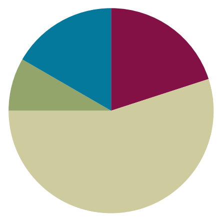
Lesson 15

Objective: Find common units or number of units to compare two fractions.

Suggested Lesson Structure

Fluency Practice	(12 minutes)
Application Problem	(5 minutes)
Concept Development	(33 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**



Fluency Practice (12 minutes)

- Count by Equivalent Fractions **4.NF.1** (4 minutes)
- Find Equivalent Fractions **4.NF.1** (4 minutes)
- Compare Fractions **4.NF.2** (4 minutes)

Count by Equivalent Fractions (4 minutes)

Materials: (S) Personal white boards

Note: This activity builds fluency with equivalent fractions. The progression builds in complexity. Work the students up to the highest level of complexity in which they can confidently participate.

$\frac{0}{4}$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{4}{4}$
$\frac{0}{4}$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	1 whole
0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1 whole

T: Count by ones to 4, starting at zero.

S: 0, 1, 2, 3, 4.

T: Count by fourths to 4 fourths. (Write as students count.)

S: $\frac{0}{4}, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}$.

T: (Point to $\frac{4}{4}$.) 4 fourths is the same as 1 of what unit?

S: 1 whole.

T: (Beneath $\frac{4}{4}$, write *1 whole*.) Count by fourths again. This time, when you come to 1 whole, say, "1 whole." Try not to look at the board.

S: $0, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}$, 1 whole.

T: (Point to $\frac{2}{4}$.) 2 fourths is the same as 1 of what unit?

S: $\frac{1}{2}$.

T: (Beneath $\frac{2}{4}$, write $\frac{1}{2}$.) Count by fourths again. This time, convert to halves and whole numbers. Try not to look at the board.

S: 0, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, 1 whole.

Direct students to count forward and backward from $\frac{1}{4}$ to 1 whole, occasionally changing directions.

Find Equivalent Fractions (4 minutes)

Materials: (S) Personal white boards

Note: This fluency activity reviews skills applied in G4–M5–Lesson 14.

T: (Write $\frac{1}{2} = \frac{—\times—}{—\times—} = \frac{2}{—}$. Point to $\frac{1}{2}$.) Say the unit fraction.

S: $\frac{1}{2}$.

T: On your boards, fill in the unknown numbers to make an equivalent fraction.

S: (Write $\frac{1}{2} = \frac{1 \times 2}{2 \times 2} = \frac{2}{4}$.)

Continue the process for the following possible suggestions: $\frac{1}{2} = \frac{4}{8}$, $\frac{1}{3} = \frac{2}{6}$, $\frac{1}{3} = \frac{3}{9}$, $\frac{1}{4} = \frac{4}{16}$, $\frac{1}{5} = \frac{3}{15}$.

NOTES ON MULTIPLE MEANS FOR ACTION AND EXPRESSION:

Fluency drills are fun, fast-paced math games, but do not leave English language learners behind. Make sure to clarify that *common unit, common denominator, like unit,* and *like denominator* are terms that refer to the same thing and are often used in math class interchangeably.

Compare Fractions (4 minutes)

Materials: (S) Personal white boards

Note: This fluency activity reviews G4–M5–Lesson 14.

T: (Write $\frac{1}{2}$ ___ $\frac{3}{4}$.) On your boards, find a common denominator, and write the greater than or less than sign.

S: (Write $\frac{1}{2}$ ___ $\frac{3}{4}$. Beneath it, write $\frac{2}{4} < \frac{3}{4}$.)

Continue the process with $\frac{1}{2}$ ___ $\frac{3}{8}$, $\frac{1}{4}$ ___ $\frac{3}{8}$, $\frac{5}{6}$ ___ $\frac{1}{3}$, $\frac{1}{4}$ ___ $\frac{5}{12}$, and $\frac{1}{3}$ ___ $\frac{2}{9}$.

Application Problem (5 minutes)

Jamal ran $\frac{2}{3}$ mile. Ming ran $\frac{2}{4}$ mile. Laina ran $\frac{7}{12}$ mile. Who ran the farthest? What do you think is the easiest way to determine the

Jamal ran the farthest.
It is easiest to form equivalent fractions since all 3 fractions have different denominators.

$\frac{2}{4} = \frac{2\times3}{4\times3} = \frac{6}{12}$

$\frac{2}{3} = \frac{2\times4}{3\times4} = \frac{8}{12}$

$\frac{7}{12}$

$\frac{6}{12} < \frac{7}{12} < \frac{8}{12}$

$\frac{2}{4} < \frac{7}{12} < \frac{2}{3}$

answer to this question? Talk with a partner about your ideas.

Note: This Application Problem reviews skills learned in G4–M5–Topic B to compare fractions and anticipates finding common units in this lesson. Be ready for conversations centered around comparing the fractions in other ways. Such conversations might include area models, tape diagrams, and finding equivalent fractions.

Concept Development (33 minutes)

Materials: (S) Personal white boards

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

When comparing fractions, we seek to make common units. We can model this by representing $\frac{3}{4}$ vertically, while representing $\frac{4}{5}$ horizontally. Then, we will decompose each model to make twentieths. Both models will then show common units of the same size and shape, even if the whole units are not drawn perfectly square.

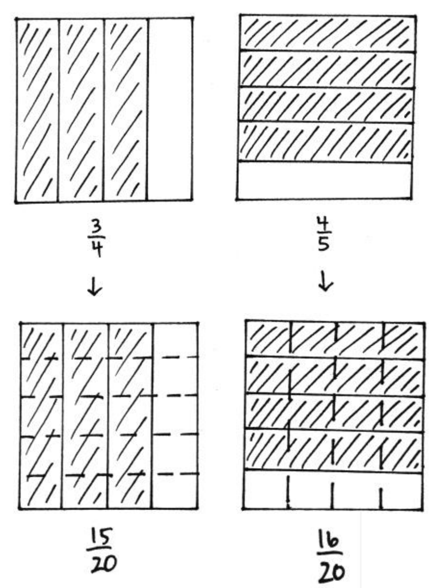
Problem 1: Compare two fractions with unrelated denominators using area models.

T: (Display $\frac{3}{4}$ and $\frac{4}{5}$.) We have compared fractions by using benchmarks to help us reason. Another way to compare fractions is to find like units.

T: Draw two almost square rectangles that are the same size. Each model is 1 whole. Partition one of the area models into fourths by drawing vertical lines. (Model.)

S: (Draw two almost square rectangles.)

T: Shade $\frac{3}{4}$ of one rectangle. Partition the other whole into fifths by drawing horizontal lines. Shade $\frac{4}{5}$. (Demonstrate.)



S: (Shade and draw lines.)

T: Do we have like denominators?

S: No.

T: Partition each fourth into 5 equal pieces. (Demonstrate.)

MP.2

T: How many units are in the whole now?

S: 20.

T: What is the value of one of the new units?

S: 1 twentieth.

T: How many twentieths are shaded?

S: 15.

T: Now, let's decompose $\frac{4}{5}$. Partition each fifth into 4 equal pieces. (Model the decomposition.) How many twentieths are the same as $\frac{4}{5}$?

$\frac{15}{20} < \frac{16}{20}$ so $\frac{3}{4} < \frac{4}{5}$

S: $\frac{16}{20}$ is the same as $\frac{4}{5}$.

T: Now that we have common units, can you compare the fractions?

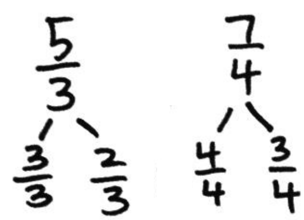
S: Yes! $\frac{15}{20}$ is less than $\frac{16}{20}$, so $\frac{3}{4}$ is less than $\frac{4}{5}$.

T: How did we decompose $\frac{4}{5}$ and $\frac{3}{4}$ to compare?

S: We made common units so that we would be able to compare the fractions. First, we drew area models to show each fraction. We partitioned one using vertical lines and the other using horizontal lines. Then, we partitioned each model again to create like units. Once we had like units, it was easy to compare the fractions. We compared $\frac{15}{20}$ and $\frac{16}{20}$. Then, we knew that $\frac{3}{4} < \frac{4}{5}$.

Repeat with $\frac{2}{3}$ and $\frac{3}{5}$, drawing thirds vertically and fifths horizontally. Then, partition the thirds into fifths and the fifths into thirds.

Problem 2: Compare two improper fractions with unrelated denominators using number bonds and area models.



T: (Display $\frac{5}{3}$ and $\frac{7}{4}$.) These fractions are greater than 1. Draw number bonds to show how $\frac{5}{3}$ and $\frac{7}{4}$ can be expressed as the sum of a whole number and a fraction.

S: $\frac{5}{3} = \frac{3}{3} + \frac{2}{3}$ and $\frac{7}{4} = \frac{4}{4} + \frac{3}{4}$.

T: Since the wholes are the same, we can just compare $\frac{2}{3}$ and $\frac{3}{4}$. Draw area models once again to help.

S: $\frac{2}{3}$ is less than $\frac{3}{4}$. → Since $\frac{2}{3}$ is less than $\frac{3}{4}$, $1\frac{2}{3}$ is less than $1\frac{3}{4}$. → $\frac{5}{3}$ is less than $\frac{7}{4}$.

Repeat with $\frac{6}{4}$ and $\frac{7}{5}$.

Problem 3: Compare two fractions with unrelated denominators without an area model.

T: We modeled common units to compare $\frac{4}{5}$ and $\frac{3}{4}$. What was the common unit?

S: Twentieths!

T: Use multiplication to show that $\frac{4}{5}$ is the same as $\frac{16}{20}$.

S: $\frac{4}{5} = \frac{4 \times 4}{5 \times 4} = \frac{16}{20}$.

T: Use multiplication to show that $\frac{3}{4}$ is the same as $\frac{15}{20}$.

S: $\frac{3}{4} = \frac{3 \times 5}{4 \times 5} = \frac{15}{20}$.

T: We decomposed by multiplying by the denominator of the other fraction.

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

It may help students represent fractions precisely to compare them if they are given a template of equally sized rectangles that can be partitioned as area models.

T: Let's compare $\frac{3}{5}$ and $\frac{8}{12}$ by multiplying the denominators. We could use area models, but that would be a lot of little boxes!

T: (Write $\frac{3}{5} = \frac{3 \times 12}{5 \times 12} = \frac{}{60}$.) How many sixtieths are the same as 3 fifths? Write your answer as a multiplication sentence.

S: $\frac{3}{5} = \frac{3 \times 12}{5 \times 12} = \frac{36}{60}$.

T: (Write $\frac{8}{12} = \frac{8 \times 5}{12 \times 5} = \frac{}{60}$.) How many sixtieths are the same as 8 twelfths? Write your answer as a multiplication sentence.

S: $\frac{8}{12} = \frac{8 \times 5}{12 \times 5} = \frac{40}{60}$.

T: Compare $\frac{3}{5}$ and $\frac{8}{12}$.

S: $\frac{36}{60} < \frac{40}{60}$, so $\frac{3}{5} < \frac{8}{12}$.

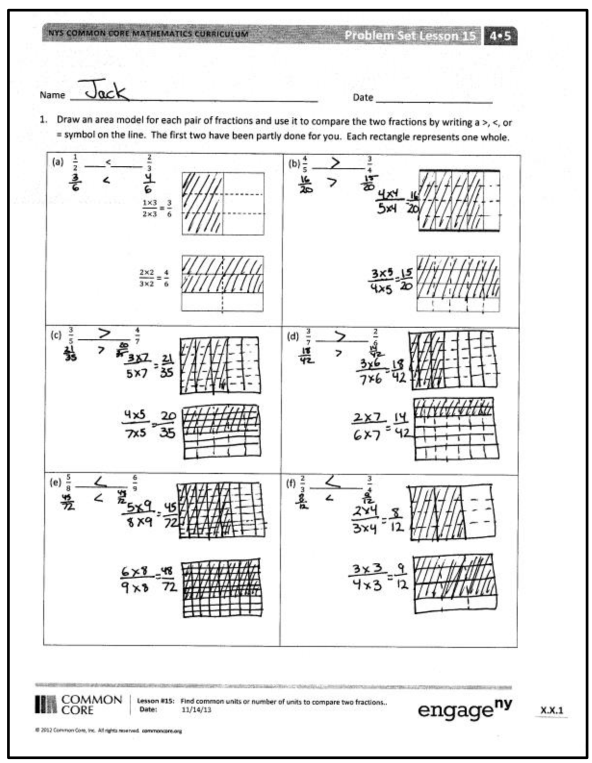
T: Write $\frac{9}{5}$ and $\frac{10}{8}$. Express each as an equivalent fraction using multiplication.

S: $\frac{9}{5} = \frac{9 \times 8}{5 \times 8} = \frac{72}{40}$.
$\frac{10}{8} = \frac{10 \times 5}{8 \times 5} = \frac{50}{40}$.

T: $\frac{72}{40} > \frac{50}{40}$. That means $\frac{9}{5} > \frac{10}{8}$.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students solve these problems using the RDW approach used for Application Problems.



Student Debrief (10 minutes)

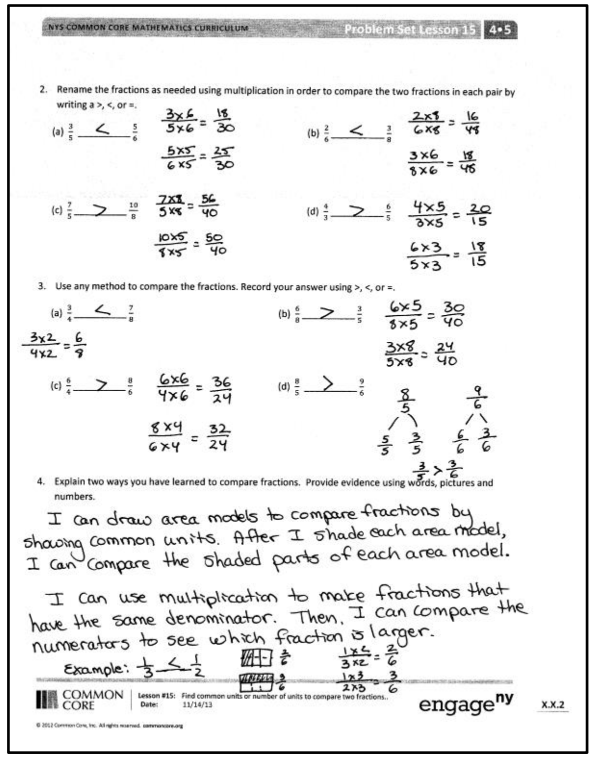
Lesson Objective: Find common units or number of units to compare two fractions.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

You may choose to use any combination of the questions below to lead the discussion.

- In Problem 2, did you need to use multiplication for every part? Why or why not? When is multiplication not needed even with different denominators?
- In Problem 2(b), did everyone use forty-eighths? Did anyone use twenty-fourths?
- In Problem 3, how did you compare the fractions? Why?
- Do we always need to multiply the denominators to make like units?
- If fractions are hard to compare, we can always get like units by multiplying denominators, a method that always works. Why is it sometimes not the best way to compare fractions?
- What new or significant math vocabulary did we use today to communicate precisely?
- How did the Application Problem connect to today's lesson?


NYS COMMON CORE MATHEMATICS CURRICULUM Problem Set Lesson 15 4•5

2. Rename the fractions as needed using multiplication in order to compare the two fractions in each pair by writing a >, <, or =.

(a) $\frac{3}{5}$ < $\frac{5}{6}$ $\frac{3\times6}{5\times6}=\frac{18}{30}$ $\frac{5\times5}{6\times5}=\frac{25}{30}$

(b) $\frac{2}{6}$ < $\frac{3}{8}$ $\frac{2\times8}{6\times8}=\frac{16}{48}$ $\frac{3\times6}{8\times6}=\frac{18}{48}$

(c) $\frac{7}{5}$ > $\frac{10}{8}$ $\frac{7\times8}{5\times8}=\frac{56}{40}$ $\frac{10\times5}{8\times5}=\frac{50}{40}$

(d) $\frac{4}{3}$ > $\frac{6}{5}$ $\frac{4\times5}{3\times5}=\frac{20}{15}$ $\frac{6\times3}{5\times3}=\frac{18}{15}$

3. Use any method to compare the fractions. Record your answer using >, <, or =.

(a) $\frac{3}{4}$ < $\frac{7}{8}$ $\frac{3\times2}{4\times2}=\frac{6}{8}$

(b) $\frac{6}{8}$ > $\frac{3}{5}$ $\frac{6\times5}{8\times5}=\frac{30}{40}$ $\frac{3\times8}{5\times8}=\frac{24}{40}$

(c) $\frac{6}{4}$ > $\frac{8}{6}$ $\frac{6\times6}{4\times6}=\frac{36}{24}$ $\frac{8\times4}{6\times4}=\frac{32}{24}$

(d) $\frac{8}{5}$ > $\frac{9}{6}$ $\frac{5}{5}$ $\frac{3}{5}$ $\frac{6}{6}$ $\frac{3}{6}$ $\frac{3}{5}>\frac{3}{6}$

4. Explain two ways you have learned to compare fractions. Provide evidence using words, pictures and numbers.

I can draw area models to compare fractions by showing common units. After I shade each area model, I can compare the shaded parts of each area model.

I can use multiplication to make fractions that have the same denominator. Then, I can compare the numerators to see which fraction is larger.

Example: $\frac{1}{3}$ < $\frac{1}{2}$ $\frac{2}{6}$ $\frac{3}{6}$ $\frac{1\times2}{3\times2}=\frac{2}{6}$ $\frac{1\times3}{2\times3}=\frac{3}{6}$

COMMON CORE Lesson #15: Find common units or number of units to compare two fractions. Date: 11/14/13 engage^ny X.X.2

Exit Ticket (3 minutes)

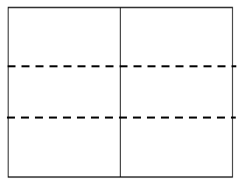
After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students' understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.

Name ____ Date ______________________

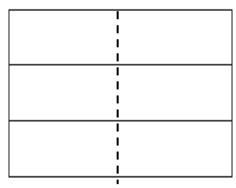
1. Draw an area model for each pair of fractions, and use it to compare the two fractions by writing a >, <, or = symbol on the line. The first two have been partly done for you. Each rectangle represents one whole.

a. $\frac{1}{2}$ _____<_______ $\frac{2}{3}$

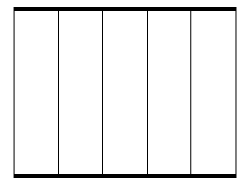
$\frac{1\times3}{2\times3}=\frac{3}{6}$



$\frac{2\times2}{3\times2}=\frac{4}{6}$



b. $\frac{4}{5}$ ______________ $\frac{3}{4}$



c. $\frac{3}{5}$ ______________ $\frac{4}{7}$

d. $\frac{3}{7}$ ______________ $\frac{2}{6}$

e. $\frac{5}{8}$ ______________ $\frac{6}{9}$

f. $\frac{2}{3}$ ______________ $\frac{3}{4}$

2. Rename the fractions, as needed, using multiplication in order to compare the two fractions in each pair by writing a >, <, or =.

a. $\frac{3}{5}$ ________ $\frac{5}{6}$

b. $\frac{2}{6}$ ________ $\frac{3}{8}$

c. $\frac{7}{5}$ ________ $\frac{10}{8}$

d. $\frac{4}{3}$ ________ $\frac{6}{5}$

3. Use any method to compare the fractions. Record your answer using >, <, or =.

a. $\frac{3}{4}$ ________ $\frac{7}{8}$

b. $\frac{6}{8}$ ________ $\frac{3}{5}$

c. $\frac{6}{4}$ ________ $\frac{8}{6}$

d. $\frac{8}{5}$ ________ $\frac{9}{6}$

4. Explain two ways you have learned to compare fractions. Provide evidence using words, pictures, and numbers.

Name ____ Date ______________________

1. Draw an area model for each pair of fractions, and use it to compare the two fractions by writing a >, <, or = symbol on the line.

   a. $\frac{3}{4}$ ________ $\frac{4}{5}$

   b. $\frac{2}{6}$ ________ $\frac{3}{5}$

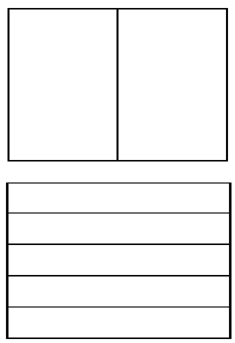
Name ______________________________ Date ________________

1. Draw an area model for each pair of fractions, and use it to compare the two fractions by writing a >, <, or = symbol on the line. The first two have been partly done for you. Each rectangle represents one whole.

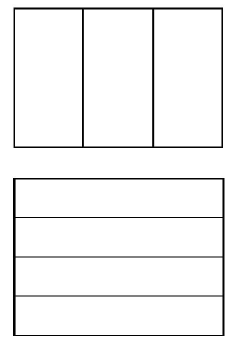
a. $\frac{1}{2}$ ____<______ $\frac{3}{5}$

$\frac{1\times5}{2\times5}=\frac{5}{10}$ $\frac{3\times2}{5\times2}=\frac{6}{10}$

$\frac{5}{10}<\frac{6}{10}$ so $\frac{1}{2}<\frac{3}{5}$



b. $\frac{2}{3}$ __________ $\frac{3}{4}$



c. $\frac{4}{6}$ __________ $\frac{5}{8}$

d. $\frac{2}{7}$ __________ $\frac{3}{5}$

e. $\frac{4}{6}$ __________ $\frac{6}{9}$

f. $\frac{4}{5}$ __________ $\frac{5}{6}$

2. Rename the fractions as needed using multiplication in order to compare the two fractions in each pair by writing a >, <, or =.

a. $\frac{2}{3}$ ____________ $\frac{2}{4}$

b. $\frac{4}{7}$ ____________ $\frac{1}{2}$

c. $\frac{5}{4}$ ____________ $\frac{9}{8}$

d. $\frac{8}{12}$ ____________ $\frac{5}{8}$

3. Use any method to compare the fractions. Record your answer using >, <, or =.

a. $\frac{8}{9}$ ____________ $\frac{2}{3}$

b. $\frac{4}{7}$ ____________ $\frac{4}{5}$

c. $\frac{3}{2}$ ____________ $\frac{9}{6}$

d. $\frac{11}{7}$ ____________ $\frac{5}{3}$

4. Explain which method you prefer to compare fractions. Provide an example using words, pictures, and numbers.