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# Examples of Functions from Geometry



[1] Each lesson is ONE day and ONE day is considered a 45-minute period.

## Grade 8 • Module 5

# Examples of Functions from Geometry

## OVERVIEW

In Topic A of Module 5, students learn the concept of a function and why functions are necessary for describing geometric concepts and occurrences in everyday life. The module begins by explaining the important role functions play in making predictions. For example, if an object is dropped, a function allows us to determine its height at a specific time. To this point, our work has relied on assumptions of constant rates; here, students are given data that shows that objects do not always travel at a constant speed. Once we explain the concept of a function, we then provide a formal definition of function. A function is defined as an assignment to each input, exactly one output (**8.F.A.1**). Students learn that the assignment of some functions can be described by a mathematical rule or formula. With the concept and definition firmly in place, students begin to work with functions in real-world contexts. For example, students relate constant speed and other proportional relationships (**8.EE.B.5**) to linear functions. Next, students consider functions of discrete and continuous rates and understand the difference between the two. For example, we ask students to explain why they can write a cost function for a book, but they cannot input 2.6 into the function and get an accurate cost as the output.

Students apply their knowledge of linear equations and their graphs from Module 4 (**8.EE.B.5**, **8.EE.B.6**) to graphs of linear functions. Students know that the definition of a graph of a function is the set of ordered pairs consisting of an input and the corresponding output (**8.F.A.1**). Students relate a function to an input-output machine: a number or piece of data, goes into the machine, known as the input, and a number or piece of data comes out of the machine, known as the output. In Module 4, students learned that a linear equation graphs as a line and that all lines are graphs of linear equations. In Module 5, students inspect the rate of change of linear functions and conclude that the rate of change is the slope of the graph of a line. They learn to interpret the equation $y = mx + b$ (**8.EE.B.6**) as defining a linear function whose graph is a line (**8.F.A.3**). Students will also gain some experience with non-linear functions, specifically by compiling and graphing a set of ordered pairs, and then by identifying the graph as something other than a straight line.

Once students understand the graph of a function, they begin comparing two functions represented in different ways (**8.EE.C.8**), similar to comparing proportional relationships in Module 4. For example, students are presented with the graph of a function and a table of values that represent a function, and are then asked to determine which function has the greater rate of change (**8.F.A.2**). Students are also presented with functions in the form of an algebraic equation or written description. In each case, students examine the average rate of change and know that the one with the greater rate of change must overtake the other at some point.

In Topic B, students apply their knowledge of volume from previous grade levels (**5.MD.C.3**, **5.MD.C.5**) to the learning of the volume formulas for cones, cylinders, and spheres (**8.G.C.9**). First, students are reminded of what they already know about volume, that volume is always a positive number that describes the hollowed out portion of a solid figure that can be filled with water. Next, students use what they learned about the area of circles (**7.G.B.4**) to determine the volume formulas of cones and cylinders. In each case, physical models will be used to explain the formulas, first with a cylinder seen as a stack of circular disks that provide the height of the cylinder. Students consider the total area of the disks in three dimensions understanding it as volume of a cylinder. Next, students make predictions about the volume of a cone that has the same dimensions as a cylinder. A demonstration shows students that the volume of a cone is one-third the volume of a cylinder with the same dimension, a fact that will be proved in Module 7. Next, students compare the volume of a sphere to its circumscribing cylinder (i.e., the cylinder of dimensions that touches the sphere at points, but does not cut off any part of it). Students learn that the formula for the volume of a sphere is two-thirds the volume of the cylinder that fits tightly around it. Students extend what they learned in Grade 7 (**7.G.B.6**) about how to solve real-world and mathematical problems related to volume from simple solids to include problems that require the formulas for cones, cylinders, and spheres.

# Focus Standards

### Define, evaluate, and compare functions.

**8.F.A.1** Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.[2]

**8.F.A.2** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*

**8.F.A.3** Interpret the equation $y = mx + b$ as defining a linear function whose graph is a straight line; give examples of functions that are not linear. *For example, the function* $A = s^2$ *giving the area of a square as a function of its side length is not linear because its graph contains the points* $(1, 1)$, $(2, 4)$ *and* $(3, 9)$ *which are not on a straight line.*

### Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

**8.G.C.9** Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

[2] Function notation is not required in Grade 8.

# Foundational Standard

### Geometric measurement: Understand concepts of volume and relate volume to multiplication and to addition.

**5.MD.C.3** Recognize volume as an attribute of solid figures and understand concepts of volume measurement.

a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.

b. A solid figure which can be packed without gaps or overlaps using $n$ unit cubes is said to have a volume of $n$ cubic units.

**5.MD.C.5** Relate volume to the operations of multiplication and addition, and solve real-world and mathematical problems involving volume.

a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.

b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole number edge lengths in the context of solving real world and mathematical problems.

c. Recognize volume as additive. Find volume of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to real world problems.

### Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

**7.G.B.4** Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

**7.G.B.6** Solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

### Understand the connections between proportional relationships, lines, and linear equations.

**8.EE.B.5** Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*

**8.EE.B.6** Use similar triangles to explain why the slope $m$ is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at $b$.

**Analyze and solve linear equations and pairs of simultaneous linear equations.**

**8.EE.C.7** Solve linear equations in one variable.

a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where $a$ and $b$ are different numbers).

b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

**8.EE.C.8** Analyze and solve pairs of simultaneous linear equations.

a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example,* $3x + 2y = 5$ *and* $3x + 2y = 6$ *have no solution because* $3x + 2y$ *cannot simultaneously be* $5$ *and* $6$.

c. Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

## Focus Standards for Mathematical Practice

**MP.2** **Reason abstractly or quantitatively**. Students examine, interpret, and represent functions symbolically. They make sense of quantities and their relationships in problem situations. For example, students make sense of values as they relate to the total cost of items purchased or a phone bill based on usage in a particular time interval. Students use what they know about rate of change to distinguish between linear and non-linear functions. Further, students contextualize information gained from the comparison of two functions.

**MP.6** **Attend to precision**. Students use notation related to functions in general, as well as volume formulas. Students are expected to clearly state the meaning of the symbols used in order to communicate effectively and precisely to others. Students attend to precision when they interpret data generated by functions. They know when claims are false; for example, calculating the height of an object after it falls for $-2$ seconds. Students also understand that a table of values is an incomplete *representation* of a continuous function, as an infinite number of values can be found for a function.

**MP.8** **Look for and express regularity in repeated reasoning**. Students will use repeated computations to determine equations from graphs or tables. While focused on the details of a specific pair of numbers related to the input and output of a function, students will maintain oversight of the process. As students develop equations from graphs or tables, they will evaluate the reasonableness of their equation as they ensure that the desired output is a function of the given input.

# Terminology

## New or Recently Introduced Terms

- **Function** (A *function* is a rule that assigns to each input exactly one output.)
- **Input** (The number or piece of data that is put into a function is the *input*.)
- **Output** (The number or piece of data that is the result of an input of a function is the *output*.)

## Familiar Terms and Symbols[3]

- Volume
- Area
- Solids
- Linear equation
- Non-linear equation
- Rate of change

# Suggested Tools and Representations

- 3D solids: cones, cylinders, and spheres.

# Assessment Summary

| Assessment Type | Administered | Format | Standards Addressed |
|---|---|---|---|
| End-of-Module Assessment Task | After Topic B | Constructed response with rubric | 8.F.A.1, 8.F.A.2, 8.F.A.3, 8.G.C.9 |

[3] These are terms and symbols students have seen previously.