall grid-paper.
- Hang posters around the room to use as a gallery walk for students who finish their exit tickets early, or use them as review for the mid-assessment later in the module.

Our Story Problem: My mom gained 5.5 pounds last month. She went on a diet and lost the weight she gained.



- *Our Scale: 1*
- *Our Units: Pounds*
- *Description: On the number line, zero represents my mom's original weight before she lost or gained any pounds. The point -5.5 represents the change in my mom's weight. The amount lost is 5.5 pounds.*
- *Other Information: A rational number to the left of 5.5 is 4.5. A rational number to the right of 5.5 is 5.75.*

Closing (2 minutes)

- How is graphing the number $\frac{4}{3}$ on a number line similar to graphing the number 4 on a number line?
  - *When graphing each number, you start at zero and move to the right 4 units.*
- How is graphing the number $\frac{4}{3}$ on a number line different from graphing the number 4 on a number line?
  - *When we graph 4, the unit length is one, and when we graph $\frac{4}{3}$ the unit length is $\frac{1}{3}$.*
- On a vertical number line, describe the location of the rational number that represents -5.1 and its opposite.
  - *The number -5.1 would be 5.1 units below zero because it is negative. Its opposite, 5.1 would be 5.1 units above zero because it is positive.*

Exit Ticket (6 minutes)

Name _____ Date_____

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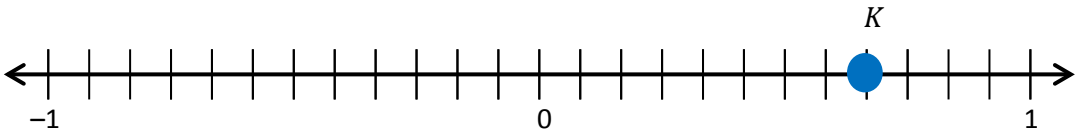
Exit Ticket

Use the number line diagram below to answer the following questions.



1. What is the length of each segment on the number line?

2. What number does point K represent?

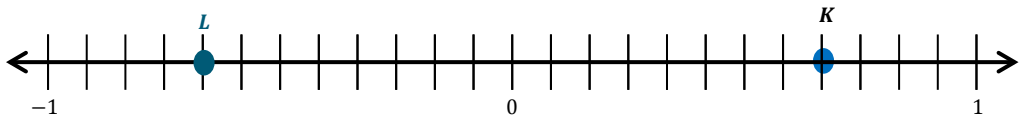
3. What is the opposite of point K?

4. Locate the opposite of point K on the number line, and label it point L.

5. In the diagram above, zero represents the location of MLK Middle School. Point K represents the library, which is located several miles away from the middle school to the east. In words, create a real-world situation that could represent point L, and describe its location in relation to 0 and point K.

Exit Ticket Sample Solutions

Use the number line diagram below to answer the following questions.



1. **What is the length of each segment on the number line?**

   $\frac{1}{12}$

2. **What number does point K represent?**

   $\frac{8}{12}$

3. **What is the opposite of point K?**

   $-\frac{8}{12}$

4. **Locate the opposite of point K on the number line, and label it point L.**

5. **In the diagram above, zero represents the location of MLK Middle School. Point K represents the library, which is located several miles away from the middle school to the east. In words, create a real-world situation that could represent point L, and describe its location in relation to 0 and point K.**

   (Answers may vary.) Point L is $\frac{8}{12}$ units to the left of 0, so it is a negative number. Point L represents the recreation center which is located $\frac{8}{12}$ mile west of MLK Middle School. This means that the recreation center and library are the same distance from the middle school but in opposite directions because the opposite of $\frac{8}{12}$ is $-\frac{8}{12}$.

Problem Set Sample Solutions

Students gain additional practice with graphing rational numbers on the number line.

1. **In the space provided, write the opposite of each number.**

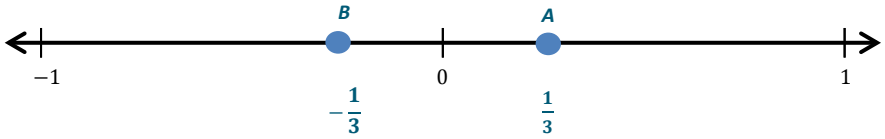
   a. $\frac{10}{7}$ $-\frac{10}{7}$

   b. $-\frac{5}{3}$ $\frac{5}{3}$

   c. 3.82 -3.82

   d. $-6\frac{1}{2}$ $6\frac{1}{2}$

2. Choose a non-integer between 0 and 1. Label it point A and its opposite point B on the number line. Write their values below the points. (Answers may vary.)



a. To draw a scale that would include both points, what could be the length of each segment?

(Answers may vary.) $\frac{1}{3}$

b. In words, create a real-world situation that could represent the number line diagram.

(Answers may vary.) Starting at home, I ran $\frac{1}{3}$ *mile. My brother ran* $\frac{1}{3}$ *mile from home in the opposite direction.*

3. Choose a value for point P that is between -6 and -7.

(Answers may vary.) $-\frac{13}{2}, -6.25, -6.8$

a. What is the opposite of P?

(Answers may vary.) $\frac{13}{2}, 6.25, 6.8$

b. Choose one possible value from part a, and describe its location on the number line in relation to zero.

$\frac{13}{2}$ *is the same distance as* $-\frac{13}{2}$ *from zero but to the right.*

c. Find the opposite of the opposite of point P. Show your work and explain your reasoning.

The opposite of an opposite of the number is the number itself. If P is $-\frac{13}{2}$*, then the opposite of the opposite of P is* $-\frac{13}{2}$*. The opposite of* $-\frac{13}{2}$ *is* $\frac{13}{2}$*. The opposite of* $\frac{13}{2}$ *is* $-\frac{13}{2}$*.*

$-\left(-\frac{13}{2}\right) = \frac{13}{2}$

$-\left(\frac{13}{2}\right) = -\frac{13}{2}$

4. Locate and label each point on the number line. Use the diagram to answer the questions.

Jill lives one block north of the pizza shop.

Janette's house is $\frac{1}{3}$ block past Jill's house.

Jeffrey and Olivia are in the park $\frac{4}{3}$ blocks south of the pizza shop.

Janet's Jazzy Jewelry Shop is located half-way between the pizza shop and the park.

a. Describe an appropriate scale to show all the points in this situation.

An appropriate scale would be $\frac{1}{3}$ because the numbers given in the example all have denominators of 3. I would divide the number line by equal segments of $\frac{1}{3}$.

b. What number represents the location of Janet's Jazzy Jewelry Shop? Explain your reasoning.

The number is $-\frac{2}{3}$. I got my answer by finding the park first. It is 4 units below 0. Since the jewelry shop is halfway between the pizza shop and the park, half of 4 is 2. Then I moved 2 units down on the number line since the shop is south of the pizza shop before the park.