Lesson 7: The Relationship Between Visual Fraction Models and Equations

Student Outcomes

- Students formally connect models of fractions to multiplication through the use of multiplicative inverses as they are represented in models.

Lesson Notes

Using pre-cut fraction strips saves time and assures more accurate models. Fraction strips are found at the end of this document and should be reproduced and cut prior to the lesson. With each example that students work through, the concept will be reinforced: dividing by a fraction yields the same result as multiplying by the inverse of that fraction.

The terms **inverse** and **reciprocal** should be defined and displayed in the classroom, perhaps as part of a Word Wall.

The reciprocal, or inverse, of a fraction is the fraction made by interchanging the numerator and denominator.

$\frac{2}{3} \Rightarrow \frac{3}{2}$

$\frac{5}{8} \Rightarrow \frac{8}{5}$

$4 \Rightarrow \frac{1}{4}$

Classwork

Opening (2 minutes)

Introduce the definition of the term Multiplicative Inverses: Two numbers whose product is 1 are multiplicative inverses of one another. For example, $\frac{3}{4}$ and $\frac{4}{3}$ are multiplicative inverses of one another because $\frac{3}{4} \times \frac{4}{3} = \frac{4}{3} \times \frac{3}{4} = 1$.

Point out and ask students to write several of their own examples of multiplicative inverses. The general form of the concept $\frac{a}{b} \times \frac{b}{a} = \frac{b}{a} \times \frac{a}{b} = 1$ should also be displayed in several places throughout the classroom.

- We already know how to make tape diagrams to model division problems. Today we will use fraction tiles or fraction strips to model dividing fractions. This will help you understand this concept more completely.

MP.1

During this lesson, students continue to make sense of problems and persevere in solving them. They use concrete representations when understanding the meaning of division and apply it to the division of fractions. They ask themselves, "What is this problem asking me to find?" For instance, when determining the quotient of fractions, students ask themselves how many sets or groups of the divisor is in the dividend. That quantity is the quotient of the problem.

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Example 1 (15 minutes)

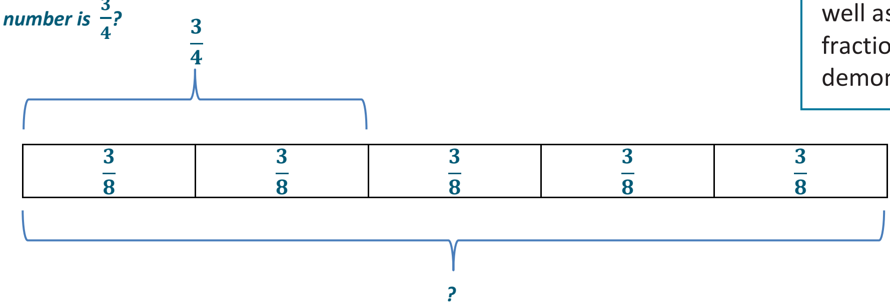
Consider the following, an example that we have worked with in previous lessons:

> *Scaffolding:*
> Students who prefer using a manipulative will benefit from having plastic models of fractions available in addition to the paper copies called for in the lesson. Students should always use the "1" piece as well as pulling as many of the fraction pieces they need to demonstrate the problem.

Example 1

$\frac{3}{4} \div \frac{2}{5}$

$\frac{2}{5}$ of what number is $\frac{3}{4}$?



Shade 2 of the 5 sections $\left(\frac{2}{5}\right)$.

Label the part that is known $\left(\frac{3}{4}\right)$.

Make notes below on the math sentences needed to solve the problem.

MP.1

- How did we choose which fraction strip to use?
  - *We are dividing by $\frac{2}{5}$. There is a 5 in the denominator, which tells us what kind of fraction, so we chose fifths.*
- How did we know to shade in two sections?
  - *There is a 2 in the numerator. That tells us how many of the fifths to shade.*
- How did we know to put the $\frac{3}{4}$ in the brace that shows $\frac{2}{5}$?
  - *That is the part we know.*
- What about the bottom brace?
  - *That is unknown right now and is ready to be calculated.*
- Think about this problem like this:
- 2 units $= \frac{3}{4}$.
- 1 unit is half of $\frac{3}{4} = \frac{1}{2} \cdot \frac{3}{4} = \frac{3}{8}$.
- 5 units $= \frac{3}{8} \cdot 5 = \frac{15}{8}$.
- So $\frac{3}{4} \div \frac{2}{5} = \frac{15}{8}$.
- We have the answer to our initial question, but perhaps there is another way to solve the problem. Do you think it is possible to solve this problem without using division?

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Extension: Allow time for students to inspect the problem and offer conjectures using the reciprocal of the divisor. Prompt them, if necessary, to consider another operation (multiplication). The following is a series of steps to explain how "invert and multiply" is justified.

- Using the Any-Order Property, we can say the following:
- $\frac{1}{2} \cdot \frac{3}{4} \cdot 5 = \frac{3}{4} \cdot \left(\frac{1}{2} \cdot 5\right) = \frac{3}{4} \cdot \frac{5}{2} = \frac{15}{8}$.
- So $\frac{3}{4} \div \frac{2}{5} = \frac{15}{8}$ and $\frac{3}{4} \cdot \frac{5}{2} = \frac{15}{8}$.
- Therefore, $\frac{3}{4} \div \frac{2}{5} = \frac{3}{4} \cdot \frac{5}{2}$.
- This method for dividing fractions by fractions is called "invert and multiply". Dividing by a fraction is the same as multiplying by its inverse. It is important to invert the second fraction (the divisor) and ***not*** the first fraction (the dividend).

Exercise 1 (10 minutes)

MP.7

During this part of the lesson, students look for and make use of structure. They continue to find patterns and connections when dividing fractions, and they recognize and make use of a related multiplication problem to determine the number of times the divisor is added to obtain the dividend.

- Does this always work? Let's find out using the example $\frac{1}{4} \div \frac{2}{3}$.
- How did we choose which fraction strip to use?
  - *We are dividing by $\frac{2}{3}$. There is a 3 in the denominator, which tells us what kind of fraction, so we chose thirds.*
- How did we know to shade in two sections?
  - *There is a 2 in the numerator. That tells us how many of the thirds to shade.*
- How did we know to put the $\frac{1}{4}$ in the brace that shows $\frac{2}{3}$?
  - *That is the part we know.*
- What about the bottom brace?
  - *That is unknown right now and is ready to be calculated.*
- Think about this problem like this:
- $2 \text{ units} = \frac{1}{4}$.
- 1 unit is half of $\frac{1}{4}$. $\frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8}$.
- 3 units $= \frac{1}{8} \cdot 3 = \frac{3}{8}$.

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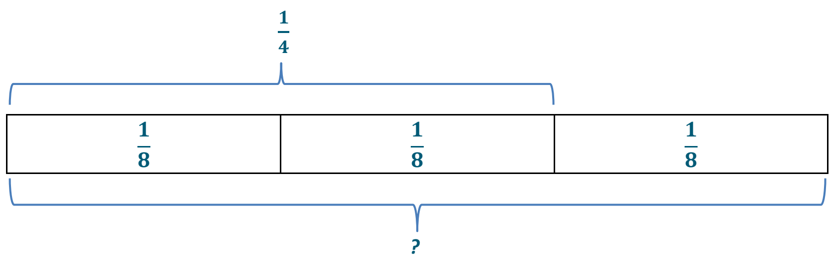
Exercise 1

$\frac{1}{4} \div \frac{2}{3}$



Show the number sentences below.

$\frac{1}{4} \div \frac{2}{3} = \frac{2}{8} \div \frac{2}{3} = \frac{3}{8}$

$\frac{1}{4} \cdot \frac{3}{2} = \frac{3}{8}$

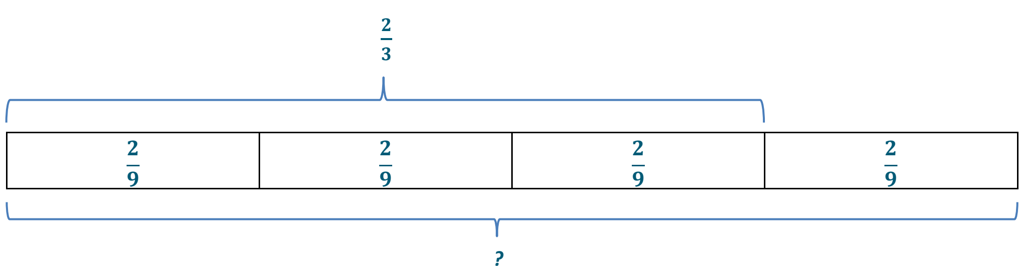
Therefore, $\frac{1}{4} \div \frac{2}{3} = \frac{1}{4} \cdot \frac{3}{2}$.

Exercise 2 (5 minutes)

Ask students to solve the following problem with both a tape diagram and the "invert and multiply" rule. They should compare the answers obtained from both methods and find them to be the same.

Exercise 2

$\frac{2}{3} \div \frac{3}{4}$



Show the number sentences below.

$\frac{2}{3} \div \frac{3}{4} = \frac{8}{12} \div \frac{9}{12} = \frac{8}{9}$

$\frac{2}{3} \div \frac{3}{4} = \frac{2}{3} \cdot \frac{4}{3} = \frac{8}{9}$

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- How did we choose which fraction strip to use?
  - *We are dividing by* $\frac{3}{4}$*. There is a* 4 *in the denominator, which tells us what kind of fraction, so we chose fourths.*
- How did we know to shade in three sections?
  - *There is a* 3 *in the numerator. That tells us how many of the fourths to shade.*
- How did we know to put the $\frac{2}{3}$ in the brace that shows $\frac{3}{4}$?
  - *That is the part we know.*
- What about the bottom brace?
  - *That is unknown right now and is ready to be calculated.*
- Think about this problem like this:
- 3 units $= \frac{2}{3}$.
- 1 unit is one third of $\frac{2}{3}$ $\qquad \frac{1}{3} \cdot \frac{2}{3} = \frac{2}{9}$.
- 4 units $= \frac{2}{9} \cdot 4 = \frac{8}{9}$.

Closing (3 minutes)

- Dividing by a fraction is equivalent to multiplying by its reciprocal, or inverse. Connecting the models of division by a fraction to multiplication by its inverse strengthens your understanding.

> **Lesson Summary**
>
> **Connecting models of fraction division to multiplication through the use of reciprocals helps in understanding the "invert and multiply" rule.**

Exit Ticket (3 minutes)

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Name ____ Date____________________

Lesson 7: The Relationship Between Visual Fraction Models and Equations

Exit Ticket

1. Write the reciprocal of the following numbers:

Number	$\frac{7}{10}$	$\frac{1}{2}$	5
Reciprocal			

2. Rewrite this division problem as a multiplication problem: $\frac{5}{8} \div \frac{2}{3}$.

3. Solve the problem 2 using models.

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Exit Ticket Sample Solutions

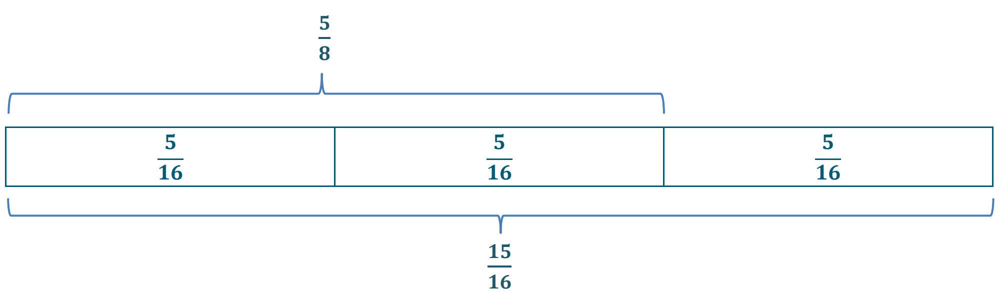
1. **Write the reciprocal of the following numbers:**

Number	$\frac{7}{10}$	$\frac{1}{2}$	5
Reciprocal	$\frac{10}{7}$	2	$\frac{1}{5}$

2. **Rewrite this division problem as a multiplication problem:** $\frac{5}{8} \div \frac{2}{3}$.

$\frac{5}{8} \cdot \frac{3}{2}$

3. **Solve the problem in #2 using models.**



Answer: $\frac{15}{16}$

$\frac{5}{8} \div \frac{2}{3} = \frac{5}{8} \cdot \frac{3}{2} = \frac{15}{16}$

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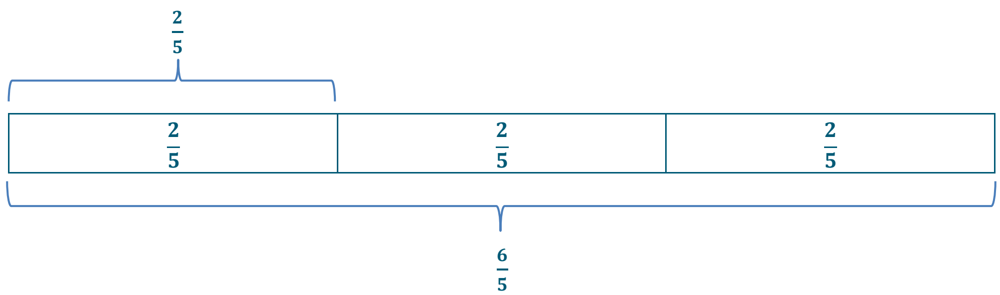
Problem Set Sample Solutions

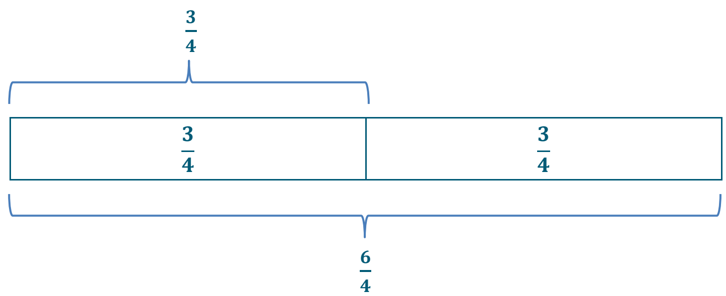
1. **Draw a model that shows $\frac{2}{5} \div \frac{1}{3}$. Find the answer as well.**



Answer: $\frac{6}{5}$

$\frac{2}{5} \div \frac{1}{3} = \frac{2}{5} \cdot \frac{3}{1} = \frac{6}{5}$

2. **Draw a model that shows $\frac{3}{4} \div \frac{1}{2}$. Find the answer as well.**



Answer: $\frac{6}{4}$

$\frac{3}{4} \div \frac{1}{2} = \frac{3}{4} \cdot \frac{2}{1} = \frac{6}{4}$

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1 whole unit											
$\frac{1}{2}$	$\frac{1}{2}$										
$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$									
$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$								
$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$							
$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$						
$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$				
$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$			
$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$		
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