Lesson 18: Least Common Multiple and Greatest Common Factor

Student Outcomes

- Students find the least common multiple and greatest common factor and apply factors to the Distributive Property

Lesson Notes

Least common multiple and greatest common factor are terms that are easily confused by young learners. A clear definition of both with several examples of each should be posted in the classroom before, during, and after the lesson. Furthermore, these skills should be practiced regularly, so the concepts do not fade or blend together from lack of use.

During this lesson, students will move around in groups to various stations where a topic is presented on chart paper. At each station, students read the directions, choose a problem, and then work collaboratively to solve the problem. Group students prior to the lesson using the most appropriate ability or social grouping.

There are four different topics: Factors and GCF, Multiples and LCM, Using Prime Factors to Determine GCF, and Applying Factors to the Distributive Property.

If two sets of chart paper are prepared for each topic, there will be 8 stations. This makes manageable groups of 3–4 students. Further, if the stations are placed in order around the room (1, 2, 3, 4, 1, 2, 3, 4) it will not matter where a group starts, and the group will finish after only three rotations. Groups should spend five minutes at each station.

Suggested Student Roles:

Marker	This student records the group's work on the chart paper. Note: Each group should use a different (unique) color when adding its work to the chart paper.
Recorder	This student records the group's work in his/her student materials and later shares this work with the other members of the group ensuring that all students finish the activity with their student materials completed.
Calculator Operator/Master Mathematician	This student uses a calculator when necessary and helps clarify the group's response

Materials: 8 pieces of chart paper with directions attached, one calculator per group, a different colored chart marker for each group, a multiplication table posted at Stations 2 and 4.

Classwork

Opening Exercise (8 minutes)

Point out the definitions on the student pages and work through the examples before assigning and releasing groups.

> **The Greatest Common Factor of two whole numbers a and b, written $GCF(a, b)$, is the greatest whole number, which is a factor of both a and b.**
>
> **The Least Common Multiple of two nonzero numbers a and b, written $LCM(a, b)$, is the least whole number (larger than zero), which is a multiple of both a and b.**

Example 1 (3 minutes)

Example 1

Greatest Common Factor: Find the greatest common factor of 12 and 18.

- **Listing these factors as pairs can help you not miss any. Start with one times the number.**
- **Circle all factors that appear on both lists.**
- **Place a triangle around the greatest of these common factors.**

GCF $(12, 18)$ 6

12

1	12
2	6
3	4

18

1	18
2	9
3	6

Example 2 (5 minutes)

Least Common Multiple: Find the least common multiple of 12 and 18.

LCM 12, 18

- List the first 10 multiples of both numbers.
- Circle all multiples that appear on both lists.
- Place a triangle around the least of these common multiples.
  - *Student circles 36, 72, and 108 and draws a triangle around 36.*
- Did you really need to write out 10 multiples of each number?
  - *No, we could have stopped as soon as a multiple appeared on both lists.*
- Should we start by writing the multiples of the larger or the smaller of the two numbers? Which will lead us to find the LCM most efficiently?
  - *If we start writing the multiples of the larger of the two numbers, we can stop when we find the first one that is a multiple of the smaller of the two numbers. In the example given, we would list the multiples of 18 first and stop at 36 because 36 is a multiple of 12. In this case we will have found the LCM 12, 18 after listing only two numbers.*

Scaffolding:

- Multiplication tables should be used by any learners that have automaticity issues. Naming this a "Multiples Table" is also effective with some students.

Example 2

Least Common Multiple: Find the least common multiple of 12 and 18.

LCM 12, 18

Write the first 10 multiples of 12.

12, 24, 36, 48, 60, 72, 84, 96, 108, 120.

Write the first 10 multiples of 18.

18, 36, 54, 72, 90, 108, 126, 144, 162, 180.

Circle the multiples that appear on both lists.

12, 24, 36, 48, 60, 72, 84, 96, 108, 120.

18, 36, 54, 72, 90, 108, 126, 144, 162, 180.

Put a triangle around the least of these common multiples.

12, 24, 36, 48, 60, 72, 84, 96, 108, 120.

18, 36, 54, 72, 90, 108, 126, 144, 162, 180.

Discussion (5 minutes)

- Today you will be visiting several stations around the room. At each station, you will have 5 minutes to read and follow directions. Only the Recorder's paper will be used at the station. Later the Recorder will share his copy of the work so each of you will leave with a record of today's classwork.
- Another person, the Marker, in the group will have the chart marker for writing on the chart paper, and a third person will serve as Calculator Operator/Master Mathematician who will use a calculator when necessary and help clarify your group's response before putting it on the chart paper.

- At each station the directions start out the same way: Choose one of the problems that have not yet been solved. Solve it together on the Recorder's page. The Marker should copy your group's work neatly on the chart paper and cross out the problem your group solved, so the next group solves a different problem.

Exploratory Challenge 1: Factors and GCF (5 minutes)

Groups will choose from the following number problems:

Find the greatest common factor of one of these pairs: $30, 50$; $30, 45$; $45, 60$; $42, 70$; $96, 144$

Next, groups will choose from the following application problems:

a. There are 18 girls and 24 boys who want to participate in a Trivia Challenge. If each team must have the same ratio of girls and boys, what is the greatest number of teams that can enter? How many boys and girls will be on each team?

b. The Ski Club members are preparing identical welcome kits for the new skiers. The Ski Club has 60 hand warmer packets and 48 foot warmer packets. What is the greatest number of identical kits they can prepare using all of the hand warmer and foot warmer packets?

c. There are 435 representatives and 100 senators serving in the United States Congress. How many identical groups with the same numbers of representatives and senators could be formed from all of Congress, if we want the largest groups possible?

d. Is the GCF of a pair of numbers ever equal to one of the numbers? Explain with an example.

e. Is the GCF of a pair of numbers ever greater than both numbers? Explain with an example.

Exploratory Challenge 1: Factors and GCF

Choose one of these problems that have not yet been solved. Solve it together on your student page. Then use your marker to copy your work neatly on the chart paper. Use your marker to cross out your choice so the next group solves a different problem.

GCF $30, 50$

Factors of 30: $1, 2, 3, 5, 6, 10, 15, 30$.
Common Factors: $1, 2, 5, 10$.

Factors of 50: $1, 2, 5, 10, 25, 50$.
Greatest Common Factor: 10.

GCF $30, 45$

Factors of 30: $1, 2, 3, 5, 6, 10, 15, 30$.
Common Factors: $1, 3, 5, 15$.

Factors of 45: $1, 3, 5, 9, 15, 45$.
Greatest Common Factor: 15.

GCF $45, 60$

Factors of 45: $1, 3, 5, 9, 15, 45$.
Common Factors: $1, 3, 5, 15$.

Factors of 60: $1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60$.
Greatest Common Factor: 15.

GCF $42, 70$

Factors of 42: $1, 2, 3, 6, 7, 14, 21, 42$.
Common Factors: $1, 2, 7, 14$.

Factors of 70: $1, 2, 5, 7, 10, 14, 35, 70$.
Greatest Common Factor: 14.

GCF $96, 144$

Factors of 96: $1, 2, 3, 4, 6, 8, 12, 16, 24, 32, 48, 96$
Common Factors: $1, 2, 3, 4, 6, 8, 12, 16, 24, 48$.

Factors of 144: $1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 36, 48, 72, 144$
Greatest Common Factor : 48.

Next, choose one of these problems that have not yet been solved:

a. There are 18 girls and 24 boys who want to participate in a Trivia Challenge. If each team must have the same number of girls and boys, what is the greatest number of teams that can enter? How many boys and girls will be on each team?

6 teams, each having 3 girls and 4 boys.

b. The Ski Club members are preparing identical welcome kits for the new skiers. They have 60 hand warmer packets and 48 foot warmer packets. What is the greatest number of kits they can prepare using all of the hand warmer and foot warmer packets?

12 welcome kits, each having 5 hand warmer packets and 4 foot warmer packets.

c. There are 435 representatives and 100 senators serving in the United States Congress. How many identical groups with the same numbers of representative and senators could be formed from all of Congress, if we want the largest groups possible??

5 groups, each with 87 representatives and 20 senators.

d. Is the GCF of a pair of numbers ever equal to one of the numbers? Explain with an example.

Yes. Valid examples will consist of a pair of numbers where the lesser of the two numbers is a factor of the greater number; the greater of the two numbers is a multiple of the lesser number.

e. Is the GCF of a pair of numbers ever greater than both numbers? Explain with an example.

No. Factors are, by definition, less than or equal to the number. Therefore, the GCF cannot be greater than both numbers.

Exploratory Challenge 2: Multiples and LCM (5 minutes)

Groups will choose from the following number problems:

Find the least common multiple of one of these pairs: 9,12; 8,18; 4,30; 12,30; 20,50

Next, groups will choose from the following application problems:

a. Hot dogs come packed 10 in a package. Hot dog buns come packed 8 in a package. If we want one hot dog for each bun for a picnic, with none left over, what is the least amount of each we need to buy?

b. Starting at 6:00 a.m., a bus makes a stop at my street corner every 15 minutes. Also starting at 6:00 a.m., a taxi cab comes by every 12 minutes. What is the next time there will be a bus and a taxi at the corner at the same time?

c. Two gears in a machine are aligned by a mark drawn from the center of one gear to the center of the other. If the first gear has 24 teeth and the second gear has 40 teeth, how many revolutions of the first gear are needed until the marks line up again?

d. Is the LCM of a pair of numbers ever equal to one of the numbers? Explain with an example.

e. Is the LCM of a pair of numbers ever less than both numbers? Explain with an example.

Exploratory Challenge 2: Multiples and LCM

Choose one of these problems that have not yet been solved. Solve it together on your student page. Then use your marker to copy your work neatly on the chart paper. Use your marker to cross out your choice so the next group solves a different problem.

LCM 9, 12

Multiples of 9: 9, 18, 27, 36.
Least Common Multiple: 36.

Multiples of 12: 12, 24, 36.

LCM 8, 18

Multiples *of* 8: 8, 16, 24, 32, 40, 48, 56, 64, 72.
Least Common Multiple: 72.

Multiples of 18: 36, 54, 72.

LCM 4, 30

Multiples of 4: 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, 52, 56, 60.
Least Common Multiple: 60.

Multiples of 30: 30, 60.

LCM 12, 30

Multiples of 12: 12, 24, 36, 48, 60
Least Common Multiple: 60.

Multiples of 30: 30, 60.

LCM 20, 50

Multiples of 20: 20, 40, 60, 80, 100.
Least Common Multiple: 100.

Multiples of 50: 50, 100.

Next, choose one of these problems that have not yet been solved. Solve it together on your student page. Then use your marker to copy your work neatly on this chart paper. Use your marker to cross out your choice so the next group solves a different problem.

a. **Hot dogs come packed 10 in a package. Hot dog buns come packed 8 in a package. If we want one hot dog for each bun for a picnic, with none left over, what is the least amount of each we need to buy?**

4 packages of hot dogs = 40 hot dogs. 5 packages of buns = 40 buns. LCM 8, 10 = 40.

b. **Starting at 6:00 a.m., A bus makes a stop at my street corner every 15 minutes. Also starting at 6:00 a.m., a taxi cab comes by every 12 minutes. What is the next time there will be a bus and a taxi at the corner at the same time?**

7:00 a.m., which is 60 minutes after 6:00 a.m. LCM 12, 15 = 60.

c. **Two gears in a machine are aligned by a mark drawn from the center of one gear to the center of the other. If the first gear has 24 teeth and the second gear has 40 teeth, how many revolutions of the first gear are needed until the marks line up again?**

5 revolutions. During this time 120 teeth will pass by. The second gear will revolve 3 times. LCM 24, 40 = 120.

d. **Is the LCM of a pair of numbers ever equal to one of the numbers? Explain with an example.**

Yes. Valid examples will consist of a pair of numbers where the lesser of the two numbers is a factor of the greater number; the greater of the two numbers is a multiple of the lesser number.

e. **Is the LCM of a pair of numbers ever less than both numbers? Explain with an example.**

No. Multiples are, by definition, equal to or greater than the number.

Exploratory Challenge 3: Using Prime Factors to Determine GCF (5 minutes)

Note: If the classroom has Internet access, a Factor Tree applet is available at:
http://nlvm.usu.edu/en/nav/frames_asid_202_g_3_t_1.html?from=category_g_3_t_1.html

Choose "Two Factor Trees" and "User Defined Problems". When both numbers are prime factored, each common prime factor is dragged into the middle of a two-circle Venn diagram. Unique prime factors are separated into the other two regions of the Venn diagram. Introducing the applet before the lesson and allowing exploration time will strengthen understanding and make this lesson more cohesive.

Groups will choose from the following number problems:

Use prime factors to find the greatest common factor of one of the following pairs of numbers:
$30, 50;\ 30, 45;\ 45, 60;\ 42, 70;\ 96, 144.$

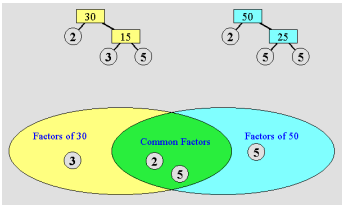
Next, groups will choose from the following application problems:

a. Would you rather find all the factors of a number or find all the prime factors of a number? Why?
b. Find the GCF of your original pair of numbers.
c. Is the product of your LCM and GCF less than, greater than, or equal to the product of your numbers?
d. Glenn's favorite number is very special because it reminds him of the day his daughter, Sarah, was born. The prime factors of this number do not repeat and are all the prime numbers less than 12. What is Glenn's number? When was Sarah born?

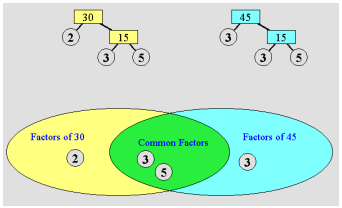
Exploratory Challenge 3: Using Prime Factors to Determine GCF

Choose one of these problems that have not yet been solved. Solve it together on your student page. Then use your marker to copy your work neatly on the chart paper. Use your marker to cross out your choice so the next group solves a different problem.

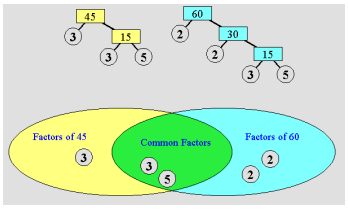
GCF 30, 50



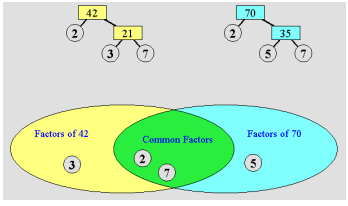
GCF 30, 45



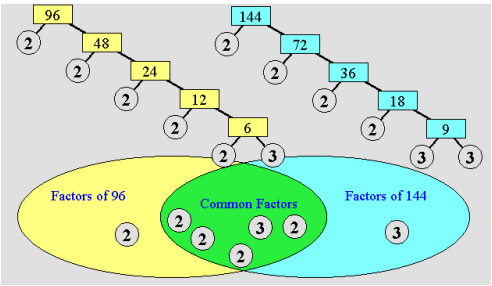
GCF 45, 60



GCF 42, 70



GCF 96, 144



Next, choose one of these problems that has not yet been solved:

a. **Would you rather find all the factors of a number or find all the prime factors of a number? Why?**

Accept opinions. Students should defend their answer, and use accurate mathematical terms in their response.

b. **Find the GCF of your original pair of numbers.**

See answers listed in Exploratory Challenge 1.

c. **Is the product of your LCM and GCF less than, greater than, or equal to the product of your numbers?**

In all cases, GCF $a, b \cdot$ LCM $a, b = a \cdot b$.

d. **Glenn's favorite number is very special because it reminds him of the day his daughter, Sarah, was born. The factors of this number do not repeat and are all the prime numbers less than 12. What is Glenn's number? When was Sarah born?**

$2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 = 2,310$ *Birthdate 2/3/10.*

Exploratory Challenge 4: Applying Factors to the Distributive Property (5 minutes)

MP.7

Study these examples of how factors apply to the Distributive Property:

$8 + 12 = 4(2) + 4(3) = 4(2 + 3) = 20$ $15 + 25 = 5(3) + 5(5) = 5(3 + 5) = 40$

Students will factor out the GCF from the two numbers and rewrite the sum using the Distributive Property.

Groups will choose from the following problems:

a. $12 + 18 =$
b. $42 + 14 =$
c. $36 + 27 =$
d. $16 + 72 =$
e. $44 + 33 =$

Next, students will add their own examples to one of two statements applying factors to the Distributive Property:

$n\,(a) + n\,(b) = n\,(a + b)$

$n\,(a) - n\,(b) = n\,(a - b)$

Exploratory Challenge 4: Applying Factors to the Distributive Property

Choose one of these problems that have not yet been solved. Solve it together on your student page. Then use your marker to copy your work neatly on the chart paper. Use your marker to cross out your choice so the next group solves a different problem.

Find the GCF from the two numbers and rewrite the sum using the Distributive Property

1. $12 + 18 =$

   $6(2) + 6(3) = 6(2 + 3) = 6(5) = 30$

2. $42 + 14 =$

   $7(6) + 7(2) = 7(6 + 2) = 7(8) = 56$

3. $36 + 27 =$

   $9(4) + 9(3) = 9(4 + 3) = 9(7) = 63$

4. $16 + 72 =$

   $8(2) + 8(9) = 8(2 + 9) = 8(11) = 88$

5. $44 + 33 =$

   $11(4) + 11(3) = 11(4 + 3) = 11(7) = 77$

Next, add another new example to one of these two statements applying factors to the Distributive Property.

Choose any numbers for n, a, and b.

$n\,(a) + n\,(b) = n\,(a + b)$

Accept all student responses that are mathematically correct.

$n\,(a) - n\,(b) = n\,(a - b)$

The Distributive Property holds for addition as well as subtraction. Accept all student responses that are mathematically correct.

Closing (6 minutes)

- Use this time to discuss each station. Assign the problem set which asks students to revisit each topic independently.

Exit Ticket (4 minutes)

Name _____ Date_____

Lesson 18: Least Common Multiple and Greatest Common Factor

Exit Ticket

1. Find the LCM and GCF of 12 and 15.

2. Write two numbers, neither of which is 8, whose GCF is 8.

3. Write two numbers, neither of which is 28, whose LCM is 28.

Rate each of the stations you visited today. Use this scale:

3 – Easy – I've got it; I don't need any help.

2 – Medium – I need more practice, but I'm understanding some of it.

1 – Hard – I'm not getting this yet.

Complete the following chart:

Station	Rating (3, 2, 1)	Comment to the Teacher
Station 1: Factors and GCF		
Station 2: Multiples and LCM		
Station 3: Using Prime Factors for GCF		
Station 4: Applying factors to the Distributive Property		

Exit Ticket Sample Solutions

1. Find the LCM and GCF of 12 and 15.

   LCM: 60; GCF: 3

2. Write two numbers, neither of which is 8, whose GCF is 8.

   Answers will vary, i.e., 16 and 24, or 24 and 32

3. Write two numbers, neither of which is 28, whose LCM is 28.

   Answers will vary, i.e., 4 and 14, or 4 and 7

Rate each of the stations you visited today. Use this scale:

3 – Easy – I've got it, I don't need any help.

2 – Medium – I need more practice, but I'm understanding some of it.

1 – Hard – I'm not getting this yet.

Complete the following chart:

Station	Rating (3, 2, 1)	Comment to the Teacher
Station 1 Factors and GCF		
Station 2 Multiples and LCM		
Station 3 Using Prime Factors for GCF		
Station 4 Applying factors to the Distributive Property		

Problem Set Sample Solutions

Students should complete the remaining exploratory challenge problems from class.

Exploratory Challenge Reproducible

Exploratory Challenge 1: Factors and GCF

Choose one of these problems that have not yet been solved. Solve it together on your student page. Then use your marker to copy your work neatly on the chart paper. Use your marker to cross out your choice so the next group solves a different problem.

Find the greatest common factor of one of these pairs: $30, 50$ $30, 45$ $45, 60$ $42, 70$ $96, 144$.

Next, choose one of these problems that have not yet been solved:

a. There are 18 girls and 24 boys who want to participate in a Trivia Challenge. If each team must have the same number of girls and boys, what is the greatest number of teams that can enter? How many boys and girls will be on each team?

b. The Ski Club members are preparing identical welcome kits for the new skiers. They have 60 hand warmer packets and 48 foot warmer packets. What is the greatest number of kits they can prepare using all of the hand warmer and foot warmer packets?

c. There are 435 representatives and 100 senators serving in the United States Congress. How many identical groups could be formed from all the senators and representatives?

d. Is the GCF of a pair of numbers ever equal to one of the numbers? Explain with an example.

e. Is the GCF of a pair of numbers ever greater than both numbers? Explain with an example.

Exploratory Challenge 2: Multiples and LCM

Choose one of these problems that have not yet been solved. Solve it together on your student page. Then use your marker to copy your work neatly on the chart paper. Use your marker to cross out your choice so the next group solves a different problem.

Find the least common multiple of one of these pairs: $9, 12$ $8, 18$ $4, 30$ $12, 30$ $20, 50$.

Next, choose one of these problems that have not yet been solved:

a. Hot dogs come packed 10 in a package. Hot dog buns come packed 8 in a package. If we want one hot dog for each bun for a picnic, with none left over, what is the least amount of each we need to buy?

b. Starting at 6:00 a.m., a bus makes a stop at my street corner every 15 minutes. Also starting at 6:00a.m., a taxi cab comes by every 12 minutes. What is the next time there will be a bus and a taxi at the corner at the same time?

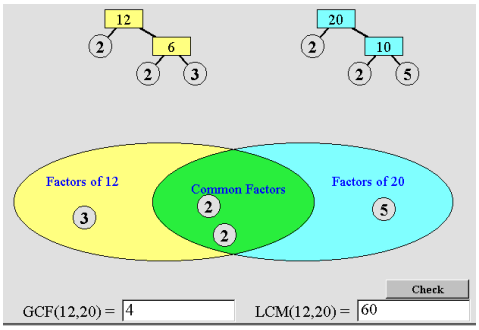
c. Two gears in a machine are aligned by a mark drawn from the center of one gear to the center of the other. If the first gear has 24 teeth and the second gear has 40 teeth, how many revolutions of the first gear are needed until the marks line up again?

d. Is the LCM of a pair of numbers ever equal to one of the numbers? Explain with an example.

e. Is the LCM of a pair of numbers ever less than both numbers? Explain with an example.

Solve it together on your student page. Then use your marker to copy your work neatly on this chart paper. Use your marker to cross out your choice so the next group solves a different problem.

Exploratory Challenge 3: Using Prime Factors to Determine GCF



Choose one of these problems that have not yet been solved. Solve it together on your student page. Then use your marker to copy your work neatly on the chart paper. Use your marker to cross out your choice so the next group solves a different problem.

Use Prime Factors to find the Greatest Common Factor of one of the following pairs of numbers:

$30, 50$ $30, 45$ $45, 60$ $42, 70$ $96, 144$

Next, choose one of these problems that have not yet been solved:

a. Would you rather find all the factors of a number or find all the prime factors of a number? Why?

b. Find the GCF of your original pair of numbers.

c. Is the product of your LCM and GCF less than, greater than, or equal to the product of your numbers?

d. Glenn's favorite number is very special because it reminds him of the day his daughter, Sarah, was born. The factors of this number do not repeat and are all the prime numbers less than 12. What is Glenn's number? When was Sarah born?

Exploratory Challenge 4: Applying Factors to the Distributive Property

Study these examples of how factors apply to the Distributive Property:

8 + 12 = 4(2) + 4(3) = 4(2+3) = 20 15 + 25 = 5(3) + 5(5) = 5(3+5) = 40 36 – 24 = 4 (9) – 4 (6) = 4 (9–6) = 12

4(2) + 4(3) = 4(5) = 20 5(3) + 5(5) = 5(8) = 40 4 (9) – 4 (6) = 4 (3) = 12

Choose one of these problems that have not yet been solved. Solve it together on your student page. Then use your marker to copy your work neatly on the chart paper. Use your marker to cross out your choice so the next group solves a different problem.

Find the GCF from the two numbers and rewrite the sum using the Distributive Property.

a. $12 + 18 =$

b. $42 + 14 =$

c. $36 + 27 =$

d. $16 + 72 =$

e. $44 + 33 =$

Next, add another new example to one of these two statements applying factors to the Distributive Property.

Choose any numbers for n, a, and b.

$n(a) + n(b) = n(a + b)$

$n(a) - n(b) = n(a - b)$