Lesson 2: Base 10 and Scientific Notation

Student Outcomes

- Students review place value and scientific notation.
- Students use scientific notation to compute with large numbers.

Lesson Notes

This lesson reviews how to express numbers using scientific notation. Students first learn about scientific notation in Grade 8 where they express numbers using scientific notation (**8.EE.A.3**) and compute with and compare numbers expressed using scientific notation (**8.EE.A.4**). Refer to Grade 8, Module 1, Topic B to review the approach to introducing scientific notation and its use in calculations with very large and very small numbers. This lesson sets the stage for the introduction of base 10 logarithms later in this module by focusing on the fact that every real number can be expressed as product of a number between 1 and 10 and a power of 10. In the paper folding activity in the last lesson, students worked with some very small and very large numbers. This lesson opens with these numbers to connect these lessons. Students also compute with numbers using scientific notation and discuss how the properties of exponents can simplify these computations (**N-RN.A.2**). In both the lesson and the problem set, students define appropriate quantities for the purpose of descriptive modeling (**N-Q.A.2**). The lesson includes a demonstration that reinforces the point that using scientific notation is convenient when working with very large or very small numbers and helps students gain some sense of the change in magnitude when we compare different powers of 10. This is an excellent time to watch the 9-minute classic film "Powers of 10" by Charles and Ray Eames, available at https://www.youtube.com/watch?v=0fKBhvDjuy0, which clearly illustrates the effect of adding another zero. The definition of scientific notation from Grade 8, Module 1, Lesson 9 is included after Example 1. You may want to allow students to use a calculator for this lesson.

Classwork

Opening (2 minutes)

In the last lesson, one of the examples gave the thickness of a sheet of gold foil as 0.28 millionths of a meter. The Exit Ticket had students calculate the size of a square sheet of paper that could be folded in half thirteen times, and it was very large. We will use these numbers in the Opening Exercise. Before students begin the Opening Exercise, briefly remind them of these numbers that they saw in the previous lesson, and tell them that this lesson will provide them with a way to conveniently represent very small or very large numbers. If you did not have an opportunity to share one of the news stories in Lesson 1, you could do that at this time as well.

Opening Exercise (5 minutes)

Students should work these exercises independently and then share their responses with a partner. Ask one or two students to explain their calculations to the rest of the class. Be sure to draw out the meaning of 0.28 millionths of a meter and how that can be expressed as a fraction. Check to make sure students know the place value for large numbers like billions. Students should be able to work the first exercise without a calculator, but on the second exercise, students should definitely use a calculator. If they do not have access to a calculator, give them the number squared and simply have them write the rounded value.

Opening Exercise

In the last lesson, you worked with the thickness of a sheet of gold foil (a very small number) and some very large numbers that gave the size of a piece of paper that actually could be folded in half more than 13 times.

a. **Convert 0.28 millionths of a meter to centimeters and express your answer as a decimal number.**

$$\frac{0.28}{1{,}000{,}000}\text{ m}\cdot\frac{100\text{ cm}}{1\text{ m}}=\frac{28}{1{,}000{,}000}\text{ cm}=0.000028\text{ cm}$$

b. **The length of a piece of square notebook paper that could be folded in half 13 times was 3294.2 in. Use this number to calculate the area of a square piece of paper that could be folded in half 14 times. Round your answer to the nearest million.**

$$(2\cdot 3294.2)^2=43407014.56$$

Rounded to the nearest million, the number is $43{,}000{,}000$.

c. **Sort the following numbers into products and single numeric expressions. Then match the equivalent expressions without using a calculator.**

3.5×10^5	-6	-6×10^0	0.6	3.5×10^{-6}
$3{,}500{,}000$	$350{,}000$	6×10^{-1}	0.0000035	3.5×10^6

Products: 3.5×10^5, -6×10^0, 3.5×10^{-6}, 6×10^{-1}, *and* 3.5×10^6

Single numeric expressions: -6, 0.6, 3500000, 350000, *and* 0.0000035

3.5×10^5 *is equal to* $350{,}000$.

-6×10^0 *is equal to* -6.

6×10^{-1} *is equal to* 0.6.

3.5×10^6 *is equal to* $3{,}500{,}000$.

3.5×10^{-6} *is equal to* 0.0000035.

As you review these solutions, point out that very large and very small numbers require us to include many digits to indicate the place value as shown in Exercises 1 and 2. Also, based on Exercise 3, it appears that you can use integer powers of 10 to express a number as a product.

Example 1 (7 minutes)

MP.3

Write the following statement on the board and ask students to consider whether or not they believe it is true. Have them discuss their thoughts with a partner, and then ask for volunteers to explain their thinking.

Students should explain that every nonzero decimal number can be expressed as the product of a number between 1 and 10 and a power of 10.

- Think of an example of a decimal number between 1 and 10. Write it down.
  - 2.5
- Think of a power of 10. Write it down.
  - 100 *or* 10^2
- What does the word product mean?
  - *It means the result of multiplying two numbers together.*
- Compute the product of the two numbers you wrote down.
  - $2.5 \cdot 10^2 = 2500$

Scaffolding:

- Have advanced learners write the number 245 as the product of a number and a power of 10 three different ways.
- To challenge advanced learners, have them make a convincing argument regarding the truth of a statement such as the following:

  Every decimal number can be expressed as the product of another decimal number and a power of 10.

First, have students share their answers with a partner. Then, put a few of the examples on the board. Finally, demonstrate how to reverse the process to express the following numbers as the product of a number between 1 and 10 and a power of 10. At this point, you can explain to students that when we write numbers in this fashion, we say that they are written using scientific notation. This is an especially convenient way to represent extremely large or extremely small numbers that otherwise would have many zero digits as place holders. Make sure to emphasize that this notation is simply rewriting a numerical expression in a different form, which helps us to quickly determine the size of the number. Students may need to be reminded of place value for numbers less than 1 and writing equivalent fractions whose denominators are powers of 10. The solutions demonstrate how any number can be expressed using what we call scientific notation.

Example 1

Write each number as a product of a decimal number between 1 and 10 and a power of 10.

a. $234,000$

$2.34 \cdot 100,000 = 2.34 \times 10^5$

b. 0.0035

$\frac{35}{10000} = \frac{3.5}{1000} = 3.5 \cdot \frac{1}{1000} = 3.5 \times 10^{-3}$

c. $532,100,000$

$5.321 \cdot 10,000,000 = 5.321 \times 10^8$

d. 0.0000000012

$\frac{12}{10,000,000,000} = \frac{1.2}{1,000,000,000} = 1.2 \cdot \frac{1}{1,000,000,000} = 1.2 \times 10^{-9}$

e. 3.331

$3.331 \cdot 1 = 3.331 \times 10^{0}$

Students may recall scientific notation from previous grades. Take time to review the definition of scientific notation provided below.

- Our knowledge of the integer powers of 10 enable us to understand the concept of scientific notation.
- Consider the estimated number of stars in the universe: 6×10^{22}. This is a 23-digit *whole number* with the **leading digit** (the leftmost digit) 6 followed by 22 zeros. When it is written in the form 6×10^{22}, it is said to be expressed in *scientific notation*.
- A positive, finite decimal[1] s is said to be written in **scientific notation** if it is expressed as a product $d \times 10^{n}$, where d is a finite decimal so that $1 \leq d < 10$, and n is an integer. That is, d is a finite decimal with only a single, nonzero digit to the left of the decimal point. The integer n is called the **order of magnitude**[2] of the decimal $d \times 10^{n}$.

A positive, finite decimal s is said to be written in scientific notation if it is expressed as a product $d \times 10^{n}$, where d is a finite decimal number so that $1 \leq d < 10$, and n is an integer.

The integer n is called the *order of magnitude* of the decimal $d \times 10^{n}$.

Exercises 1–6 (4 minutes)

Students should work these exercises independently as you monitor their progress. Encourage students to work quickly and begin generalizing a process for quickly writing numbers using scientific notation (such as counting the number of digits between the leading digit and the ones digit). After a few minutes, share the solutions with students so they can check their work.

Exercises 1–6

For Exercises 1–6, write each decimal in scientific notation.

1. $532,000,000$

   5.32×10^{8}

2. 0.0000000000000000123 (16 zeros after the decimal place)

   1.23×10^{-17}

3. $8,900,000,000,000,000$ (14 zeros after the 9)

   8.9×10^{15}

4. 0.00003382

   3.382×10^{-5}

[1] Recall that every whole number is a finite decimal.
[2] Sometimes the value of 10^{n} is known as the order of magnitude of the number $d \times 10^{n}$, but we will use n.

5. $34,000,000,000,000,000,000,000,000$ (24 zeros after the 4)
3.4×10^{25}

6. 0.0000000000000000000004 (21 zeros after the decimal place)
4×10^{-22}

To help students quickly write these problems using scientific notation, the number of zeros is written above for each problem. Be very careful that students are not using this number as the exponent on the base 10. Lead a discussion to clarify that difference for all students who make this careless mistake.

Exercises 7–8 (5 minutes)

After students practice writing numbers in scientific notation, you can really drive the point home that scientific notation is useful when working with very large or very small numbers by showing this demonstration: http://joshworth.com/dev/pixelspace/pixelspace_solarsystem.html, which illustrates just how far the planets in our solar system are from each other. After the demonstration, write down the distances between Earth and the Sun, between Jupiter and the Sun, and between Pluto and the Sun on the board, and have students work with a partner to answer Exercise 7. Be sure to mention that these distances are averages; he distances between the planets and the Sun are constantly changing as the planets complete their orbits. The average distance from the Sun to Earth is $151{,}268{,}468$ km. The average distance from the Sun to Jupiter is $780{,}179{,}470$ km. The average distance between the Sun and Pluto is $5{,}908{,}039{,}124$ km. In these exercises, students will round the distances to the nearest tenth to minimize all the writing and help them focus more readily on the magnitude of the numbers relative to one another.

Exercises 7–8

7. **Approximate the average distances between the Sun and Earth, Jupiter, and Pluto. Express your answers in scientific notation ($d \times 10^n$), where d is rounded to the nearest tenth.**

   a. **Sun to Earth:**

   1.5×10^8 km

   b. **Sun to Jupiter:**

   7.8×10^8 km

   c. **Sun to Pluto:**

   5.9×10^9 km

   d. **Earth to Jupiter:**

   $780,179,470 \text{ km} - 151,268,468 \text{ km} = 628,911,002 \text{ km}$

   6.3×10^8 km

   e. **Jupiter to Pluto:**

   $5,908,039,124 - 780,179,470 = 5,127,859,654$ km

   5.1×10^9 km

MP.7

8. **Order the numbers in Exercise 7 from smallest to largest. Explain how writing the numbers in scientific notation helps you to quickly compare and order them.**

The numbers from smallest to largest are 1.5×10^8, 6.3×10^8, 7.8×10^8, 5.1×10^9, and 5.9×10^9. The power of 10 helps to quickly sort the numbers by their order of magnitude, and then it is easy to quickly compare the numbers with the same order of magnitude because they are only written as a number between one and ten.

Example 2 (10 minutes): Arithmetic Operations With Numbers Written Using Scientific Notation

Model the solutions to the following example problems. Be sure to emphasize that final answers should be expressed using the scientific notation convention of a number between 1 and 10 and a power of 10. On part (a), you may need to provide some additional scaffolding if students are struggling to rewrite the numbers using the same order of magnitude. Have students practice writing a number as a product of a power of 10 in three different ways. For example, $15{,}000 = 1.5 \times 10^4 = 15 \times 10^3 = 150 \times 10^2$, etc. The lessons of Algebra I, Module 1, Topic B provide some suggestions and fluency exercises if students need additional practice on arithmetic operations with numbers in scientific notation. Be sure that students understand that the properties of exponents allow you to quickly perform the indicated operations.

Scaffolding:

- For Example 2, part (a), students will want to add the exponents as they do when multiplying numbers written using scientific notation. Take time to discuss the differences in the three expressions if you notice students making this type of mistake.

Example 2: Arithmetic Operations with Numbers Written Using Scientific Notation

a. $(2.4 \times 10^{20}) + (4.5 \times 10^{21})$

$(2.4 \times 10^{20}) + (45 \times 10^{20}) = 47.4 \times 10^{20} = 4.74 \times 10^{21}$

b. $(7 \times 10^{-9})(5 \times 10^{5})$

$(7 \cdot 5) \times (10^{-9} \cdot 10^{5}) = 35 \times 10^{-4} = 3.5 \times 10^{-3}$

c. $\dfrac{1.2 \times 10^{15}}{3 \times 10^{7}}$

$\dfrac{1.2}{3} \times 10^{15-7} = 0.4 \times 10^{8} = 4 \times 10^{7}$

Debrief with the questions designed to help students see that the order of magnitude and the properties of exponents greatly simplify calculations.

- How do the properties of exponents help to simplify these calculations?
- How can you quickly estimate the size of your answer?

Exercises 9–11 (5 minutes)

Exercises 9–11

9. Perform the following calculations without rewriting the numbers in decimal form.

a. $(1.42 \times 10^{15}) - (2 \times 10^{13})$

$142 \times 10^{13} - 2 \times 10^{13} = (142 - 2) \times 10^{13} = 140 \times 10^{13} = 1.4 \times 10^{15}$

b. $(1.42 \times 10^{15})(2.4 \times 10^{13})$

$(1.42 \cdot 2.4) \times (10^{15} \cdot 10^{13}) = 3.408 \times 10^{28}$

c. $\dfrac{1.42 \times 10^{-5}}{2 \times 10^{13}}$

$\dfrac{1.42 \times 10^{-5}}{2 \times 10^{13}} = 0.71 \times 10^{-5-13} = 0.71 \times 10^{-18} = 7.1 \times 10^{-19}$

10. Estimate how many times farther Jupiter is from the Sun than Earth is from the Sun. Estimate how many times farther Pluto is from the Sun than Earth is from the Sun.

Earth is approximately 1.5×10^8 km *from the Sun, and Jupiter is approximately* 7.8×10^8 km *from the Sun. Therefore, Jupiter is about* 5 *times as far from the Sun as Earth is from the sun. Pluto is approximately* 5.9×10^9 km *from the Sun. Therefore, Pluto is approximately* 40 *times as far from the Sun as Earth is, since*

$$\frac{59 \times 10^8}{1.5 \times 10^8} = 59 \div 1.5 \approx 40.$$

11. Estimate the distance between Earth and Jupiter and between Jupiter and Pluto.

The distance between Earth and Jupiter is approximately $(7.8 - 1.5) \times 10^8$ km, *which is equal to* 6.3×10^8 km. *The distance between Jupiter and Pluto is approximately* $(59 - 7.8) \times 10^8 = 51.2 \times 10^8 = 5.12 \times 10^9$ km.

Closing (3 minutes)

Have students discuss the following question with a partner and record the definition of scientific notation in their mathematics notebook. Debrief by asking a few students to share their responses with the entire class.

- List two advantages of writing numbers using scientific notation.
  - *You do not have to write as many zeros when working with very large or very small numbers, and you can quickly multiply and divide numbers using the properties of exponents.*

Exit Ticket (4 minutes)

Name ____ Date ______________

Lesson 2: Base 10 and Scientific Notation

Exit Ticket

1. A sheet of gold foil is 0.28 millionths of a meter thick. Write the thickness of a gold foil sheet measured in centimeters using scientific notation.

2. Without performing the calculation, estimate which expression is larger. Explain how you know.

$$(4 \times 10^{10})(2 \times 10^{5}) \qquad \text{and} \qquad \frac{4 \times 10^{12}}{2 \times 10^{-4}}$$

Exit Ticket Sample Solutions

1. **A sheet of gold foil is 0.28 millionths of a meter thick. Write the thickness of a gold foil sheet measured in centimeters using scientific notation.**

   The thickness is 0.28×10^{-6} m. In scientific notation, the thickness of a gold foil sheet is 2.8×10^{-7} m, which is 2.8×10^{-5} cm.

2. **Without performing the calculation, estimate which expression is larger. Explain how you know.**

   $$(4 \times 10^{10})(2 \times 10^{5}) \quad \text{and} \quad \frac{4 \times 10^{12}}{2 \times 10^{-4}}$$

   The order of magnitude on the first expression is 15, and the order of magnitude on the second expression is 16. The product and quotient of the number between 1 and 10 in each expression is a number between 1 and 10. Therefore, the second expression is larger than the first one.

Problem Set Sample Solutions

1. **Write the following numbers used in these statements in scientific notation. (Note: Some of these numbers have been rounded.)**

   a. **The density of helium is 0.0001785 grams per cubic centimeter.**

      1.785×10^{-4}

   b. **The boiling point of gold is $5200°\text{F}$.**

      5.2×10^{3}

   c. **The speed of light is $186,000$ miles per second.**

      1.86×10^{5}

   d. **One second is 0.000278 hours.**

      2.78×10^{-4}

   e. **The acceleration due to gravity on the Sun is $900\ \text{ft/s}^2$.**

      9×10^{2}

   f. **One cubic inch is 0.0000214 cubic yards.**

      2.14×10^{-5}

   g. **Earth's population in 2012 was $7,046,000,000$ people.**

      7.046×10^{9}

   h. **Earth's distance from the Sun is $93,000,000$ miles.**

      9.3×10^{7}

i. Earth's radius is $4,000$ miles.

4×10^3

j. The diameter of a water molecule is 0.000000028 cm.

2.8×10^{-8}

2. Write the following numbers in decimal form. (Note: Some of these numbers have been rounded.)

a. A light year is 9.46×10^{15} m.

$9,460,000,000,000,000$

b. Avogadro's number is $6.02 \times 10^{23}\ \text{mol}^{-1}$.

$602,000,000,000,000,000,000,000$

c. The universal gravitational constant is $.674 \times 10^{-11}\ \text{N}\left(\frac{\text{m}}{\text{kg}}\right)^2$.

0.00000000006674

d. Earth's age is 4.54×10^9 years.

$4,540,000,000$

e. Earth's mass is 5.97×10^{24} kg.

$5,970,000,000,000,000,000,000,000$

f. A foot is 1.9×10^{-4} miles.

0.00019

g. The population of China in 2014 was 1.354×10^9 people.

$1,354,000,000$

h. The density of oxygen is 1.429×10^{-4} grams per liter.

0.0001429

i. The width of a pixel on a smartphone is 7.8×10^{-2} mm.

0.078

j. The wavelength of light used in optic fibers is 1.55×10^{-6} m.

0.00000155

3. **State the necessary value of n that will make each statement true.**

a. $0.000027 = 2.7 \times 10^n$

-5

b. $-3.125 = -3.125 \times 10^n$

0

c. $7,540,000,000 = 7.54 \times 10^n$

9

d. $0.033 = 3.3 \times 10^n$

-2

e. $15 = 1.5 \times 10^n$

1

f. $26,000 \times 200 = 5.2 \times 10^n$

6

g. $3000 \times 0.0003 = 9 \times 10^n$

-1

h. $0.0004 \times 0.002 = 8 \times 10^n$

-7

i. $\frac{16000}{80} = 2 \times 10^n$

2

j. $\frac{500}{0.002} = 2.5 \times 10^n$

5

Perform the following calculations without rewriting the numbers in decimal form.

k. $(2.5 \times 10^4) + (3.7 \times 10^3)$

2.87×10^4

l. $(6.9 \times 10^{-3}) - (8.1 \times 10^{-3})$

-1.2×10^{-3}

m. $(6 \times 10^{11})(2.5 \times 10^{-5})$

1.5×10^{7}

n. $\dfrac{4.5 \times 10^{8}}{2 \times 10^{10}}$

2.25×10^{-2}

4. **The wavelength of visible light ranges from 650 nanometers to 850 nanometers, where $1\text{ nm} = 1 \times 10^{-7}\text{ cm}$. Express the wavelength range of visible light in centimeters.**

Convert 650 nm to centimeters: $(6.5 \times 10^{2})(1 \times 10^{-7}) = 6.5 \times 10^{-5}$

The wavelength of visible light in centimeters is 6.5×10^{-5} cm *to* 8.5×10^{-5} cm.

5. **In 1694, the Dutch scientist Antonie van Leeuwenhoek was one of the first scientists to see a red blood cell in a microscope. He approximated that a red blood cell was "$25,000$ times as small as a grain of sand." Assume a grain of sand is $\frac{1}{2}$ mm wide and a red blood cell is approximately 7 micrometers wide. One micrometer is 1×10^{-6} m. Support or refute Leeuwenhoek's claim. Use scientific notation in your calculations.**

Convert millimeters to meters: $(5 \times 10^{-1})(1 \times 10^{-3}) = 5 \times 10^{-4}$. *A medium size grain of sand measures* 5×10^{-4} m *across. Similarly, a red blood cell is approximately* 7×10^{-6} m *across. If you divide these numbers, you get*

$$\frac{5 \times 10^{-4}}{7 \times 10^{-6}} = 0.714 \times 10^{2} = 7.14 \times 10^{1}.$$

So, a red blood cell is 71.4 *times as small as a grain of sand. Leeuwenhoek's claim was off by approximately a factor of* 350.

6. **When the Mars Curiosity Rover entered the atmosphere of Mars on its descent in 2012, it was traveling roughly $13,200$ mph. On the surface of Mars, its speed averaged 0.00073 mph. How many times faster was the speed when it entered the atmosphere than its typical speed on the planet's surface? Use scientific notation in your calculations.**

$$\frac{1.32 \times 10^{4}}{7.3 \times 10^{-4}} = 0.18 \times 10^{8} = 1.8 \times 10^{7}$$

The speed when it entered the atmosphere is greater than its surface speed by an order of magnitude of 7.

7. **Earth's surface is approximately 70% water. There is no water on the surface of Mars, and its diameter is roughly half of Earth's diameter. Assume both planets are spherical. The surface area of a sphere is given by the formula $SA = 4\pi r^{2}$, where r is the radius of the sphere. Which has more land mass, Earth or Mars? Use scientific notation in your calculations.**

The surface area of Earth is $4\pi(4000)^{2} \approx 2 \times 10^{8}$ sq. mi., *and the surface area of Mars is* $4\pi(2000)^{2} \approx 5 \times 10^{7}$ sq. mi. *Thirty percent of Earth's surface area is approximately* 6×10^{7} sq. mi. *Earth has more land mass by approximately* 20%.

8. **There are approximately 25 trillion (2.5×10^{13}) red blood cells in the human body at any one time. A red blood cell is approximately 7×10^{-6} m wide. Imagine if you could line up all your red blood cells end to end. How long would the line of cells be? Use scientific notation in your calculations.**

$(2.5 \times 10^{13})(7 \times 10^{-6}) = 1.75 \times 10^{8}$ m. *That means the line of cells would be* 1.75×10^{5} km *long. One mile is equivalent to* 1.6 km, *so the line of blood cells measures* $\frac{1.75 \times 10^{5}}{1.6}$ km $\approx 109,375$ km, *which is almost halfway to the moon!*

9. Assume each person needs approximately $100\ \text{sq. ft}$. of living space. Now imagine that we are going to build a giant apartment building that will be 1 mile wide and 1 mile long to house all the people in the United States, estimated to be 313.9 million people in 2012. If each floor of the apartment building is $10\ \text{ft}$. high; how tall will the apartment building be?

We need $(3.139 \times 10^{8})(100) = 3.139 \times 10^{10}\ \text{ft}^2$ *of living space.* $1\ \text{mi}^2 = 5280^2\ \text{ft}^2 = 27878400\ \text{ft}^2$.

Next, divide the total number of square feet by the number of square feet per floor to get the number of needed floors.

$$\frac{3.139 \times 10^{10}}{2.78784 \times 10^{7}} \approx 1.126 \times 10^{3}$$

Multiplying the number of floors by $10\ \text{ft}$. *per floor gives a height of* $11{,}260\ \text{ft}$.*, which is approximately* $2.13\ \text{mi}$.