



Lesson 8: Ordering Integers and Other Rational Numbers

Student Outcomes

- Students write, interpret, and explain statements of order for rational numbers in the real-world.
- Students recognize that if $a < b$, then $-a > -b$, because a number and its opposite are equal distances from zero; and moving along the horizontal number line to the right means the numbers are increasing.

Lesson Notes

As a continuation of Lesson 7, students order rational numbers from least to greatest and from greatest to least. They relate the orderings to numbers' location on the number line.

Classwork

Opening Exercise (6 minutes)

For this warm-up exercise, students work in groups of 3 or 4 to order the following rational numbers from least to greatest. Each group of students may be provided with cards to put in order, or the numbers may be displayed on the board where the students work at their seats, recording them in the correct order. As an alternative, the numbers may be displayed on an interactive board along with a number line, and students or teams come up to the board and slide the numbers onto the number line into the correct order. Allow time for the class to come to a consensus on the correct order, and for students to share with the class their strategies and thought processes.

The following are examples of rational numbers to sort and order:

$0, -4, \frac{1}{4}, -\frac{1}{2}, 1, -3\frac{3}{5}, 2, -4.1, -0.6, \frac{23}{5}, 6, -1, 4.5, -5, 2.1$

Solution:

$-5, -4.1, -4, -3\frac{3}{5}, -1, -0.6, -\frac{1}{2}, 0, \frac{1}{4}, 1, 2, 2.1, 4.5, \frac{23}{5}, 6$

Scaffolding:

Adjust the number of cards given to students depending on their ability level. The types of rational numbers given to each group of students may also be differentiated.

The following line of questioning can be used to elicit student responses:

- How did you begin to sort and order the numbers? What was your first step?
 - Our group began by separating them into two groups: negative numbers and positive numbers. Zero was not in either group, but we knew it fell in between the negative numbers and positive numbers.*
- What was your next step? What did you do with the two groups of numbers?
 - We ordered the positive whole numbers 1, 2, 4, 5, 6 and then took the remaining positive numbers and determined which two whole numbers they fell in between.*

- How did you know where to place $\frac{1}{4}$ and $\frac{23}{5}$?
 - Since $\frac{1}{4}$ is less than a whole (1), but greater than zero, we arrived at the following: $0, \frac{1}{4}, 1, 2, 4, 5, 6$.
 - We know that $\frac{23}{5}$ is the same as $4\frac{3}{5}$, which is more than 4 but less than 5, and so we arrived at: $0, \frac{1}{4}, 1, 2, 4, 4\frac{3}{5}, 5, 6$.
- How did you order the negative numbers?
 - First we started with the negative integers: $-5, -4$, and -1 . Negative 5 is the least because it is farthest left at 5 units to the left of zero. Then came -4 , and then came -1 , which is only 1 unit to the left of zero.
- How did you order the negative non-integers?
 - We know $-\frac{1}{2}$ is equivalent to $-\frac{5}{10}$, which is to the right of -0.6 (or $-\frac{6}{10}$) since -0.5 is closer to zero than -0.6 . And then we ordered -4.1 and $-3\frac{3}{5}$. Both numbers are close to -4 , but -4.1 is to the left of -4 and $-3\frac{3}{5}$ is to the right of -4 and to the left of -3 . Lastly we put our ordered group of negative numbers to the left of zero, and our ordered group of positive numbers to the right of zero and ended up with:

$$-5, -4.1, -4, -3\frac{3}{5}, -1, -0.6, -\frac{1}{2}, 0, \frac{1}{4}, 1, 2, 2.1, 4.5, \frac{23}{5}, 6$$

Exercise 1 (8 minutes)

Exercise 1

- i. Students are each given four index cards or small slips of paper. Each student must independently choose four non-integer rational numbers, and write each one on a slip of paper. At least two of the numbers must be negative.
- ii. Students order their rational numbers from least to greatest by sliding their slips of paper into the correct order. The teacher walks around the room to check for understanding and provide individual assistance. Students may use the number line in their student materials to help determine the order.
- iii. Once all students have arranged their numbers into the correct order, they shuffle them and switch with another student.
- iv. Students arrange the new set of cards they received into the correct order from least to greatest.
- v. The pairs of students who exchanged cards discuss their solutions and come to a consensus.

Example 1 (3 minutes): Ordering Rational Numbers from Least to Greatest

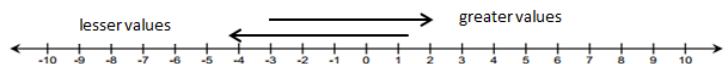
Example 1: Ordering Rational Numbers from Least to Greatest

Sam has \$10 in the bank. He owes his friend Hank \$2.25. He owes his sister \$1.75. Consider the three rational numbers related to this story of Sam’s money. Write and order them from least to greatest.

$-2.25, -1.75, 10$

Scaffolding:

Provide a number line diagram for visual learners to help them determine the numbers’ order.



- Explain the process you used to determine the order of the numbers.
 - *There was only one positive number, 10, so I know that 10 is the greatest. I know 2.25 is farther to the right on the number line than 1.75, so its opposite, -2.25 , will be farther to the left than the opposite of 1.75. This means -2.25 is the least, and -1.75 would be in between -2.25 and 10.*
- How would the order change if you were asked to write the numbers from greatest to least?
 - *The order would be reversed. I would list the numbers so that the number that comes first is the one furthest to the right on the number line, and the number that comes last is the one furthest to the left on the number line. The order would be 10 (the greatest), followed by -1.75 , and then followed by -2.25 (the least).*

Exercises 2–4 (10 minutes)

Allow time for students to share their answers with the class and explain their reasoning.

Exercises 2–4

For each problem, list the rational numbers that relate to each situation. Then, order them from least to greatest; and explain how you made your determination.

2. During their most recent visit to the optometrist (eye doctor), Kadijsha and her sister Beth had their vision tested. Kadijsha's vision in her left eye was -1.50 and her vision in her right eye was the opposite number. Beth's vision was -1.00 in her left eye and $+0.25$ in her right eye.

$-1.50, -1.00, 0.25, 1.50$

The opposite of -1.50 is 1.50 and 1.50 is furthest right on the number line, so it is the greatest. -1.50 is the same distance from zero but on the other side. So it is the least number. Negative one is to the right of -1.50 , so it is greater, and then 0.25 is to the right of -1.00 , so it is greater, then comes 1.50 , which is the greatest.

3. There are three letters in Ms. Thomas's mailbox: a bill from the phone company for $\$38.12$, a bill from the electric company for $\$67.55$, and a tax refund check for $\$25.89$. (A bill is money that you owe someone and a tax refund check is money that you receive from someone.)

$-67.55, -38.12, 25.89$

The change in Ms. Thomas' money is represented by -38.12 due to the phone bill, and -67.55 represents the change in her money due to the electric bill. Since -67.55 is furthest to the left on the number line, it is the least number. Since -38.12 is to the right of -67.55 , it comes next. The check she has to deposit for $\$25.89$ can be represented by 25.89 which is to the right of -38.12 , and so it is the greatest number.

4. Monica, Jack and Destiny each had their arm length measured for an experiment in science class. They compared their arm lengths to a standard of 22 inches. The listing below shows in inches how each student's arm length compares to 22 inches. Order these rational numbers from least to greatest.

Monica: $-\frac{1}{8}$

Jack: $1\frac{3}{4}$

Destiny: $-\frac{1}{2}$

$-\frac{1}{2}, -\frac{1}{8}, 1\frac{3}{4}$

I ordered the numbers on a number line and $-\frac{1}{2}$ was furthest to the left. To the right of that was $-\frac{1}{8}$. Last is $1\frac{3}{4}$ which is to the right of $-\frac{1}{8}$, so $1\frac{3}{4}$ is the greatest.

MP.2

Example 2 (3 minutes): Ordering Rational Numbers from Greatest to Least**Example 2**

Jason is entering college and has opened a checking account, which he will use for college expenses. His parents gave him \$200 to deposit into the account. Jason wrote a check for \$85.00 to pay for his Calculus book and a check for \$25.34 to pay for miscellaneous school supplies. Write the three rational numbers related to the balance in Jason's checking account in order from greatest to least.

200, -25.34, -85.00

- Explain the process you used to determine the order of the numbers.
 - *There was only one positive number, 200, so I know that 200 is the greatest. I know 85.00 is farther to the right on the number line than 25.34, so its opposite, -85.00, will be farther to the left than the opposite of 25.34. This means -85.00 is the least, and -25.34 would be in between -85.00 and 200.*

Exercises 5–6 (6 minutes)

Allow time for students to share their answers with the class and explain their reasoning.

Exercise 5–6

For each problem, list the rational numbers that relate to each situation in order from greatest to least. Lastly, explain how you arrived at their order.

5. The following are the current monthly bills that Mr. McGraw must pay:

\$122.00 Cable and Internet

\$73.45 Gas and Electric

\$45.00 Cell phone

-45, -73.45, -122

Because Mr. McGraw owes the money, I represented the amount of each bill as a negative number. Ordering them from greatest to least means I have to move from right to left on a number line. Since -45 is furthest right it is the greatest. To the left of that is -73.45, and to the left of that is -122, which means -122 is the least.

6. Arrange the following rational numbers in order from greatest to least: $-\frac{1}{3}$, 0 , $-\frac{1}{5}$, $\frac{1}{8}$.

$\frac{1}{8}$, 0 , $-\frac{1}{5}$, $-\frac{1}{3}$

I graphed them on the number line. Since I needed to order them from greatest to least, I moved from right to left to record the order. Farthest to the right is $\frac{1}{8}$, so that is the greatest value. To the left of that number is 0. To the left of 0 is $-\frac{1}{5}$ and the furthest left is $-\frac{1}{3}$, so that it is the least.

Closing (3 minutes)

- If three numbers are ordered from least to greatest and the order is a, b, c , what would the order be if the same three numbers were arranged in order from greatest to least? How did you determine the new order?
 - c, b, a
- How does graphing numbers on a number line help us determine the order when arranging the numbers from greatest to least or least to greatest?

Lesson Summary

When we order rational numbers, their opposites will be in the opposite order. For example, if seven is greater than five, negative seven is less than negative five.

Exit Ticket (6 minutes)



Name _____

Date _____

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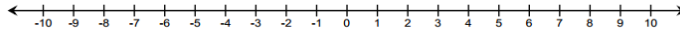
Exit Ticket

Order the following set of rational numbers from least to greatest, and explain how you determined their order.

0, $-\frac{1}{2}$, 1, $-3\frac{1}{3}$, 6, 5, -1, $\frac{21}{5}$, 4

Exit Ticket Sample Solutions

Order the following set of rational numbers from least to greatest, and explain how you determined their order.



$$-3, 0, -\frac{1}{2}, 1, -3\frac{1}{3}, 6, 5, -1, \frac{21}{5}, 4$$

$$-3\frac{1}{3}, -3, -1, -\frac{1}{2}, 0, 1, 4, \frac{21}{5}, 5, 6$$

I drew a number line and started at zero. I located the positive numbers to the right and their opposites (the negative numbers) to the left of zero. The positive integers listed in order from left to right are: 1, 4, 5, 6. And since $\frac{21}{5}$ is equal to $4\frac{1}{5}$, I know that it is $\frac{1}{5}$ more than 4, but less than 5. So I arrived at 0, 1, 4, $\frac{21}{5}$, 5, 6. Next, I ordered the negative numbers. Since -1 and -3 are the opposites of 1 and 3, they are 1 unit and 3 units from zero but to the left of zero. And $-3\frac{1}{3}$ is even farther left, since it is $3\frac{1}{3}$ units to the left of zero. The smallest number is farthest to the left, so I arrive at the following order: $-3\frac{1}{3}, -3, -1, -\frac{1}{2}, 0, 1, 4, \frac{21}{5}, 5, 6$.

Problem Set Sample Solutions

1. In the table below, list each set of rational numbers from greatest to least. Then, in the appropriate column, state which number was farthest right and which number was farthest left on the number line.

Column 1	Column 2	Column 3	Column 4
Rational Numbers	Ordered from Greatest to Least	Farthest Right on the Number Line	Farthest Left on the Number Line
-1.75, -3.25	-1.75, -3.25	-1.75	-3.25
-9.7, -9	-9, -9.7	-9	-9.7
$\frac{4}{5}, 0$	$\frac{4}{5}, 0$	$\frac{4}{5}$	0
$-70, -70\frac{4}{5}$	$-70, -70\frac{4}{5}$	-70	$-70\frac{4}{5}$
-15, -5	-5, -15	-5	-15
$\frac{1}{2}, -2$	$\frac{1}{2}, -2$	$\frac{1}{2}$	-2
-99, -100, -99.3	-99, -99.3, -100	-99	-100
0.05, 0.5	0.5, 0.05	0.5	0.05
$0, -\frac{3}{4}, -\frac{1}{4}$	$0, -\frac{1}{4}, -\frac{3}{4}$	0	$-\frac{3}{4}$
-0.02, -0.04	-0.02, -0.04	-0.02	-0.04

2. For each row, describe the relationship between the number in Column 3 and its order in Column 2. Why is this?

The number in Column 3 is the first number listed in Column 2. Since it is farthest right on the number line, it will be the greatest, and so it comes first when ordering the numbers from greatest to least.

3. For each row, describe the relationship between the number in Column 4 and its order in Column 2. Why is this?

The number in Column 4 is the last number listed in Column 2. Since it is farthest left on the number line, it will be the smallest, and so it comes last when ordering the numbers from greatest to least.

4. If two rational numbers, a and b , are ordered such that a is less than b , then what must be true about the order for their opposites: $-a$ and $-b$?

The order will be reversed for the opposites which means $-a$ is greater than $-b$.

5. Read each statement and then write a statement relating the *opposites* of each of the given numbers:

- a. 7 is greater than 6.

-7 is less than -6 .

- b. 39.2 is greater than 30.

-39.2 is less than -30 .

- c. $-\frac{1}{5}$ is less than $\frac{1}{3}$.

$\frac{1}{5}$ is greater than $-\frac{1}{3}$.

6. Order the following from least to greatest: -8 , -19 , 0 , $\frac{1}{2}$, $\frac{1}{4}$

$-19, -8, 0, \frac{1}{4}, \frac{1}{2}$

7. Order the following from greatest to least: -12 , 12 , -19 , $1\frac{1}{2}$, 5

$12, 5, 1\frac{1}{2}, -12, -19$